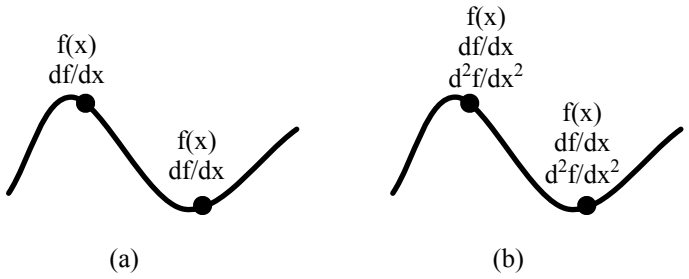
Homework 2
15-462: Computer Graphics

1. Suppose you wanted to build a spline curve out of polynomial segments.

   (a) Your requirement is that each curve pass through its control end points and have pre-specified derivatives at those endpoints. What degree polynomial is required for your spline segments? (Hint: These are nothing but Hermite Splines.) Assuming adjacent splines agree in position and derivative, what degree of continuity will the whole curve have?

   (b) Now suppose the curve must also respect pre-specified *second* derivatives at it's endpoints, and that these second derivatives will also agree across adjacent splines? What degree polynomial must the spline segments be? What degree of continuity will the whole curve have?



2. Suppose you have two 3D shapes, A and B, described by implicit functions f_A and f_B (respecting the inside/outside convention), i.e.:

$$\begin{aligned} A &= \left\{ \mathbf{x} \in \mathbf{R}^3 \;\middle|\; f_A(\mathbf{x}) \leq 0 \right\} \\ B &= \left\{ \mathbf{x} \in \mathbf{R}^3 \;\middle|\; f_B(\mathbf{x}) \leq 0 \right\}. \end{aligned}$$

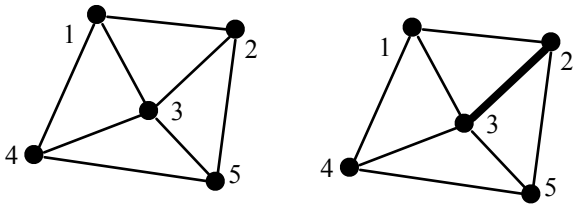
   Please give an implicit function for their union $f_{A \cup B}$, another for their intersection $f_{A \cap B}$ (hint: we talked about these in class), one for A's complement $f_{\bar{A}}$, and one for the set difference f_{A-B}. For example, for the last implicit function, we want:

$$A - B = \left\{ \mathbf{x} \in \mathbf{R}^3 \;\middle|\; f_{A-B}(\mathbf{x}) \leq 0 \right\}.$$

3. Laplacian Mesh Editing:

   (a) Please write down the 5×5 Laplacian matrix for the mesh on the left.

   (b) Suppose that we wanted to increase the "stiffness" of the edge between vertices 2 and 3 by a factor of 2. How would we modify the algorithm to do this? Please write down the 5×5 Laplacian matrix for this new system.



(a) (b)