Spatial Data Structures

Hierarchical Bounding Volumes
Grids
Octrees
BSP Trees

Speeding Up Computations

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n^2 computations

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n^2 computations

Spatial data-structures

Spatial Data Structures

- We'll look at
  - Hierarchical bounding volumes
  - Grids
  - Octrees
  - K-d trees and BSP trees
- Good data structures can give speed up ray tracing by 10x or 100x

Bounding Volumes

- Wrap things that are hard to check for intersection in things that are easy to check
  - Example: wrap a complicated polygonal mesh in a box
  - Ray can't hit the real object unless it hits the box
  - Adds some overhead, but generally pays for itself.

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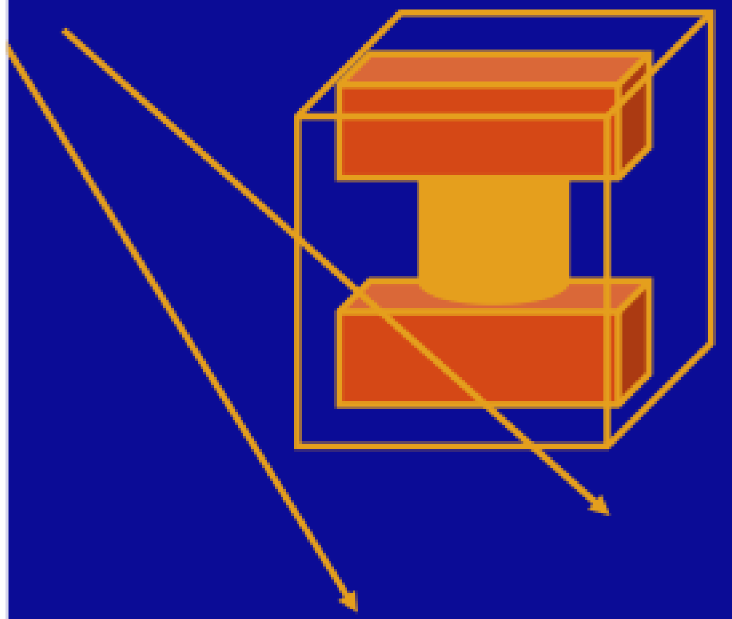
  - box can be axis-aligned or not



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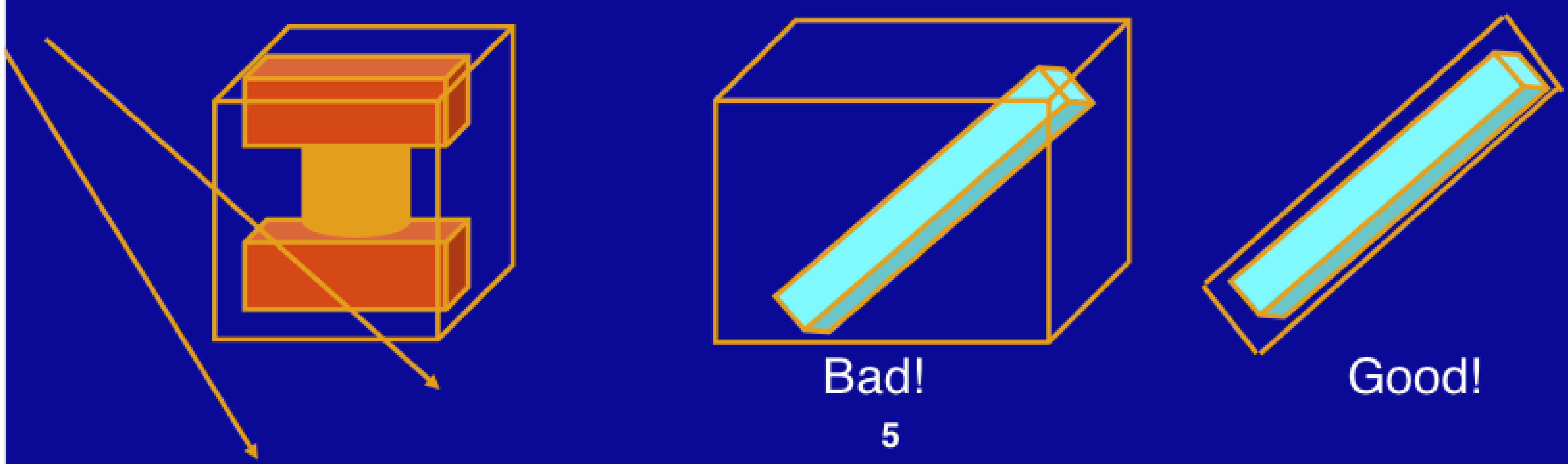
- But you don't want expensive intersection tests!



