Image Processing
Overview

Images

Pixel Filters

Neighborhood Filters

Dithering
Image as a Function

- We can think of an **image** as a function, \( f \),
- \( f : \mathbb{R}^2 \to \mathbb{R} \)
  - \( f(x, y) \) gives the **intensity** at position \((x, y)\)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - \( f : [a,b] \times [c,d] \to [0,1] \)

- A color image is just three functions pasted together. We can write this as a “vector-valued” function:
  \[
  f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}
  \]
Image as a Function
Image Processing

• Define a new image $g$ in terms of an existing image $f$
  – We can transform either the domain or the range of $f$

• Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?
Some operations preserve the range but change the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

Still other operations operate on both the domain and the range of $f$. 

Image Processing
Point Operations
Point Processing

Original

Darken

Lower Contrast

Nonlinear Lower Contrast

Invert

Lighten

Raise Contrast

Nonlinear Raise Contrast
Point Processing

Original  Darken  Lower Contrast  Nonlinear Lower Contrast

\[ x \]
\[ x - 128 \]
\[ x \div 2 \]
\[ ((x \div 255.0)^{0.33}) \times 255.0 \]

Invert  Lighten  Raise Contrast  Nonlinear Raise Contrast

\[ 255 - x \]
\[ x + 128 \]
\[ x \times 2 \]
\[ ((x \div 255.0)^2) \times 255.0 \]
Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function. Send $v \rightarrow$ actually display a pixel which has intensity equal to $v^{2.5}$

$\varphi = 1.0; f(v) = v$

$\varphi = 2.5; f(v) = v^{1/2.5} = v^{0.4}$
Neighborhood Operations
Convolution

\[
F = \begin{bmatrix}
0.2 & 0.1 & -1.0 \\
0.3 & 0.0 & 0.9 \\
0.1 & 0.3 & -1.0
\end{bmatrix}
\]

\[I' = F \ast I\]
Properties of Convolution

- Commutative
  \[ a \ast b = b \ast a \]

- Associative
  \[ (a \ast b) \ast c = a \ast (b \ast c) \]

- Cascade system
  \[ f \rightarrow h_1 \rightarrow h_2 \rightarrow g \]

  \[ = f \rightarrow h_1 \ast h_2 \rightarrow g \]

  \[ = f \rightarrow h_2 \ast h_1 \rightarrow g \]
Convolution

Convolution is linear and shift invariant

\[ g \ast f = \int_{-\infty}^{\infty} f \ast h \ast (-\tau) \; d\tau \quad g = f \ast h \]

kernel \( h \)
Convolution - Example

\[ c = a \ast b \]
Point Spread Function

scene \rightarrow \text{Optical System} \rightarrow \text{image}

- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.

$\delta \xrightarrow{\text{point source}} \text{Optical System} \xrightarrow{PSF} \text{point spread function}$

- Point spread function of Human Eyes
Point Spread Function

normal vision  myopia  hyperopia

astigmatism

Images by Richmond Eye Associates
Original Image
Blurred Image
Gaussian Smoothing

by Charles Allen Gillbert

by Harmon & Julesz

http://www.michaelbach.de/ot/cog_blureffects
Gaussian Smoothing

http://www.michaelbach.de/ot/cog_blureffects
Sharpened Image
Noise
Blurred Noise
**Median Filter**

- **Smoothing is averaging**
  (a) Blurs edges
  (b) Sensitive to outliers

- **Median filtering**
  - Sort $N^2 - 1$ values around the pixel
  - Select middle value (median)
  - Non-linear (Cannot be implemented with convolution)
Can this be described as a convolution?
Original Image
Example: Noise Reduction

Image with noise

Median filter (5x5)
Salt and pepper noise

Gaussian  Median

Gaussian  Median

3x3

5x5

7x7
Example: Noise Reduction

Original image

Image with noise

Median filter (5x5)
X-Edge Detection
Y-Edge Detection
General Edge Detection

Can this be described as a convolution?
Some operations preserve the range but change the domain of $f$:

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What kinds of operations can this perform?

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Image Processing
Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?
Image Sub-Sampling

Throw away every other row and column to create a $1/2$ size image - called *image sub-sampling*
Image Sub-Sampling

1/2

1/4 (2x zoom)

1/8 (4x zoom)
Good and Bad Sampling

Good sampling:
• Sample often or,
• Sample wisely

Bad sampling:
• see aliasing in action!
Aliasing
Alias: n., an assumed name

Picket fence receding into the distance will produce aliasing...

WHY?

Input signal:

Matlab output:

\[ x = 0:.05:5; \ \text{imagesc}(\sin((2^x) \cdot x)) \]

Alias!

Not enough samples
Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Sub-Sampling with Gaussian Pre-Filtering

- Solution: filter the image, *then* subsample
  - Filter size should double for each $\frac{1}{2}$ size reduction. Why?
Sub-Sampling with Gaussian Pre-Filtering

Gaussian 1/2

G 1/4

G 1/8
Compare with...

1/2

1/4 (2x zoom)

1/8 (4x zoom)
Canon D60 (w/ anti-alias filter)

Sigma SD9 (w/o anti-alias filter)

From Rick Matthews website, images by Dave Etchells
Warped Image
Warped Image

orig + vector field = warped

how?
Advection (just like a fluid)