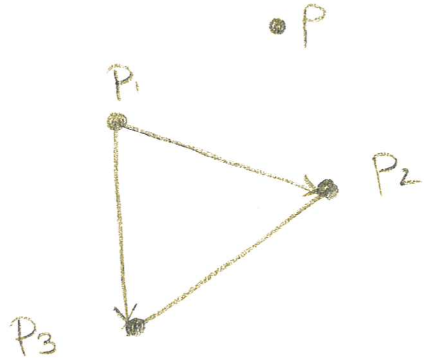
HW 1 Solutions

Part 1



1. $$\boxed{F(p) = (p - p_1) \bullet n = 0 \quad \text{where } n = (p_3 - p_1) \times (p_2 - p_1)}$$

2. $F(p_1) = (p_1 - p_1) \bullet n = 0$ ok

$F(p_2) = (p_2 - p_1) \bullet n = 0$ b/c n is orthogonal to $(p_2 - p_1)$

$F(p_3) = (p_3 - p_1) \bullet n = 0$ b/c n is orthogonal to $(p_3 - p_1)$

3. $$\boxed{P(s,t) = p_1 + s(p_2 - p_1) + t(p_3 - p_1)}$$

4. $\boxed{\begin{array}{l} \quad\ \ s \quad t \\ p_1 = (0, 0) \\ p_2 = (1, 0) \\ p_3 = (0, 1) \end{array}}$

5. $\boxed{\begin{array}{l} D(p) = (p - p_1) \cdot \hat{n} \\ \quad \text{where } \hat{n} = n/\|n\| \text{ and } n = (p_3 - p_1) \times (p_2 - p_1) \end{array}}$

6. $\boxed{p = \frac{1}{3} p_1 + \frac{1}{3} p_2 + \frac{1}{3} p_3}$

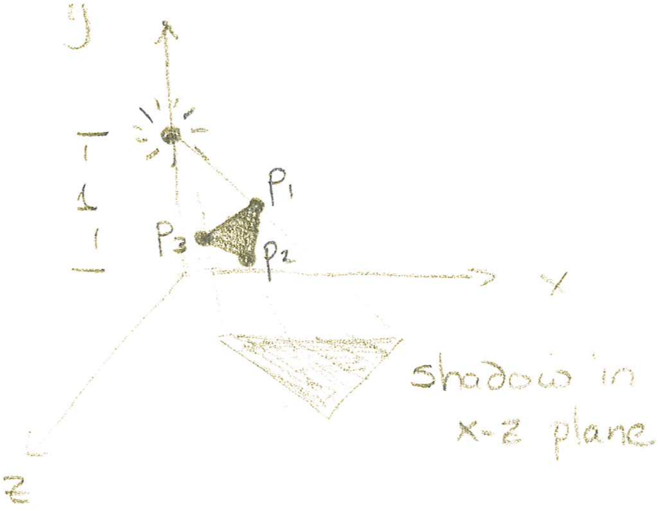
7. $P(s,t) = p_1 + s(p_2 - p_1) + t(p_3 - p_1)$

$= (1 - s - t) p_1 + s p_2 + t p_3 = \frac{1}{3} p_1 + \frac{1}{3} p_2 + \frac{1}{3} p_3$

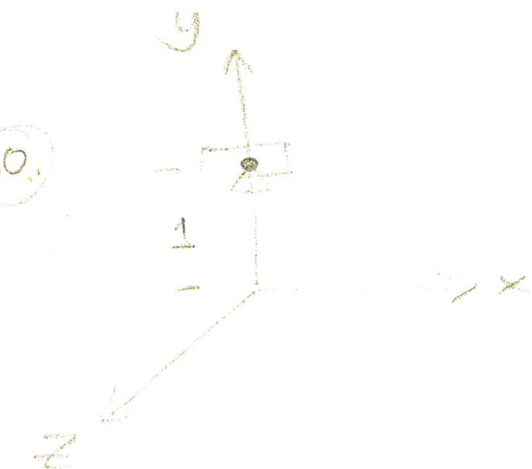
$\boxed{s = t = \frac{1}{3}}$

8. $P(s,t) = p_1 + \frac{1}{3}(p_2 - p_1) + \frac{1}{3}(p_3 - p_1) = \frac{1}{3} p_1 + \frac{1}{3} p_2 + \frac{1}{3} p_3$ ok ✓

Part 2



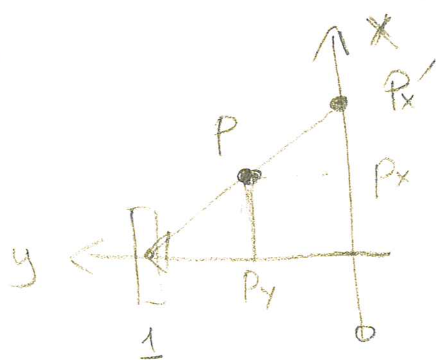
9. As at left



10. Place camera at y=1, looking down.

11. The image plane is at $\boxed{y=0}$ (the x-z plane)

12.