Bounding Volumes

- You want a snug fit!
- But you don't want expensive intersection tests!
- Use the ratio of the object volume to the enclosed volume as a measure of fit.

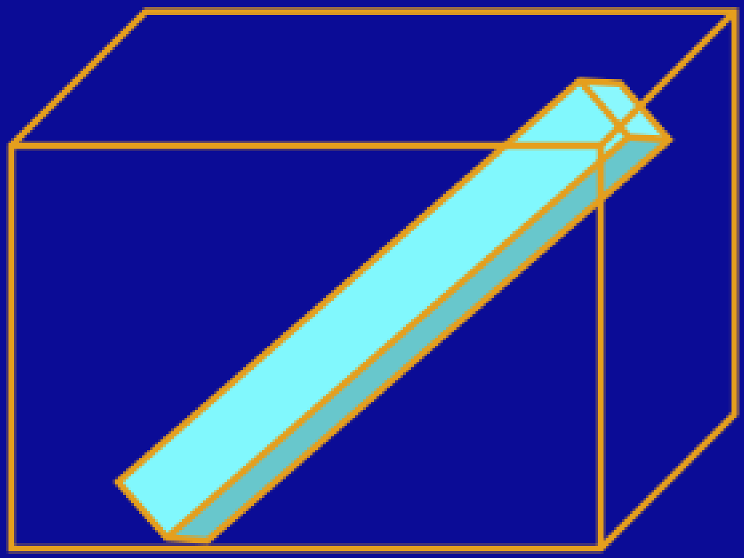
- Cost = n*B + m*I

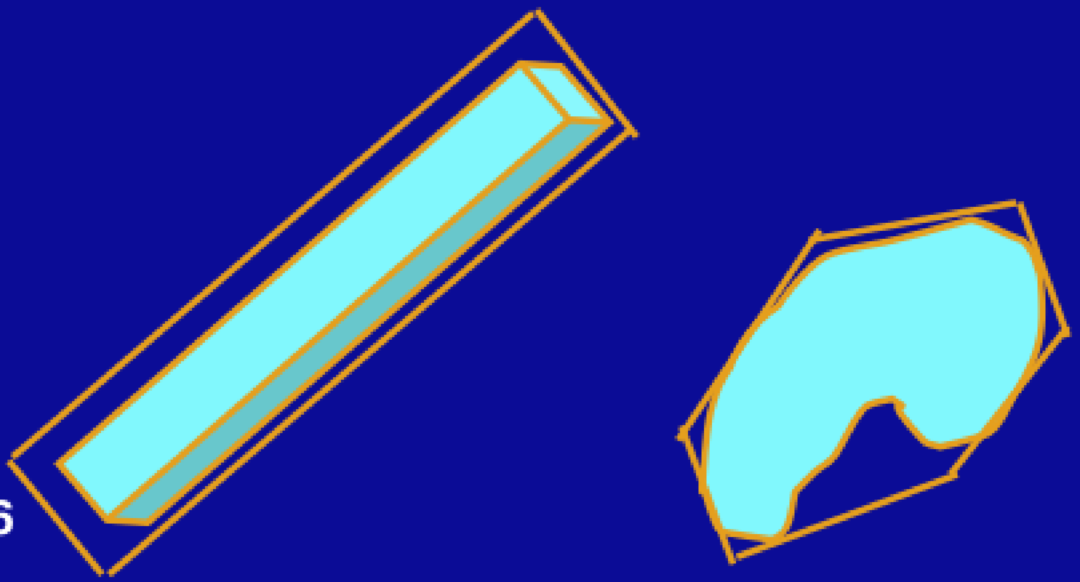
  n - is the number of rays tested against the bounding volume

