

Image Processing



Overview



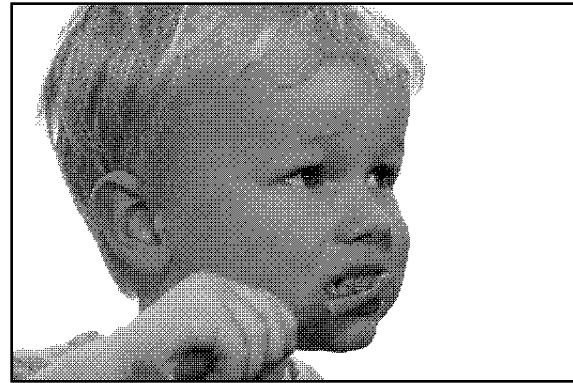
Images



Pixel Filters



Neighborhood Filters



Dithering

Image as a Function

- We can think of an **image** as a function, f ,
- $f: \mathbb{R}^2 \rightarrow \mathbb{R}$
 - $f(x, y)$ gives the **intensity** at position (x, y)
 - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
 - $f: [a,b] \times [c,d] \rightarrow [0,1]$
- A color image is just three functions pasted together. We can write this as a “vector-valued” function:
$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Image as a Function

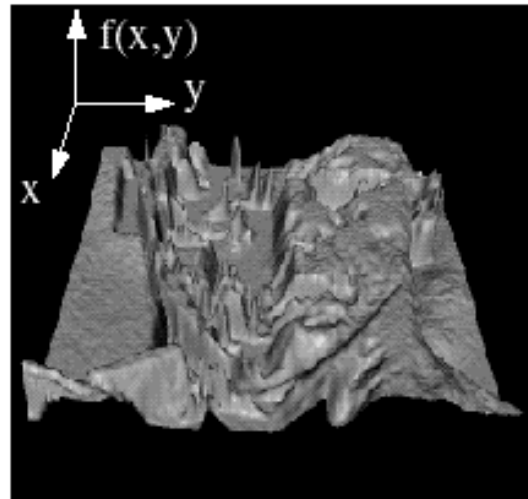
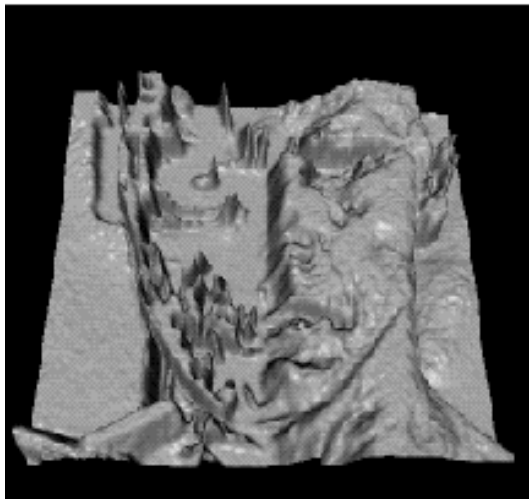
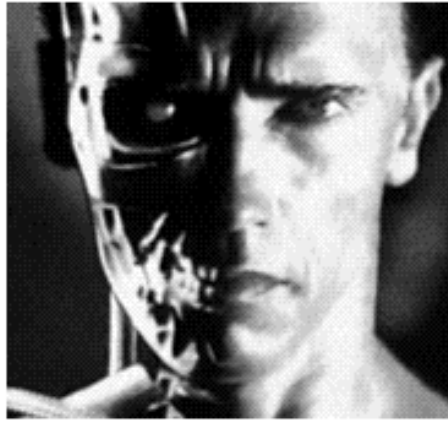


Image Processing

- Define a new image g in terms of an existing image f
 - We can transform either the domain or the range of f
- Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?

Image Processing

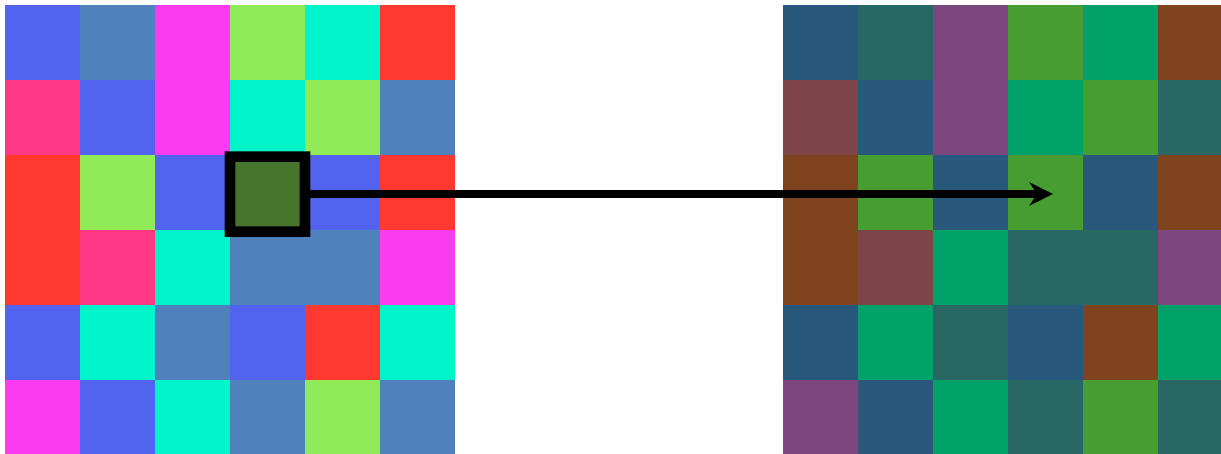
- Some operations preserve the range but change the domain of f :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

- Still other operations operate on both the domain and the range of f .

Point Operations



Point Processing

Original



Darken



Lower Contrast



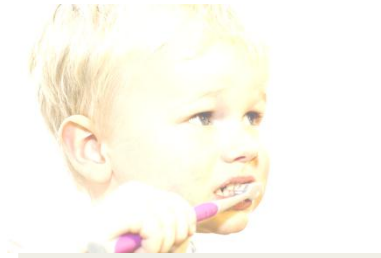
Nonlinear Lower Contrast



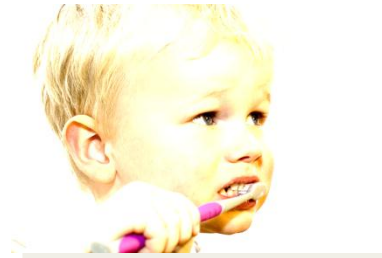
Invert



Lighten



Raise Contrast



Nonlinear Raise Contrast



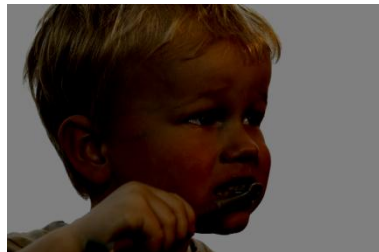
Point Processing

Original



$$x$$

Darken



$$x - 128$$

Lower Contrast



$$x / 2$$

Nonlinear Lower Contrast



$$((x / 255.0) ^ 0.33) * 255.0$$

Invert



$$255 - x$$

Lighten



$$x + 128$$

Raise Contrast



$$x * 2$$

Nonlinear Raise Contrast

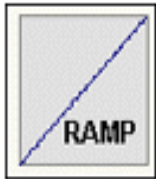


$$((x / 255.0) ^ 2) * 255.0$$

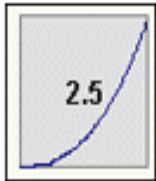
Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function

Send $v \rightarrow$ actually display a pixel which has intensity equal to $v^{2.5}$



