Homework 1 solutions
15-462
Fall 2013

1a)

$$\begin{bmatrix} 12 \\ 48 \\ -48 \end{bmatrix}$$

b)

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ 1 & 6 & -1 \\ 5 & 9 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Since $\alpha + \beta + \gamma < 1$, and $\alpha, \beta, \gamma \in [0, 1]$, the point lies within the triangle.

c)

$$\begin{bmatrix} \frac{455}{3} \\ \frac{355}{3} \\ \frac{100}{3} \end{bmatrix}$$

2)

$$w = \frac{-g}{||g||} = \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$u = \frac{t \times w}{||t \times w||} = \langle 0, 0, 1 \rangle$$

$$v = w \times u = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$M_{cam} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3)

$$\max(|x|^2 + |y|^2 - r, |z| - h) = 0$$

4)

$$P(s, t) = \langle 5, 2, 3 \rangle + s(\langle 3, 0, 4 \rangle - \langle 5, 2, 3 \rangle) + t(\langle 2, 0, 1 \rangle - \langle 5, 2, 3 \rangle)$$

5)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \sin t \\ s \\ r \cos t + 3 \end{bmatrix}$$

6a) [-1, 0]

b) Use the formula $y_s = d\frac{y}{z}$
Plugging in the first point gives us:
$y_s = 2 \cdot \frac{3.0}{1+2.5} = \frac{12}{7}$
So the projection is $[1, \frac{12}{7}]$.
The second point is already on the viewing line.

c) Point [2.5, 3.0] lies on the line.

7)

1.
$$\begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ -8 & 3 & 0 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix}$$

2. None of the points are interpolated.