  B - is the cost of each test (Do not need to compute exact intersection!)

  m - is the number of rays which actually hit the bounding volume

  I - is the cost of intersecting the object within





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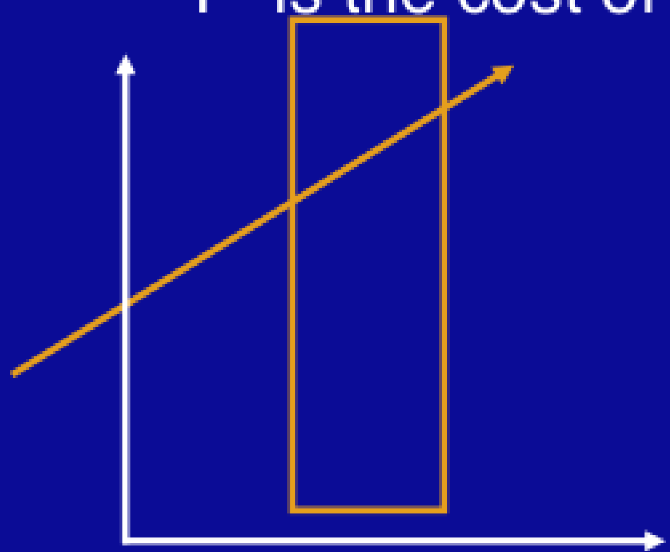
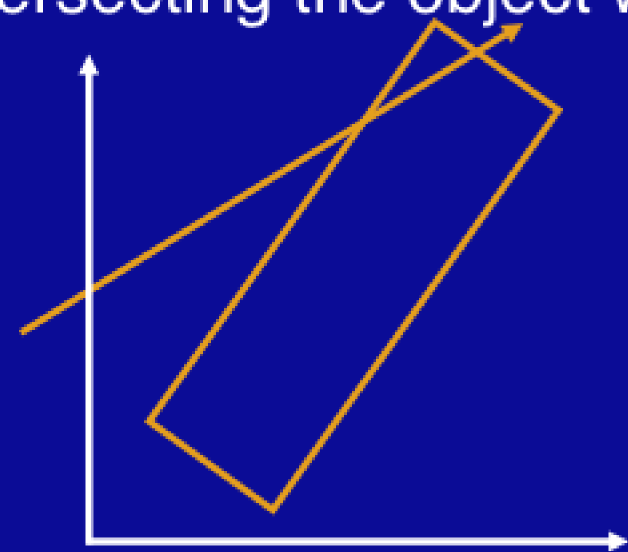
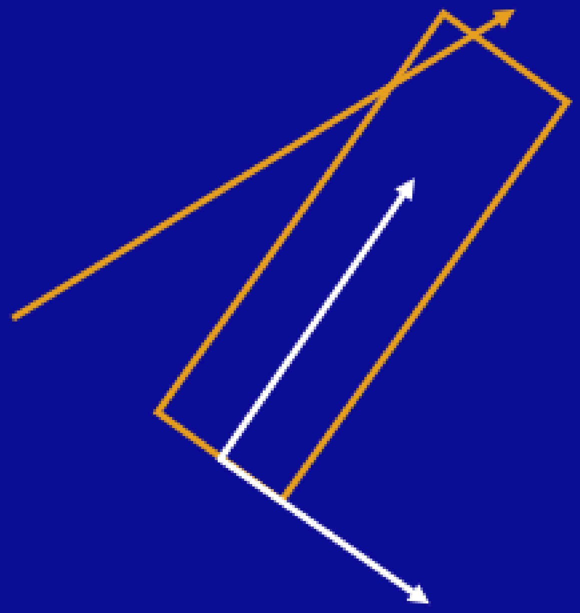
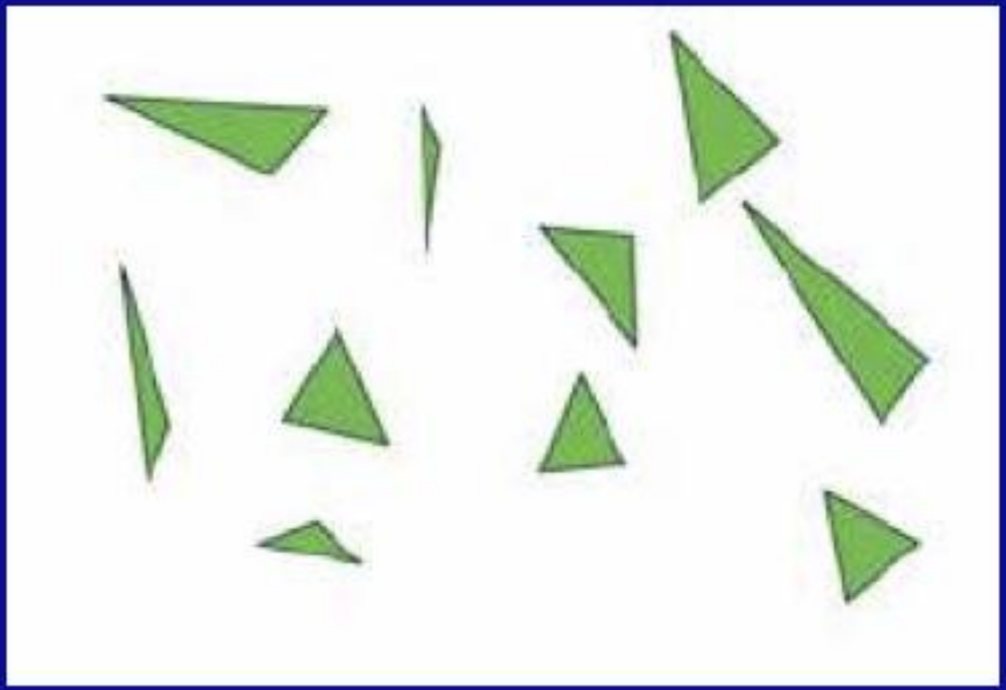




