Spatial Data Structures

- Hierarchical Bounding Volumes
- Grids
- Octrees
- BSP Trees
Speeding Up Computations
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• Ray Tracing
  – Spend a lot of time doing ray object intersection tests
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• Hidden Surface Removal – painters algorithm
  – Sorting polygons front to back
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  – Quickly determine if two objects collide

\[ n^2 \text{ computations} \]
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Spatial data-structures

\[ n^2 \text{ computations} \]
Spatial Data Structures

- We’ll look at
  - Hierarchical bounding volumes
  - Grids
  - Octrees
  - K-d trees and BSP trees

- Good data structures can give speed up ray tracing by 10x or 100x
Bounding Volumes

- Wrap things that are hard to check for intersection in things that are easy to check
  - Example: wrap a complicated polygonal mesh in a box
  - Ray can’t hit the real object unless it hits the box
  - Adds some overhead, but generally pays for itself.
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• But you don’t want expensive intersection tests!
Bounding Volumes

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- Use the ratio of the object volume to the enclosed volume as a measure of fit.

- Cost = n*B + m*I
  
  n - is the number of rays tested against the bounding volume
  B - is the cost of each test  (Do not need to compute exact intersection!)
  m - is the number of rays which actually hit the bounding volume
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Hierarchical Bounding Volumes

- Still need to check ray against every object --- $O(n)$
- Use tree data structure
  - Larger bounding volumes contain smaller ones
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Check intersect root
If not return no intersections
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If intersect
check intersect left sub-tree
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Hierarchical Bounding Volumes

• Many ways to build a tree for the hierarchy
• Works well:
  – Binary
  – Roughly balanced
  – Boxes of sibling trees not overlap too much
Hierarchical Bounding Volumes

- Sort the surfaces along the axis before dividing into two boxes
- Carefully choose axis each time
- Choose axis that minimizes sum of volumes
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Hierarchical Bounding Volumes

• Works well if you use good (appropriate) bounding volumes and hierarchy

• Should give $O(\log n)$ rather than $O(n)$ complexity ($n=\#$ of objects)

• Can have multiple classes of bounding volumes and pick the best for each enclosed object
Hierarchical bounding volumes

**Spatial Subdivision**

- Grids
- Octrees
- K-d trees and BSP trees
3D Spatial Subdivision

• Bounding volumes enclose the objects (object-centric)

• Instead could divide up the space—the further an object is from the ray the less time we want to spend checking it
  – Grids
  – Octrees
  – K-d trees and BSP trees
Grids

- Data structure: a 3-D array of cells (voxels) that tile space
  - Each cell points to list of all surfaces intersecting that cell

- Intersection testing:
  - Start tracing at cell where ray begins
  - Step from cell to cell, searching for the first intersection point
  - At each cell, test for intersection with all surfaces pointed to by that cell
  - If there is an intersection, return the closest one
Grids

- Cells are traversed in an incremental fashion
- Hits of sets of parallel lines are very regular
More on Grids

• Be Careful! The fact that a ray passes through a cell and hits an object doesn’t mean the ray hit that object in *that* cell

• Optimization: cache intersection point and ray id in “mailbox” associated with each object

- Step from cell to cell
- Get object intersecting cell
- Find closest intersection
- If found intersection --- done
More on Grids

• Grids are a poor choice when the world is nonhomogeneous (clumpy)
  – many polygons clustered in a small space

• How many cells to use?
  – too few ⇒ many objects per cell ⇒ slow
  – too many ⇒ many empty cells to step through ⇒ slow

• Non-uniform spatial subdivision is better!
Octrees

- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”
Octrees

- **Quadtree is the 2-D generalization of binary tree**
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”

- **Octree is the 3-D generalization of quadtree**
  - node (cell) is a cube, recursively split into eight equal sub-cubes
  - for ray tracing:
    - stop subdivision based on number of objects
    - internal nodes store pointers to children, leaves store list of surfaces
  - **more expensive to traverse than a grid**
  - but an octree adapts to non-homogeneous scenes better

```c
trace(cell, ray) { // returns object hit or NONE
    if cell is leaf, return closest(objects_in_cell(cell))
    for child cells pierced by ray, in order // 1 to 4 of these
        obj = trace(child, ray)
    if obj!=NONE return obj
    return NONE
}
```
Which Data Structure is Best for Ray Tracing?

