Name: _____

Homework 1

15-462/15-662 Computer Graphics, Spring 2013

Due 2/26 at the beginning of class

120 points

Please show all of your work.
Even if you work things out in your head, write it down so we can see what you are doing.
Please list any references that you used to research or obtain your solution.

Part 1. You are given a triangle with points p_1, p_2, and p_3.

1. **(5 points)** Give an **implicit function** F(p) for the plane that contains this triangle.

2. **(5 points)** Test that $F(p_1) == F(p_2) == F(p_3) == 0$.

3. **(5 points)** Give a **parametric function** P(s,t) for the same plane.

4. **(5 points)** Give coordinates s and t for each of the three vertices of the triangle, using your parametric function P(s,t).

5. **(5 points)** If our implicit function is linear,we can scale it to compute distance from the plane. Adjust your implicit function F as needed to create distance function D(p) which returns the signed distance of point p from the given plane. In other words, for a point p at distance 2 from the plane in the positive normal direction, D(p) should equal 2.

6. **(5 points)** What is the point in triangle p_1, p_2, p_3 having **barycentric coordinates** (1/3, 1/3, 1/3)?

7. **(5 points)** What are parameters s and t for this point, given your parametric function P(s,t)?

8. **(5 points)** Plug your s and t parameters into your P(s,t) function to confirm your result is correct.

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Part 2. You wish to compute the shadow cast by triangle p_1, p_2, p_3 on the floor. The shadow is cast by a point light source located at [0,1,0]. The floor is the x-z plane, at y=0. The exact vertices of the triangle are not needed for this problem, but you can assume they are located between 0<y<1.

9. **(5 points)** Sketch the scene described above.

10. **(5 points)** You can compute the desired shadow by creating a projection of the triangle onto the ground plane from the point of view of the light source. In this scheme, where should your camera eye point be placed? It may help if you imagine the camera pointing downward.

11. **(5 points)** Where is your image plane?

12. **(5 points)** Give an expression for the x value of the projection of a vertex p onto the floor given your camera eye point. (Hint: use similar triangles.)

13. **(5 points)** Give an expression for the z value of the projection of a vertex p onto the floor.

14. **(5 points)** Explicitly test at least two sample points to show that your results in 12 and 13 are correct.

15. **(5 points)** Give a constant 4x4 projection matrix that results in correct projected x and z values and sets the y value of the projection to zero.

16. **(5 points)** Give a constant 4x4 projection matrix that also preserves some depth information. Specifiically, design your matrix to project a vertex at p_y=0.5 to p_y'=0.5 and to project p_y=0 to p_y'=0.

17. **(5 points)** Give the projected value p_y' of a point with p_y=0.25 using your matrix in 18.

18. **(5 points)** Is the depth information better (i.e., higher resolution) closer to the midpoint (e.g., closer to p_y=0.5) or closer to the floor? Explain your answer based on your results in 16 and 17.

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Part 3. A quadratic Bezier curve can be expressed as follows for parameter u ranging from 0 through 1:

$$B(u) = (1-u)\,[(1-u)P_0 + uP_1] + u\,[(1-u)P_1 + uP_2]$$

19. **(5 points)** Write this B(u) in matrix form as we saw in class.

20. **(5 points)** What is the Geometry matrix?

21. **(5 points)** What are the blend functions?

22. **(5 points)** Does it have the convex hull property? Give a "yes" or "no" answer and explain what this means.

23. **(5 points)** If you answered "yes" to 22, prove that it is true. All blend functions must be positive and sum to 1 for all values of u along the curve. If you answered "no" to 22, try to find a counterexample.

24. **(5 points)** The quadratic Bezier spline may seem very similar to the cubic Bezier spline. Why do you think that we use cubic instead of quadratic Bezier curves more commonly for graphics applications?