Hierarchical Bounding Volumes

- Still need to check ray against every object --- O(n)
- Use tree data structure
  - Larger bounding volumes contain smaller ones



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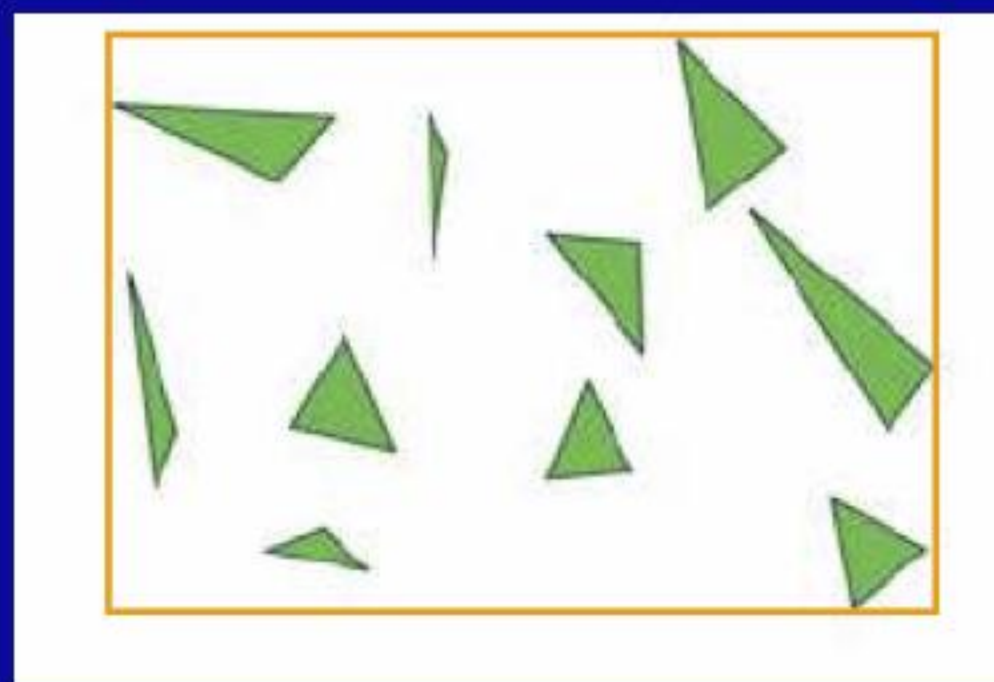