Graph of Input

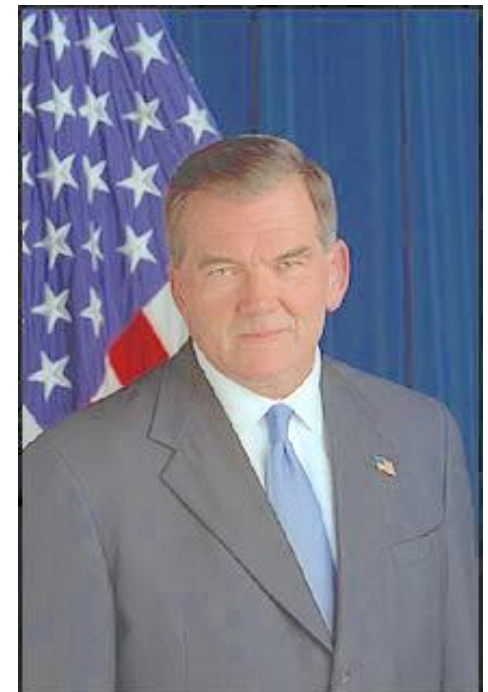


Graph of Output $L = V ^ 2.5$



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

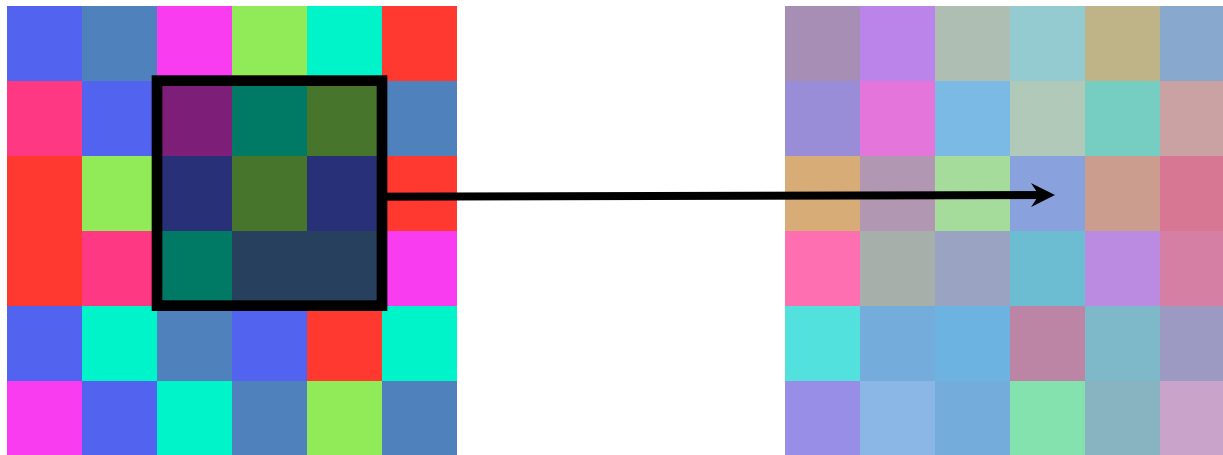
$$\beta = 1.0; f(v) = v$$



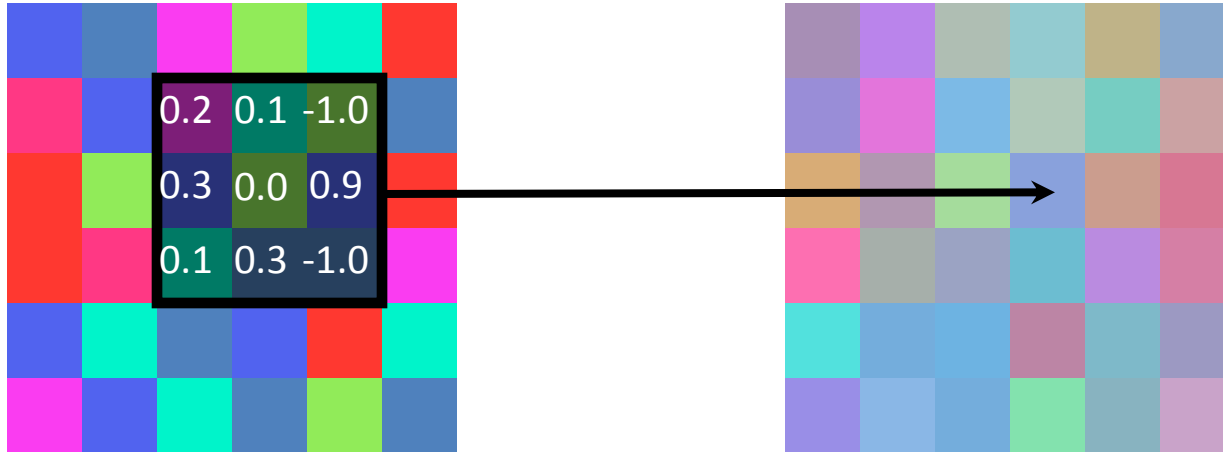
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

$$\beta = 2.5; f(v) = v^{1/2.5} = v^{0.4}$$

Neighborhood Operations



Convolution

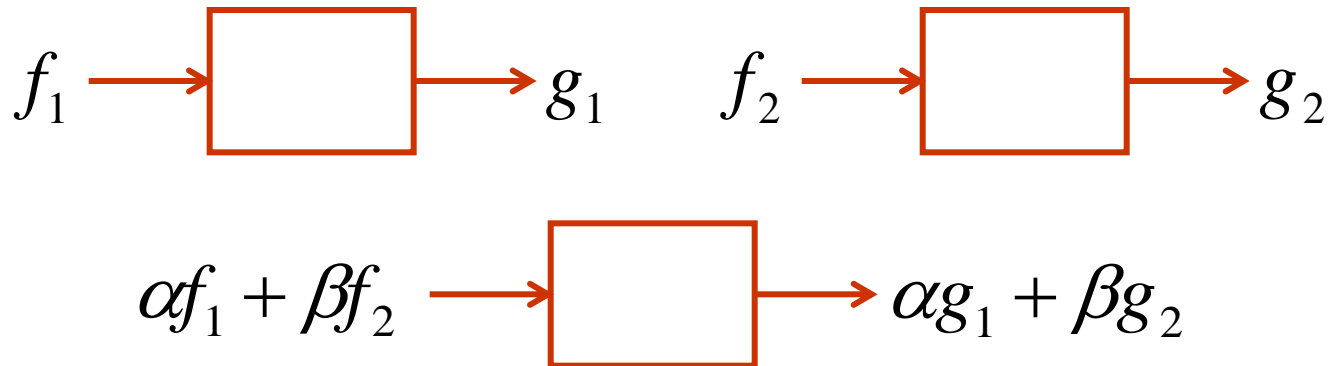


$$F = \begin{bmatrix} 0.2 & 0.1 & -1.0 \\ 0.3 & 0.0 & 0.9 \\ 0.1 & 0.3 & -1.0 \end{bmatrix}$$

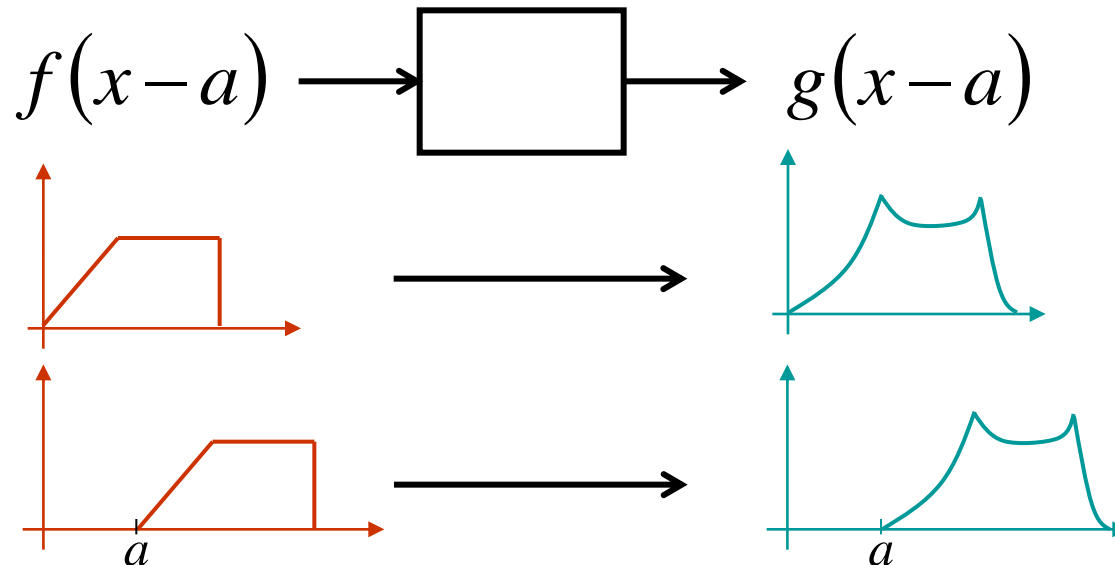
$$I' = F * I$$

Linear Shift Invariant Systems (LSIS)

Linearity:



