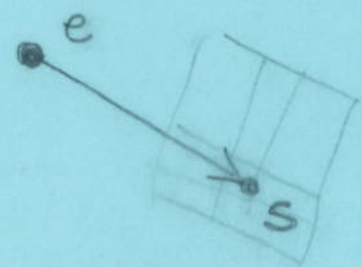
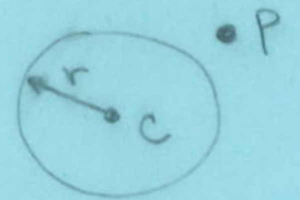
Ray sphere intersection



Ray $e + t(s-e)$

Parametric form



sphere – how to represent?

$$(p-c)\cdot(p-c) - r^2 = 0$$

Implicit form

Now, intersect the two $\Rightarrow$ find the points which fall on <u>both</u> the line and sphere.

Basic plan: Plug the ray <u>expression</u> into the sphere <u>equation</u> and solve for t.

Let's do it :

Let $p = e + t(s-e)$

Substituting into $(p-c)\cdot(p-c) - r^2 = 0$

we get:

$$(e+t(s-e)-c)\cdot(e+t(s-e)-c) - r^2 = 0$$

$$[(e-c)+t(s-e)] \cdot [(e-c)+t(s-e)] - r^2 = 0$$

$$(e-c)\cdot(e-c) + 2t(s-e)\cdot(e-c) + t^2(s-e)\cdot(s-e) - r^2 = 0$$

Note that $(e-c)\cdot(e-c) = \|(e-c)\|^2$

Let's put this into familiar form

$$At^2 + Bt + C = 0$$

Where we know the solutions

$$t = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

In our case,

$$A = \|s-e\|^2$$

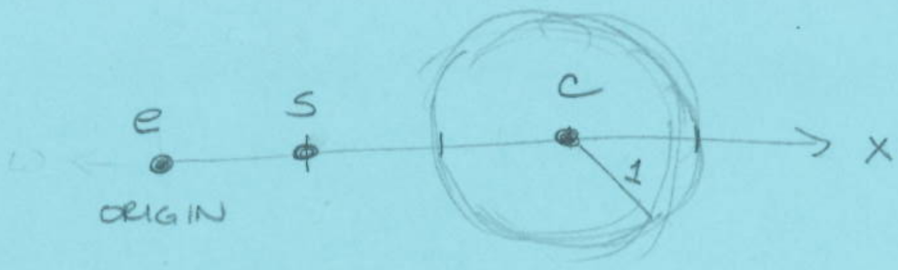
$$B = 2(s-e)\cdot(e-c)$$

$$C = \|e-c\|^2 - r^2$$

Plug in an solve for t.

Checking: ——

$$(s-e) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



$$(e-c) = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \|s-e\|^2 = 1$$

$$B = 2(s-e)\cdot(e-c) = 2(-3) = -6$$

$$C = \|e-c\|^2 - r^2 = 8$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{+6 \pm \sqrt{36 - 4(8)}}{2} = +3 \pm 1$$

$$t_1 = +2 \qquad t_2 = 4$$

$$p = e + t(s-e)$$

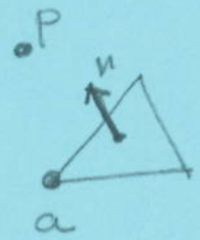
$$p_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

GOOD!

$$p_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Ray - triangle intersection

Ray $e + t(s-e)$



Triangle $(p-a)\cdot n = 0$

Let $p = e + t(s-e)$

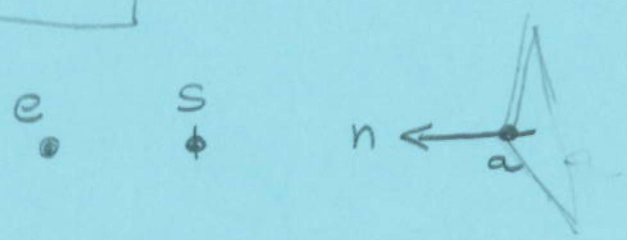
$$(e + t(s-e) - a)\cdot n = 0$$

$$(e-a)\cdot n + t\,(s-e)\cdot n = 0$$

$$t = \frac{-(e-a)\cdot n}{(s-e)\cdot n}$$

$$\boxed{p = e - \frac{(e-a)\cdot n}{(s-e)\cdot n}\,(s-e)}$$

Checking



$$p = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \frac{3}{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \quad \text{ok!}$$

Inside or outside?
Barycentric coords
See Shirley