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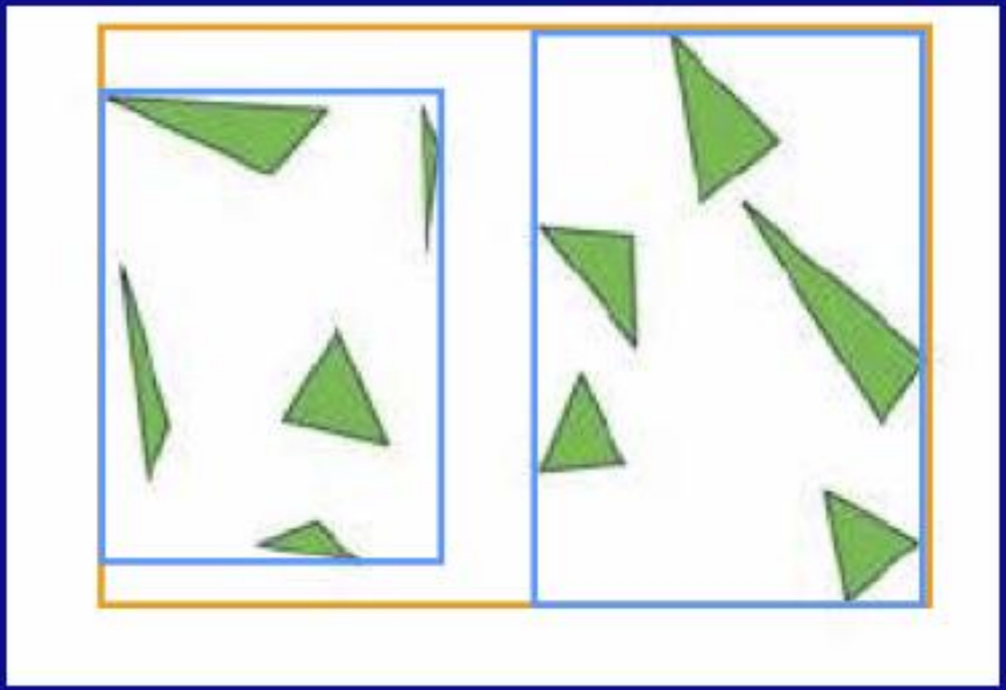