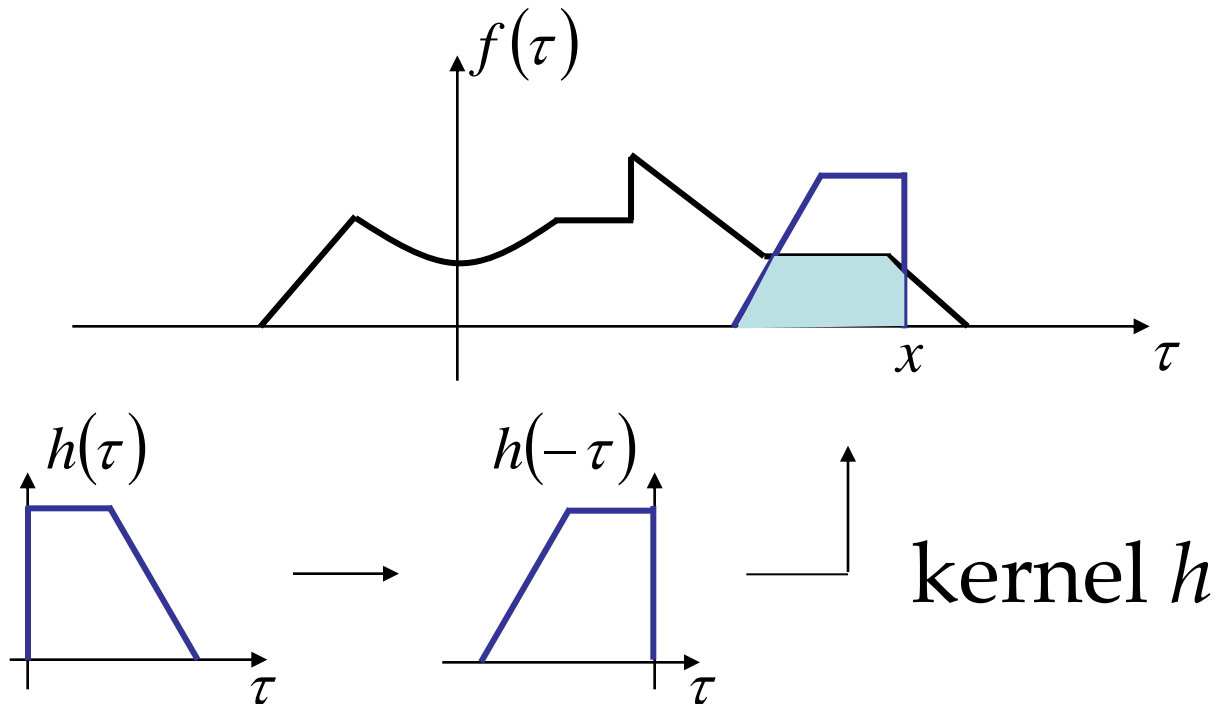
Shift invariance:



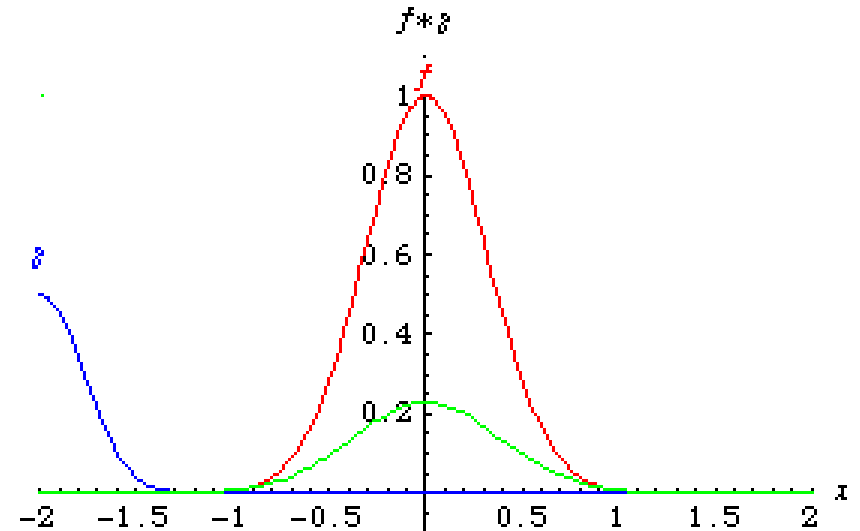
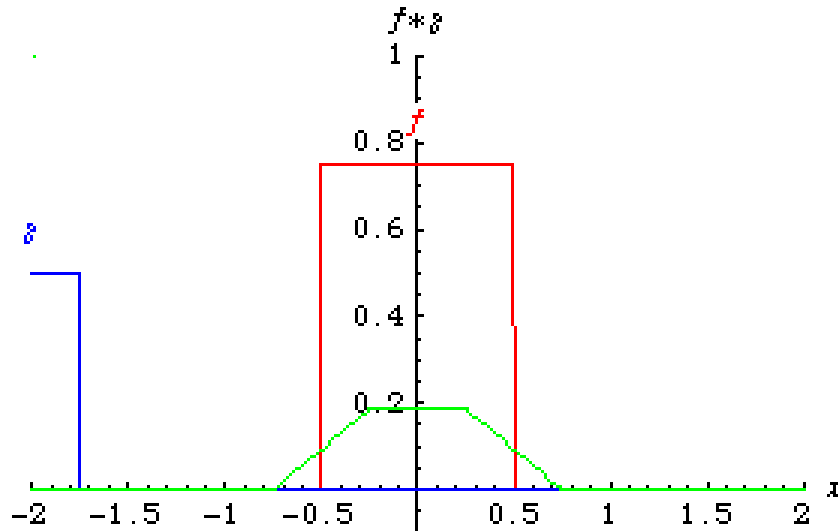
Convolution

LSIS is doing convolution; convolution is linear and shift invariant

$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)d\tau \quad g = f * h$$

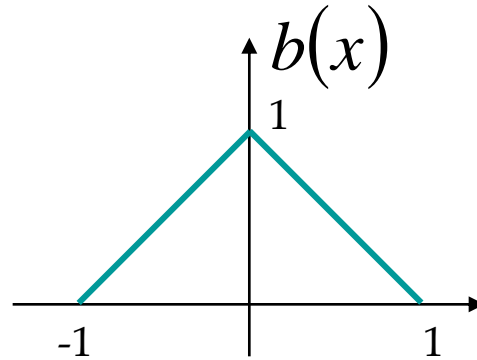
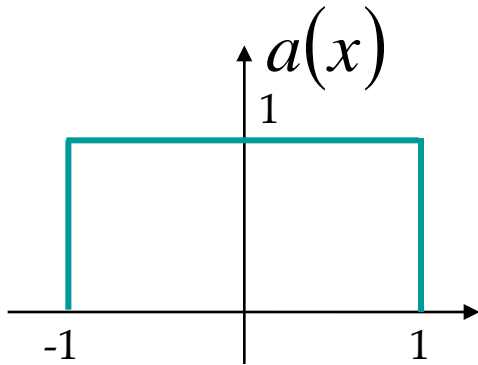


Convolution - Example

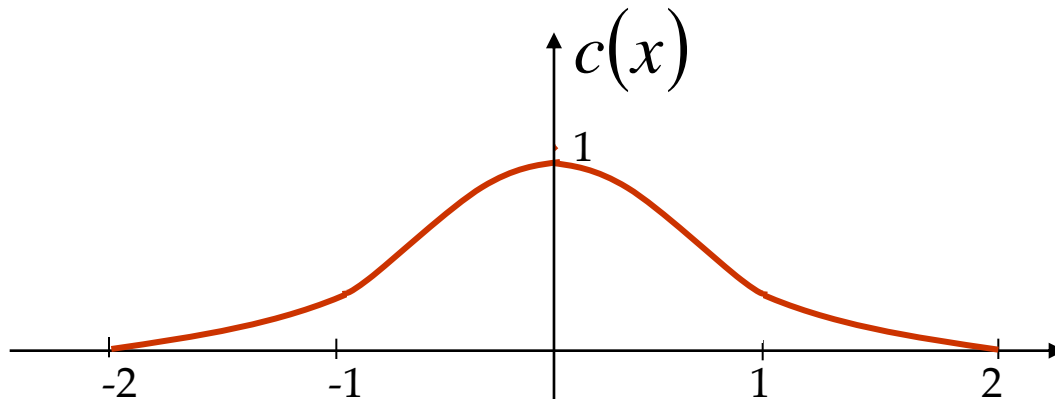


— f
— g
— $f * g$

Convolution - Example



$$\downarrow c = a * b$$



Properties of Convolution

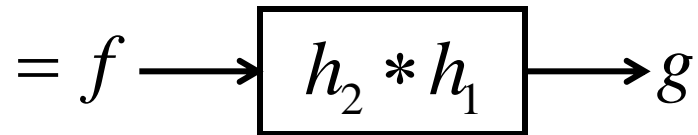
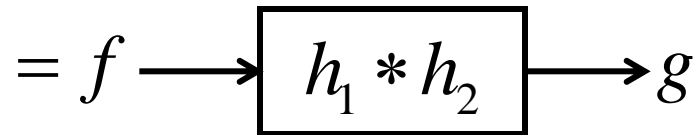
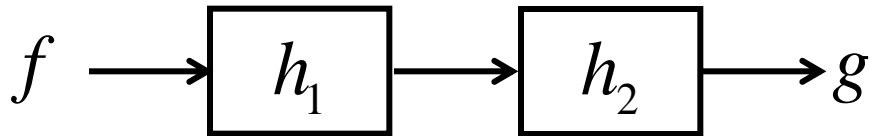
- Commutative

