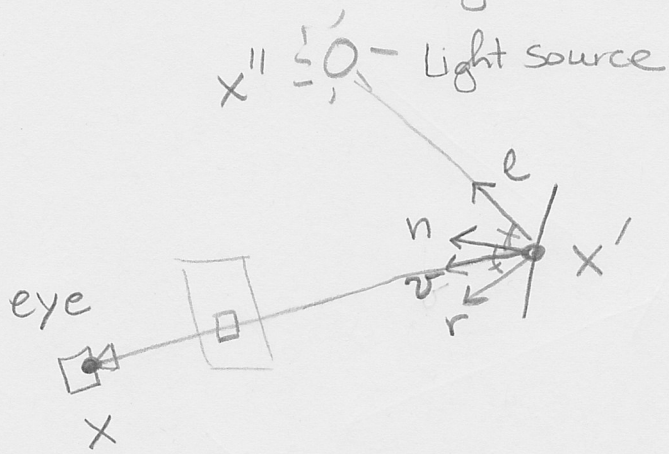
e $I(x', x'')$ = light reaching x' from x''

3



4 a. $g(x, x') = 1$ because x' is the first object struck by the ray

b. $\epsilon(x, x') = 0$ because we assume we have not struck an emitter

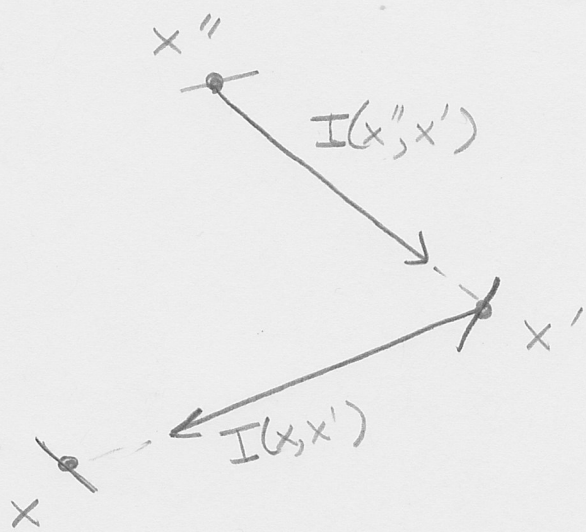
c. $\int_S$ becomes the $\sum_S$ where S are the point light sources

d. $\rho(x, x', x'') = K_d (n \cdot l) + K_s (r \cdot v)^{\alpha}$

e. $I(x', x'') = i_d = i_s$ as there is no mechanism in the rendering equation for i_d and i_s to be different

Homework 2 Solutions

1.



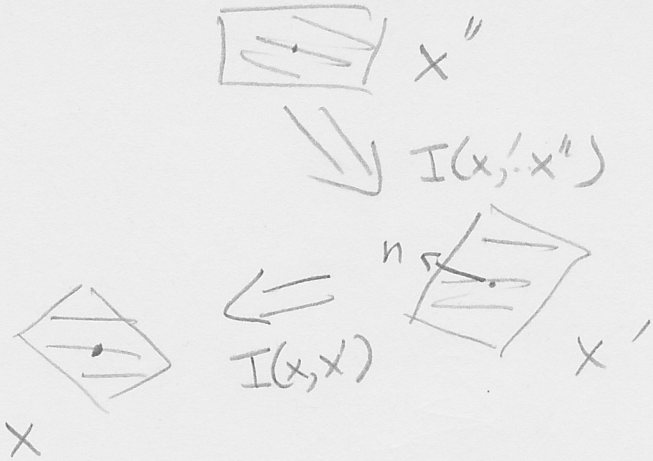
2. a. $g(x,x') = \begin{cases} 0 & \text{if light from } x' \text{ to } x \text{ is occluded} \\ 1 & \text{if light can reach } x \text{ from } x' \end{cases}$

b. $\epsilon(x,x')$ = light emitted from x' in the direction towards x

c. $\int_S$ = integral over points x'' on all surfaces S in the scene

d. $\rho(x,x',x'')$ = the fraction of light reaching x' from x'' that is reflected, refracted, or transmitted in the direction towards x

5.



surface patches

6. a $g(x, x')$ = the fraction of light from x' reaching x. It is related to, but not exactly, the Form Factor $F(x' \to x)$

b $\epsilon(x, x') = \epsilon(x')$ = light emitted by patch x'

Note that emitted light is the same in all directions

c $\int_S = \sum_S$ where S are all surface patches in the scene

d $\rho(x, x', x'') = K_d(x')\,(n \cdot (x'' - x'))$ for infinitely small patches

Otherwise, a more complex expression that is typically accommodated in the form factor

e. $I(x', x'')$ light intensity reaching x' from x''

7. We have, from the rendering equation:

Let $\Phi(x') = \epsilon(x, x') + \sum_S \rho(x, x', x'') I(x', x'')$

(x direction does not matter for ϵ)

$$\Phi(x') = \epsilon(x') + \sum_S \rho(x, x', x'') g(x', x'') \Phi(x'')$$

(replace I with rendering eq. RHS)

$$\Phi(x') = \epsilon(x') + \sum_S \rho(x') F(x'' \to x') \Phi(x'')$$

(old ρ and g equal simple ρ times form factor F)

$$\Phi(x') = \Phi_e(x') + \rho(x') \sum_{x''} \Phi(x'') F(x'' \to x')$$

($\epsilon(x')$ is $\Phi_e(x')$, pull $\rho(x')$ out of sum)

$$\Phi(x) = \Phi_e(x) + \rho(x) \sum_{x'} \Phi(x') F(x' \to x)$$

(switch variables)

TestNode(node)

8. Check bounding sphere of node for intersection

If intersection,

Run TestNode on children, testing closest to the eye first

Traversal order is: A C E D B

(assuming all intersection tests are positive & entire tree must be traversed)

---

9. Terminate when a leaf intersection is found and no untested bounding sphere on the frontier of the expansion contains points closer to the eye than the identified intersection.

10. TestNode(node)

Run TestNode on child closer to eye

Test geometry stored at node

Run TestNode on child farther from the eye

Traversal order is: E C D A B

(assuming entire tree must be traversed)

11. Terminate when an intersection is found and all remaining regions on the frontier of the expansion are farther from the eye than the intersection

12.

- Suppose we have an outdoor scene,
- Suppose we have an occluder that occludes half the light

- Take many photographs of the scene as we pass the occluder over it.
- Keep track of the min pixel color $L_{min}(p)$ & the Max pixel color $L_{Max}(p)$ at each point p
- The min light must be from global illumination only, & it is half the global component, due to our occluder

$$\boxed{L_g(p) = 2L_{min}(p)}$$

- The max light must include the direct component, & the direct component can be isolated by separating out the global component

$$\boxed{L_d(p) = L_{max}(p) - L_{min}(p)}$$