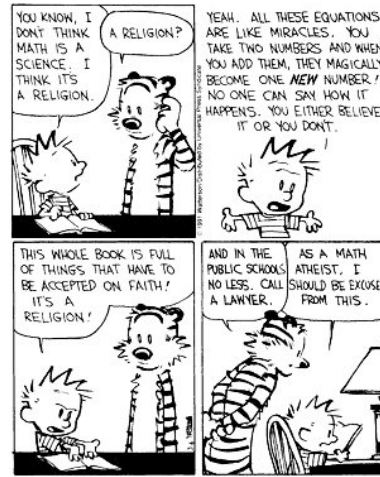


# Homework 1

## 15-642: Computer Graphics

### Due: February 4th, 2010



<http://econstudentlog.files.wordpress.com/2009/06/calvin-and-hobbes.jpg>

1. For what values of  $\alpha$  is  $\mathbf{a}$  orthogonal to  $\mathbf{b} - \alpha\mathbf{a}$ ? What about the special case where  $\|\mathbf{a}\| = 1$ ? The case where  $\|\mathbf{a}\| = 0$ ?
2. Are these a pure rotation matrices? Why or why not?

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & -\sqrt{2} & 0 \end{bmatrix} \quad \frac{1}{3\sqrt{2}} \begin{bmatrix} 2 & 3 & -4 \\ 4 & 0 & 4 \\ 2 & -3 & -4 \end{bmatrix}$$

3. Please sketch the 2D parametric curve for:

$$x(t) = \begin{cases} \frac{1}{5} & \text{if } 0 \leq t < \frac{1}{5} \\ \frac{2}{5} - t & \text{if } \frac{1}{5} \leq t < \frac{3}{5} \\ -\frac{1}{5} & \text{if } \frac{3}{5} \leq t < \frac{4}{5} \\ -\frac{9}{5} + 2t & \text{if } \frac{4}{5} \leq t < 1 \end{cases}$$

$$y(t) = \begin{cases} t & \text{if } 0 \leq t < \frac{2}{5} \\ \frac{4}{5} - t & \text{if } \frac{2}{5} \leq t < \frac{4}{5} \\ 0 & \text{if } \frac{4}{5} \leq t < 1 \end{cases}$$

Also please label the 2D coordinates of the endpoints. What shape does this form? What is the normal at  $t = \frac{1}{10}$ ? How about at  $t = \frac{7}{10}$ ?

4. Please sketch the 2D implicit curve for the function  $f : \mathbf{R}^2 \rightarrow \mathbf{R}$  where:

$$f(x, y) = |x| + |y| - 1$$

Please label 2D coordinates as appropriate. What is the normal at  $[\frac{1}{2}, \frac{1}{2}]$ ? Does this function satisfy the inside/outside convention for implicit surfaces? Why?

5. Given parallel lines  $\mathbf{f}_1(t) = [x_1, y_1, z_1]^T + t[u, v, w]^T$  and  $\mathbf{f}_2(t) = [x_2, y_2, z_2]^T + t[u, v, w]^T$ , what is their common 2D vanishing point under the perspective projection  $p(x, y, z) = [\frac{x}{z}, \frac{y}{z}]^T$ ?
6. Given a continuous parametric function  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^3$  specifying a 2D surface in 3D space, define a continuous implicit function  $g : \mathbf{R}^3 \rightarrow \mathbf{R}$  corresponding to the same surface. *Notes:*

- You do not need to prove the continuity of your function. (But you can for extra credit!)
- You'll likely want to use the *infimum* function.<sup>1</sup>
- You can ignore the inside/outside convention:  $g$  can be everywhere nonnegative.

<sup>1</sup><http://en.wikipedia.org/wiki/Infimum>