$$a * b = b * a$$

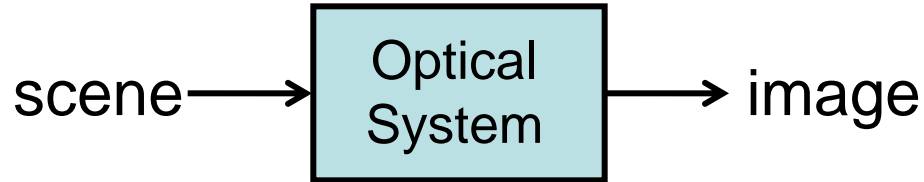
- Associative

$$(a * b) * c = a * (b * c)$$

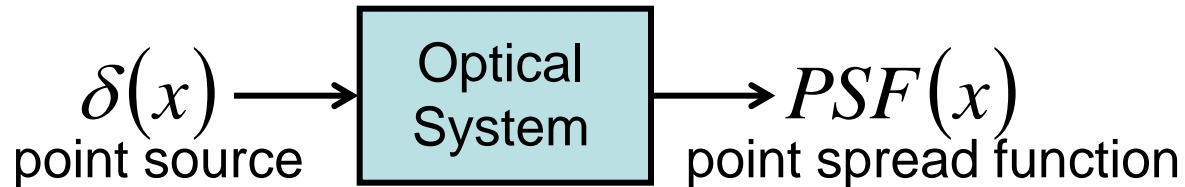
- Cascade system



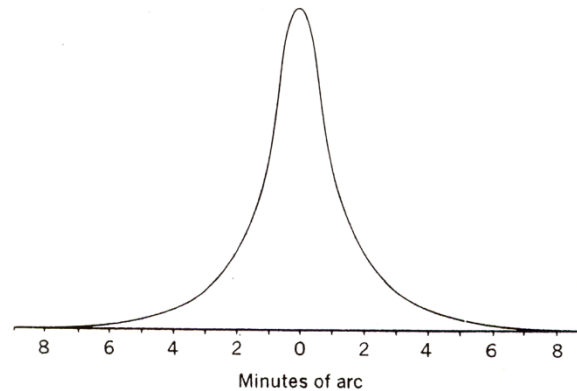
Point Spread Function



- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.