Similar triangles

$$\frac{P_x}{(1-P_y)} = \frac{P_x'}{1}$$

$$\boxed{P_x' = \frac{P_x}{(1-P_y)}}$$

13. Using exactly the same diagram as in 12., with z swapped for x,
$$\boxed{p_z' = \frac{p_z}{(1-p_y)}}$$

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14. Let $p = \begin{bmatrix} 1 \\ \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$ $\quad p_x' = \frac{p_x}{(1-p_y)} = \frac{1}{(1-\frac{1}{2})} = 2$ $\quad$ ok

Let $p = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad p_x' = p_x/(1-p_y) = 1/(1-0) = 1$ $\quad$ ok

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15. $$\boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}}$$
M

M takes $\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$ to $\begin{bmatrix} p_x \\ 0 \\ p_z \\ (1-p_y) \end{bmatrix} = \begin{bmatrix} p_x/(1-p_y) \\ 0 \\ p_z/(1-p_y) \\ 1 \end{bmatrix}$

ok

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16. Setup

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$
M

M takes p_y to $\frac{a p_y + b}{(1-p_y)}$

We want to map $p_y = 0 \rightarrow p_y' = 0$

$p_y = 0.5 \rightarrow p_y' = 0.5$

$$\frac{a(0)+b}{(1-0)} = 0 \quad \Rightarrow \quad b = 0$$

$$\frac{a(\frac{1}{2})+b}{(1-\frac{1}{2})} = \frac{1}{2} \quad \Rightarrow \quad a = \frac{1}{2}$$

$$\boxed{M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}}$$

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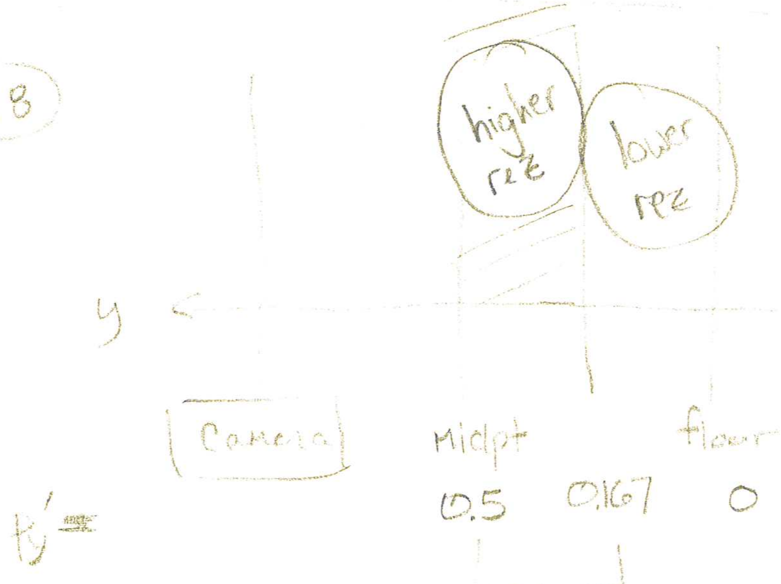
17. $$p_y' = \frac{\frac{1}{2}p_y}{(1-p_y)} = \frac{\frac{1}{2}(\frac{1}{4})}{(1-\frac{1}{4})} = \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{8}\cdot\frac{4}{3} = \boxed{\frac{1}{6}}$$

0.1666...

18.



greater range of depth values is available for the same amount of space

Part 3

19. $$B(u) = (1-u)\left[(1-u)P_0 + uP_1\right] + u\left[(1-u)P_1 + uP_2\right]$$

$$= (1-u)(1-u)P_0 + (1-u)uP_1 + u(1-u)P_1 + u^2P_2$$

$$= (1-2u+u^2)P_0 + (2u-2u^2)P_1 + u^2P_2$$

$$\boxed{\begin{bmatrix} u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}}$$

20. $$\boxed{\begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

21. $$\boxed{\begin{bmatrix} (1-2u+u^2) \\ (2u-2u^2) \\ u^2 \end{bmatrix}^T} \begin{array}{l} \leftarrow \text{amount of } P_0 \\ \leftarrow \text{amount of } P_1 \\ \leftarrow \text{amount of } P_2 \end{array}$$

22. $\boxed{\text{Yes. All points on the quadratic spline } B(u) \text{ for } 0 \le u \le 1 \text{ will be on or inside the convex hull of } P_0, P_1, \& P_2}$

23. First, show that blend functions sum to 1:

$$(1 - 2u + \cancel{u^2}) + (2u - \cancel{2u^2}) + \cancel{u^2} = 1 \quad \text{OK}$$

Second, show that blend functions are non-negative for $0 \le u \le 1$

$\boxed{f_1(u) = (1 - 2u + u^2)}$ $f_1(0) = 1$ is positive

zero crossings at $1 - 2u + u^2 = 0$ $u = \frac{2 \pm \sqrt{4-4}}{2} = 1$

The only zero crossing is at $u=1 \Rightarrow$

$\boxed{f_1(u) \text{ non-negative } 0 \le u \le 1}$ OK

$\boxed{f_2(u) = 2u - 2u^2}$ $f_2(\frac{1}{2}) = \frac{1}{2}$ is positive

zero crossings at $2u(1-u) = 0$ $u = 0, u = 1$

$\Rightarrow \boxed{f_2(u) \text{ is non-negative for } 0 \le u \le 1}$ OK

$\boxed{f_3(u) = u^2}$ $f_3(\frac{1}{2}) = \frac{1}{4}$ is positive

zero crossing at $u = 0 \Rightarrow$

$\boxed{f_3(u) \text{ is non-negative for } 0 \le u \le 1}$ OK

24. Cubic Bezier splines allow independent control of tangents at all interpolated points. Quadratic Bezier splines do not.

$$B'(u) = (-2+2u)P_0 + (2-4u)P_1 + 2uP_2$$

$$\left.\begin{aligned} B'(0) &= -2P_0 + \underline{2P_1} \\ B'(1) &= \underline{-2P_1} + 2P_2 \end{aligned}\right\}$$

In general, cannot find a P_1 to satisfy desired tangents at both ends.