**Grids**
- Easy to implement
- Require a lot of memory
- Poor results for inhomogeneous scenes

**Octrees**
- Better on most scenes (more adaptive)

**Spatial subdivision expensive for animations**
- Hierarchical bounding volumes
- Better for dynamic scenes
- Natural for hierarchical objects
k-d Trees and BSP Trees

- Relax the rules for quadtrees and octrees:

- k-dimensional (k-d) tree
  - don’t always split at midpoint
  - split only one dimension at a time (i.e. x or y or z)

- binary space partitioning (BSP) tree
  - permit splits with any line
  - In 2-D space split with lines (most of our examples)
  - 3-D space split with planes
  - K-D space split with k-1 dimensional hyperplanes

- useful for Painter’s algorithm (hidden surface removal)
Painters Algorithm

Hidden Surface Elimination
Painters Algorithm

- Need to sort objects back to front
- Order depends on the view point
- Partition objects using BSP tree
- View independent
Building a BSP Tree

- Let’s look at simple example with 3 line segments
- Arrowheads are to show left and right sides of lines.
- Using line 1 or 2 as root is easy.
- (examples from http://www.geocities.com/SiliconValley/2151/bsp.html)
Drawing Objects

- Traverse the tree from the root
- If view point is on the left of the line --- traverse right sub-tree first
- Draw the root
- Traverse left sub-tree

Line 1
Line 2
Line 3

Viewpoint

a BSP tree using 2 as root
Building the Tree 2

Using line 3 for the root requires a split
Drawing Back to Front

• Use Painter’s Algorithm for hidden surface removal

Steps:

– Draw objects on far side of line 3
  » Draw objects on far side of line 2a
    – Draw line 1
      » Draw line 2a
    – Draw line 3
    – Draw objects on near side of line 3
      » Draw line 2b
Triangles

Use plane containing triangle $T_1$ to split the space.
If view point is on one side of the plane draw polygons on the other side first.
$T_2$ does not intersect plane of $T_1$. 

Diagram: Plane containing $T_1$, $T_2$ on the other side.
Triangles

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Triangles

Split Triangle
Painter’s Algorithm with BSP trees

• Build the tree
  – Involves splitting some polygons
  – Slow, but done only once for static scene

• Correct traversal lets you draw in back-to-front or front-to-back order for any viewpoint
  – Order is view-dependent
  – Pre-compute tree once
  – Do the “sort” on the fly

• Will not work for changing scenes
Drawing a BSP Tree

- Each polygon has a set of coefficients:
  \[Ax + By + Cz + D\]
- Plug the coordinates of the viewpoint in and see:
  - >0 : front side
  - <0 : back facing
  - =0 : on plane of polygon
- Back-to-front draw: inorder traversal, do farther child first
- Front-to-back draw: inorder traversal, do near child first

```c
front_to_back(tree, viewpt) {
    if (tree == null) return;
    if (positive_side_of(root(tree), viewpt)) {
        front_to_back(positive_branch(tree, viewpt);
        display_polygon(root(tree));
        front_to_back(negative_branch(tree, viewpt);
    } else { ...draw negative branch first...}
}
```
Building a Good Tree - the tricky part

• A naïve partitioning of \( n \) polygons will yield \( O(n^3) \) polygons because of splitting!

• Algorithms exist to find partitionings that produce \( O(n^2) \).
  – For example, try all remaining polygons and add the one which causes the fewest splits
  – Fewer splits \( \rightarrow \) larger polygons \( \rightarrow \) better polygon fill efficiency

• Also, we want a balanced tree.
Demos

BSP Tree construction
http://symbolcraft.com/graphics/bsp/index.html

• KD Tree construction
Real-time and Interactive Ray Tracing

The OpenRT Real-Time Ray-Tracing Project
http://www.openrt.de/index.php

• Interactive ray tracing via space subdivision
  http://www.cs.utah.edu/~reinhard/egwr/

• Interactive ray tracing with good hardware
  http://www.cs.utah.edu/vissim/projects/raytracing/