- Point spread function of Human Eyes



Point Spread Function



normal vision



myopia



hyperopia



astigmatism

Original Image



Blurred Image



Gaussian Smoothing

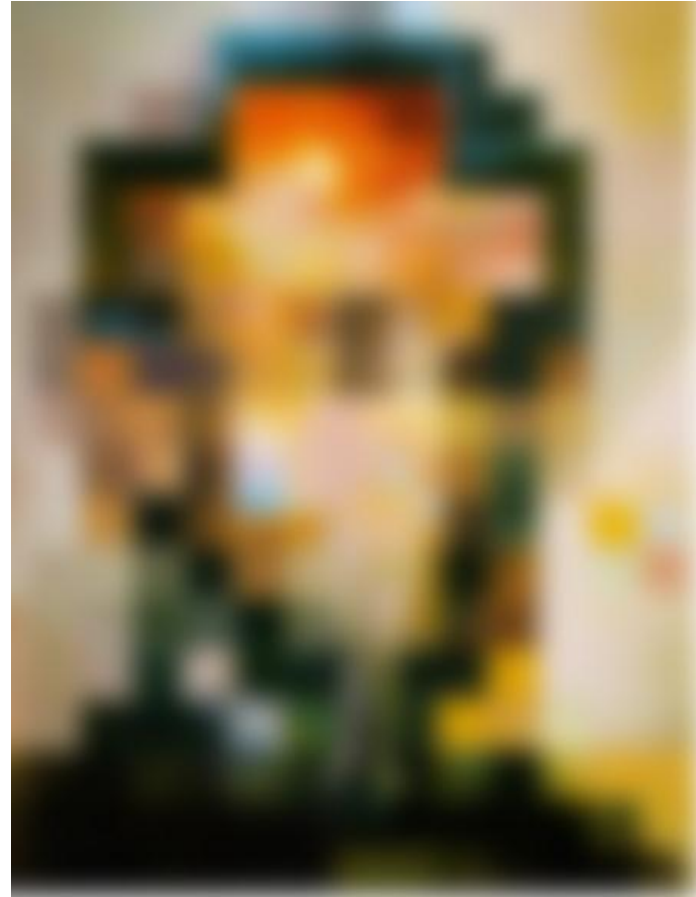


by Charles Allen Gillbert



by Harmon & Julesz

Gaussian Smoothing



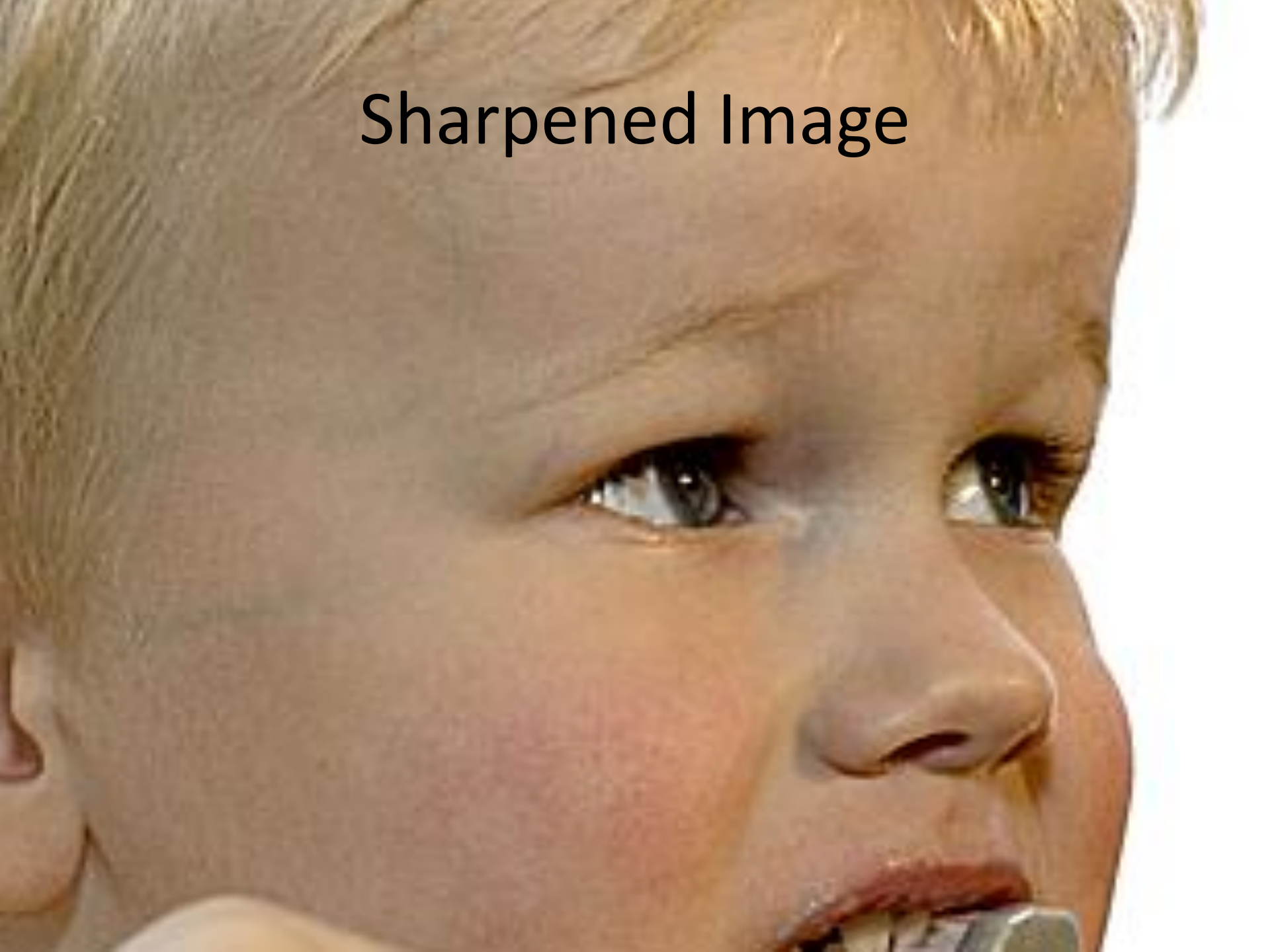
Original Image



Sharpened Image



Sharpened Image





Noise



Blurred Noise

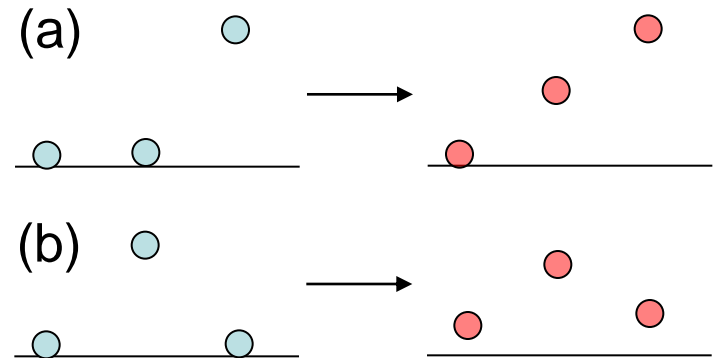


Median Filter

- Smoothing is averaging

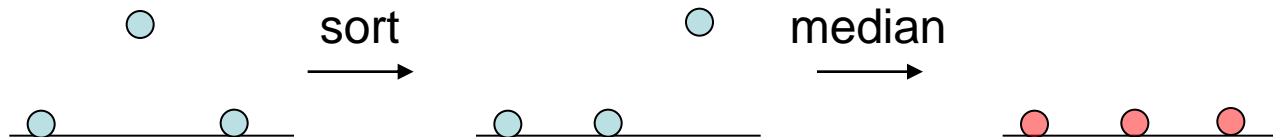
(a) Blurs edges

(b) Sensitive to outliers



- Median filtering

- Sort $N^2 - 1$ values around the pixel
- Select middle value (median)



- Non-linear (Cannot be implemented with convolution)

Median Filter



Can this be described as a convolution?

Original Image



Example: Noise Reduction



Image with noise



Median filter (5x5)

Salt and pepper noise

Gaussian

Median

3x3



5x5



7x7



Gaussian noise

Gaussian

Median



Example: Noise Reduction



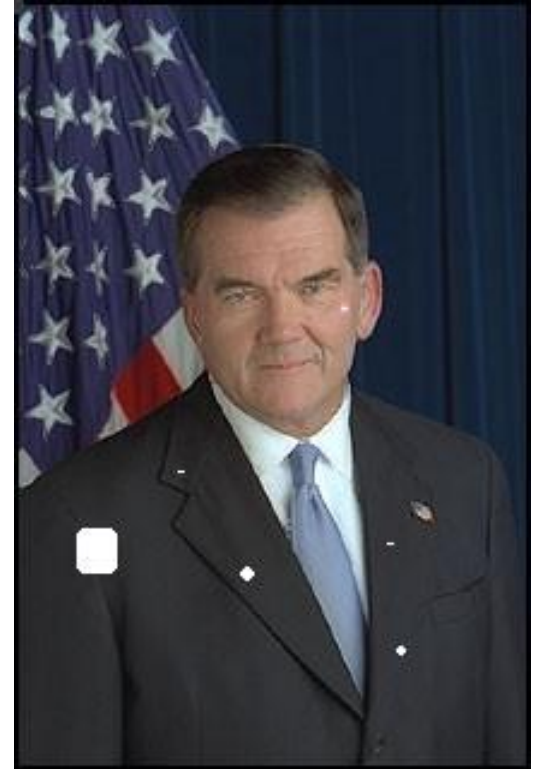
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Original image



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Image with noise



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Median filter (5x5)

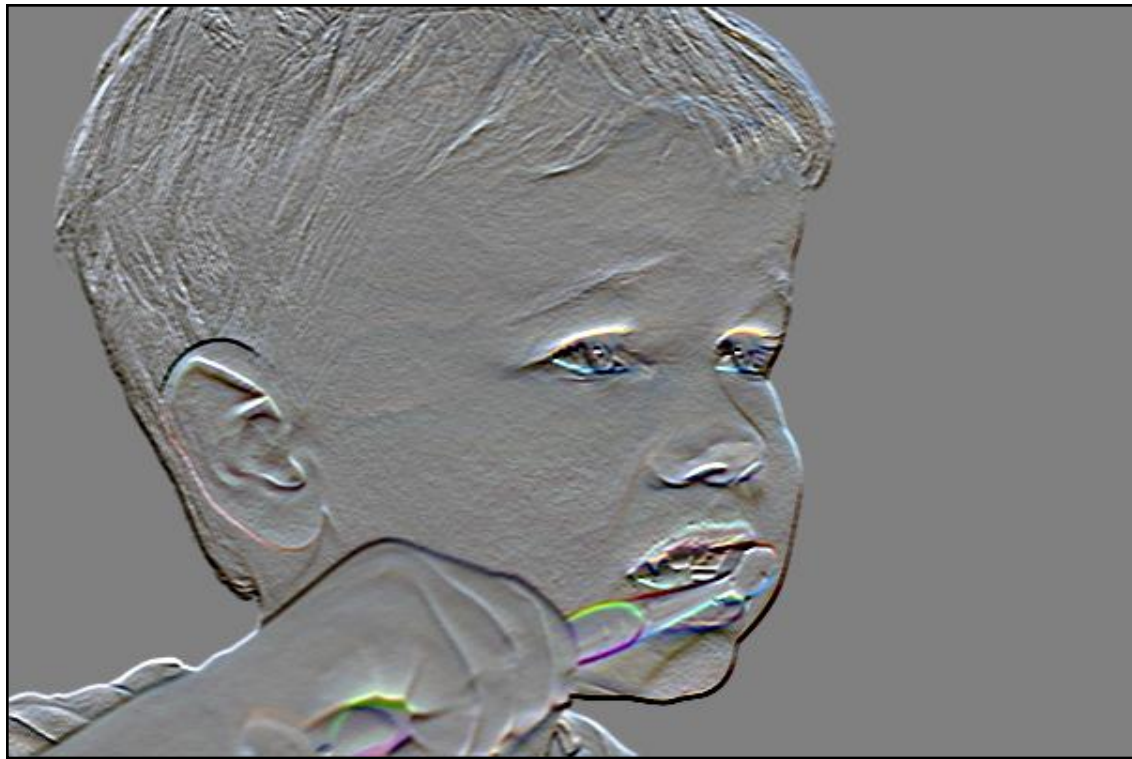
Original Image



X-Edge Detection



Y-Edge Detection



Edge detection filters

Roberts (2 x 2):

0	1
-1	0

1	0
0	-1

Sobel (3 x 3):

-1	0	1
-1	0	1
-1	0	1

1	1	1
0	0	0
-1	-1	1

Sobel (5 x 5):

-1	-2	0	2	1
-2	-3	0	3	2
-3	-5	0	5	3
-2	-3	0	3	2
-1	-2	0	2	1

1	2	3	2	1
2	3	5	3	2
0	0	0	0	0
-2	-3	-5	-3	-2
-1	-2	-3	-2	-1

General Edge Detection



Can this be described as a convolution?

Image Processing

- Some operations preserve the range but change the domain of f :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

- Still other operations operate on both the domain and the range of f .

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

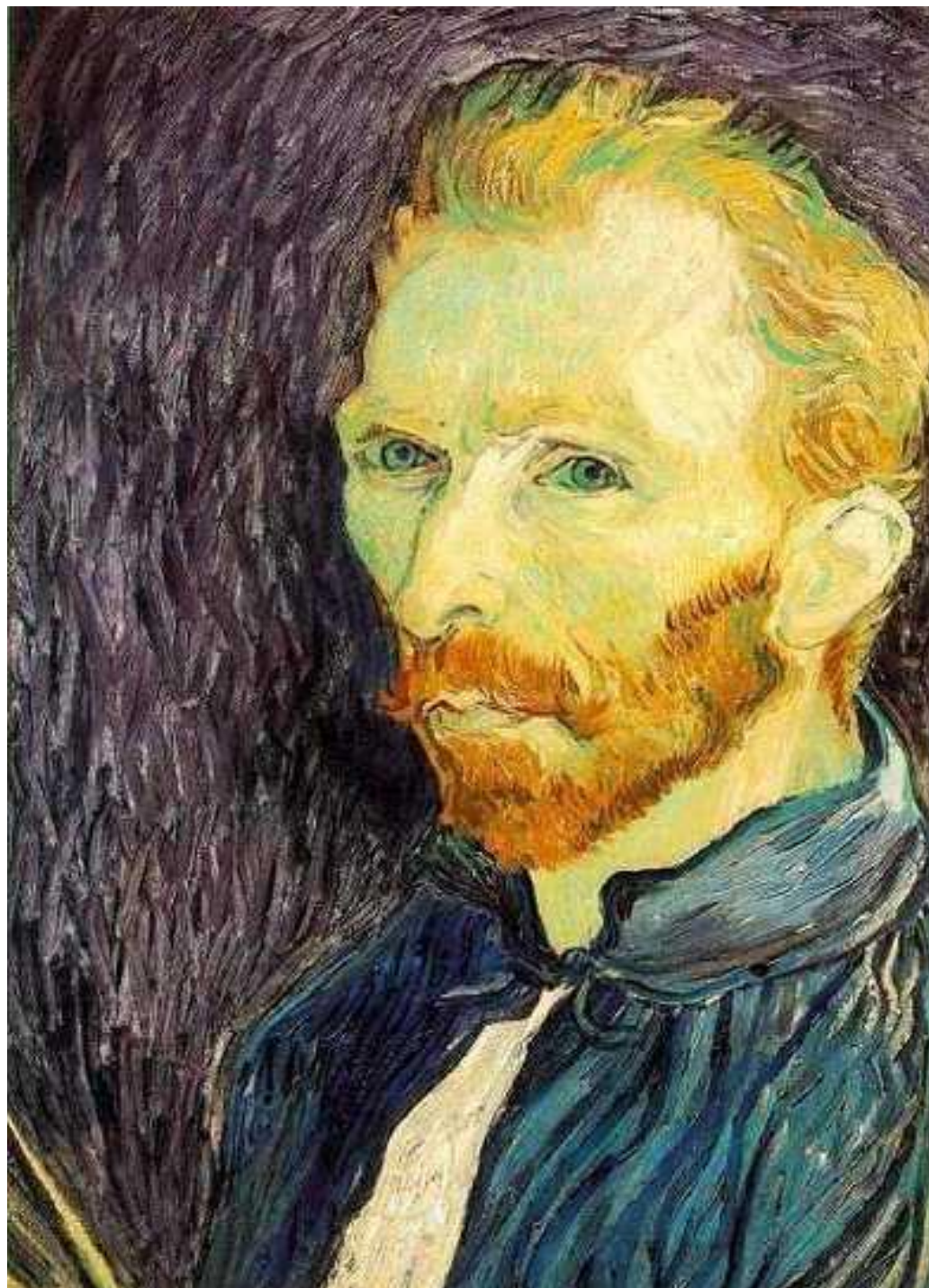
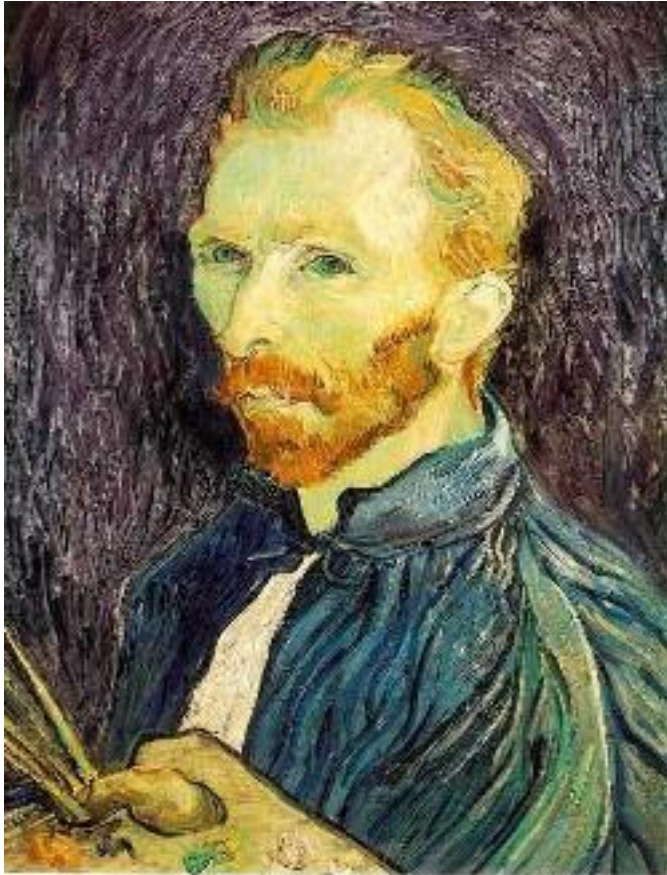


Image Sub-Sampling



1/4



1/8

Throw away every other row and column to create a $1/2$ size image
- called *image sub-sampling*

Image Sub-Sampling



1/2

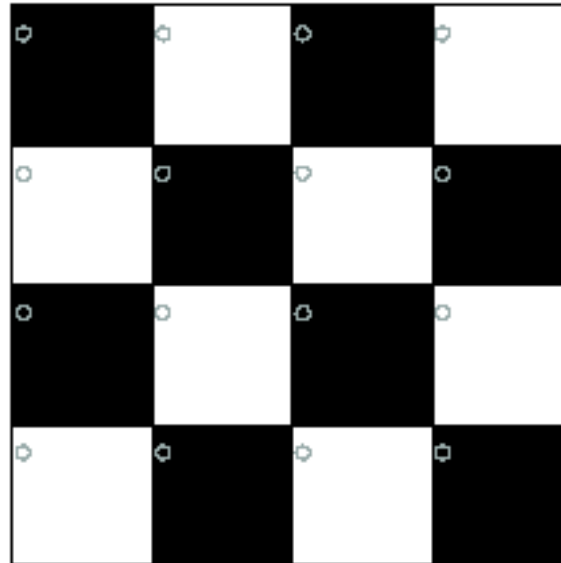
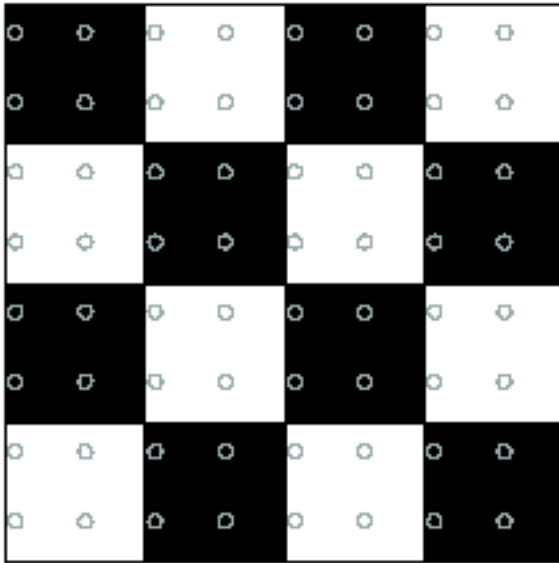


1/4 (2x zoom)

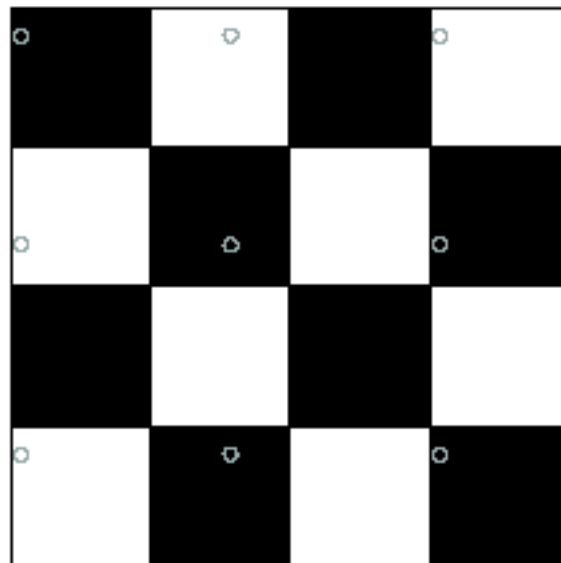
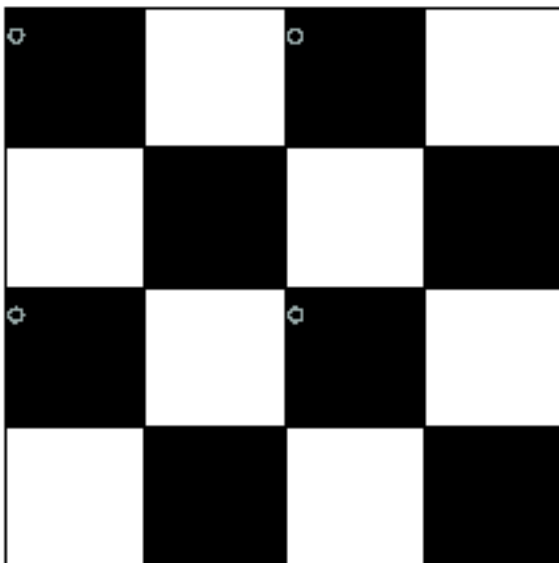


1/8 (4x zoom)

Good and Bad Sampling

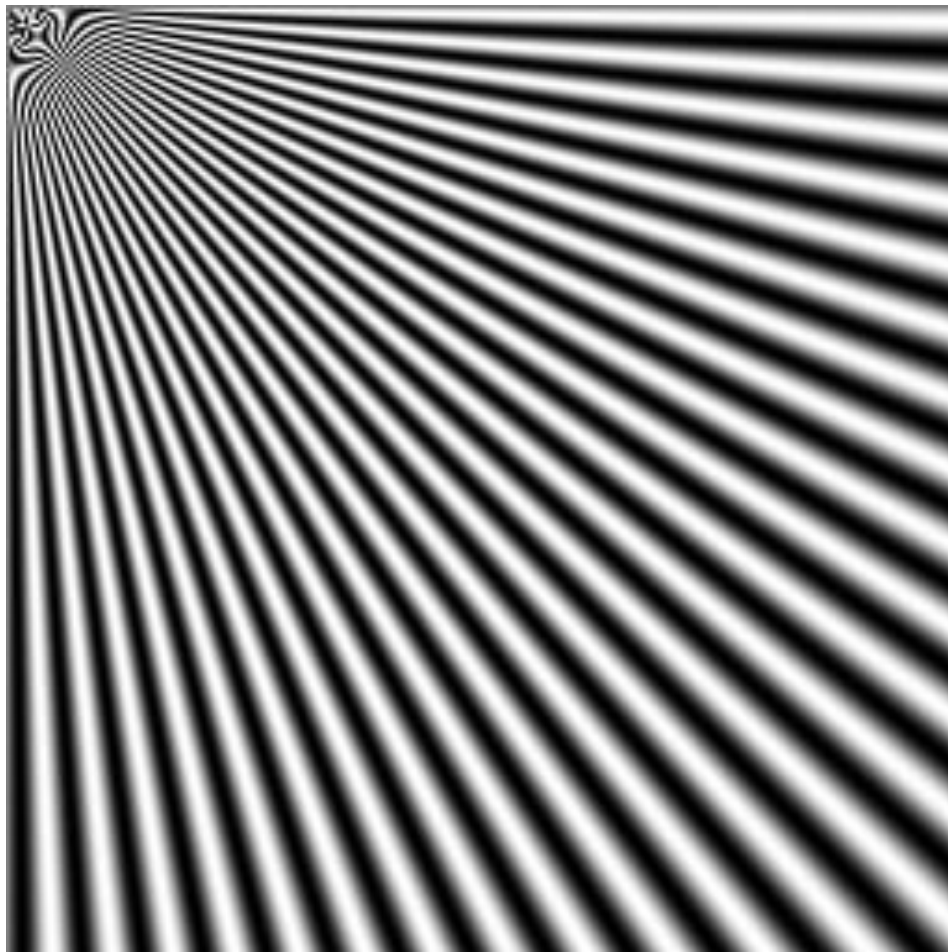


Good sampling:
• Sample often or,
• Sample wisely

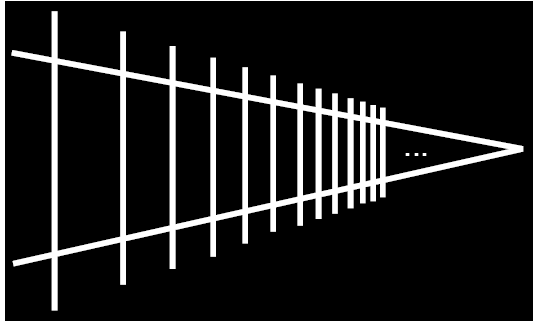


Bad sampling:
• see aliasing in action!

Aliasing

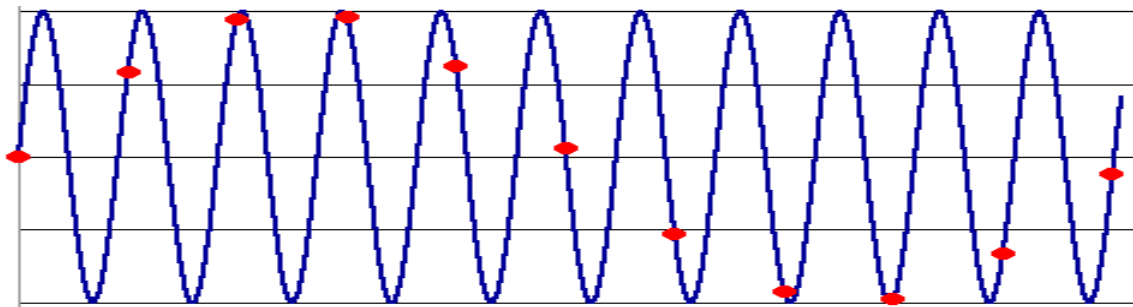


Alias: n., an assumed name



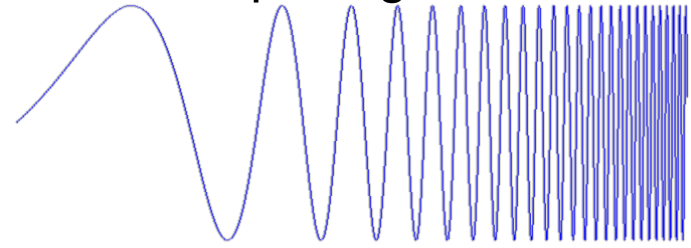
Picket fence receding into the distance will produce aliasing...

WHY?

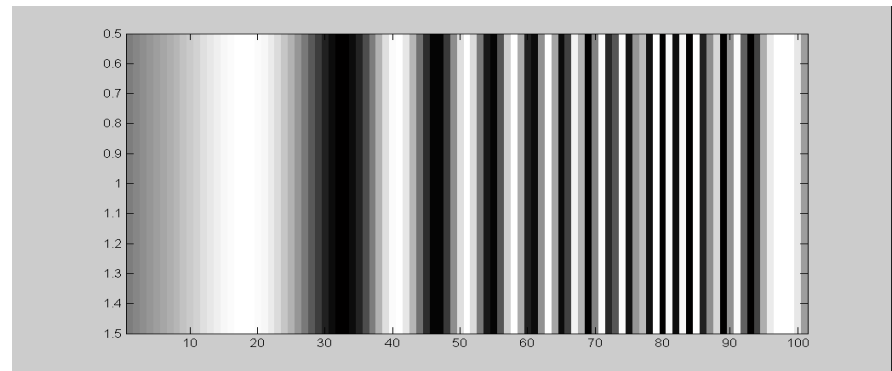


Not enough samples

Input signal:



Matlab output:



```
x = 0:.05:5; imagesc(sin((2.^x).*x))
```

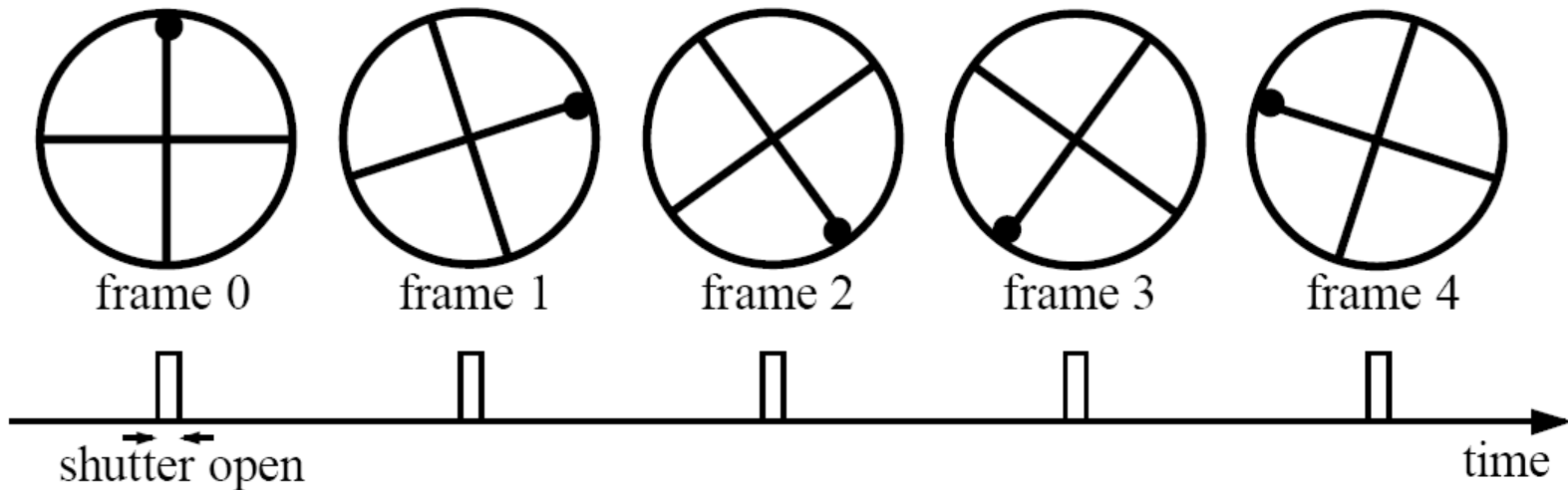
Alias!

Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise).

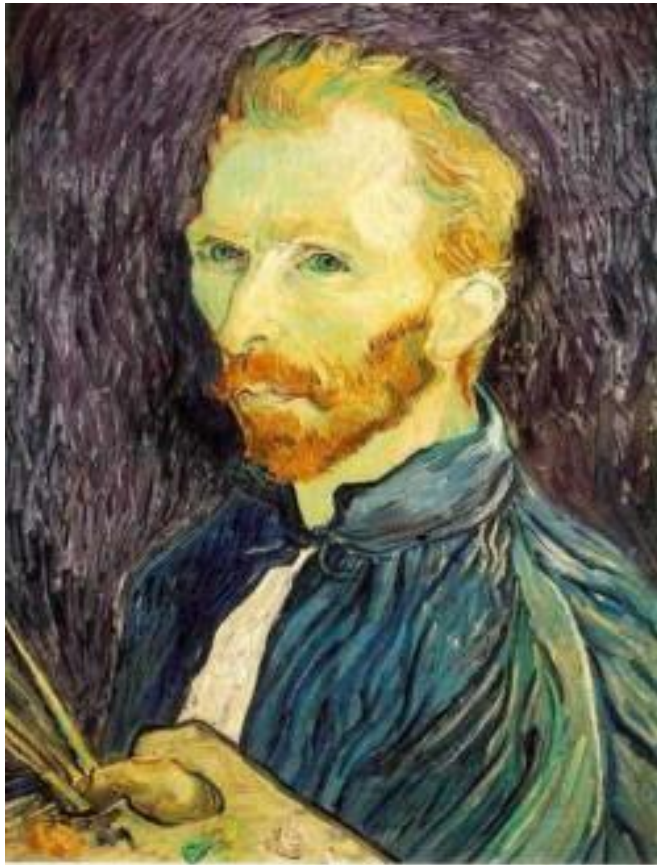
Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = $1/30$ sec. for video, $1/24$ sec. for film):



Without dot, wheel appears to be rotating slowly backwards!
(counterclockwise)

Sub-Sampling with Gaussian Pre-Filtering



Gaussian 1/2



G 1/4



G 1/8

- Solution: filter the image, *then* subsample
 - Filter size should double for each $\frac{1}{2}$ size reduction. Why?

Sub-Sampling with Gaussian Pre-Filtering



Gaussian $1/2$



G $1/4$



G $1/8$

Compare with...



1/2

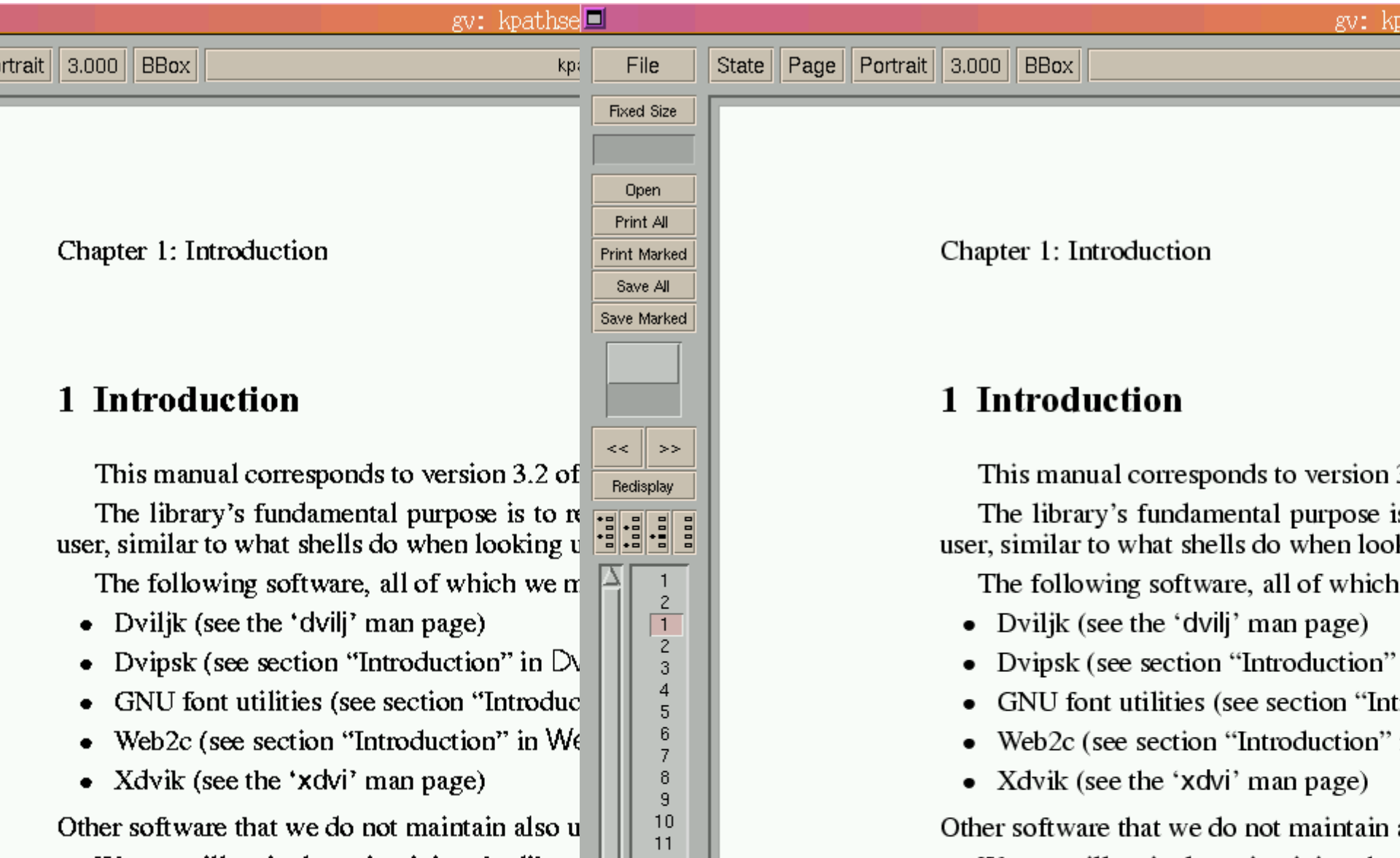


1/4 (2x zoom)



1/8 (4x zoom)

Aliasing





Canon D60 (w/ anti-alias filter)



Artifact not detail

SD9

Sigma SD9 (w/o anti-alias filter)



512

256

128

64

32

16

8



Figure from David Forsyth

Original Image



Warped Image

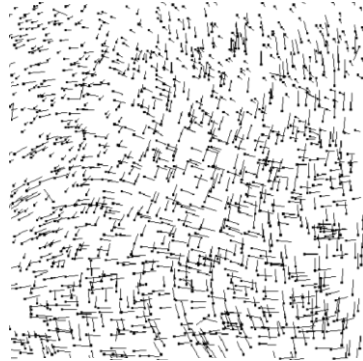


Warped Image



orig

+



vector field

=



warped

how?

Advection (just like a fluid)