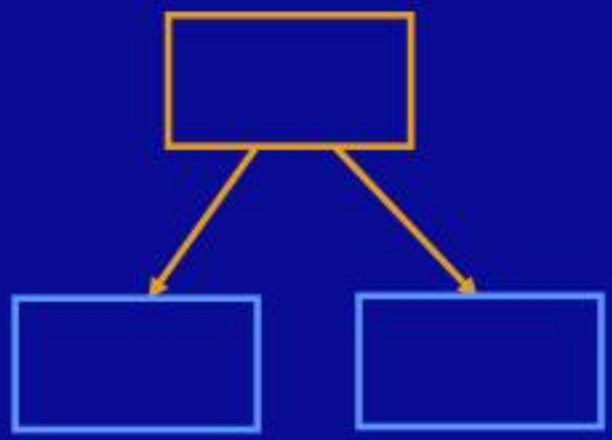

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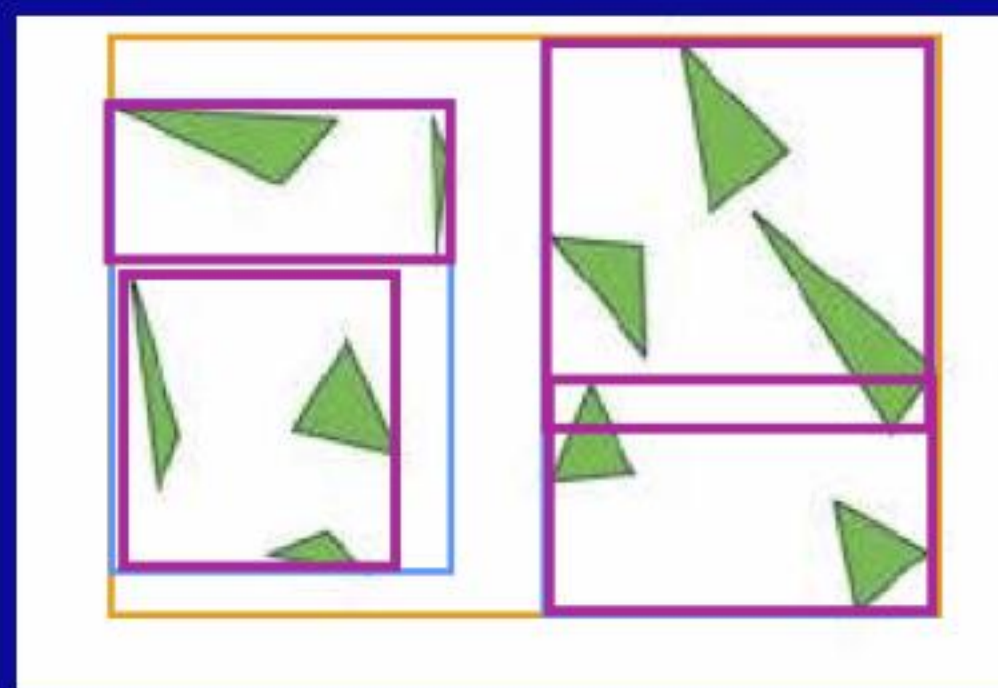

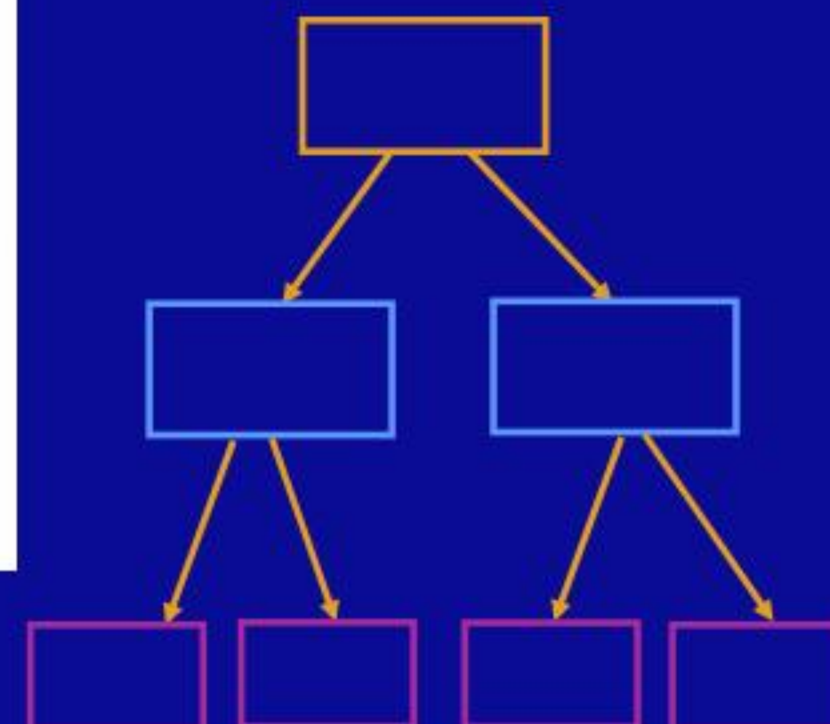

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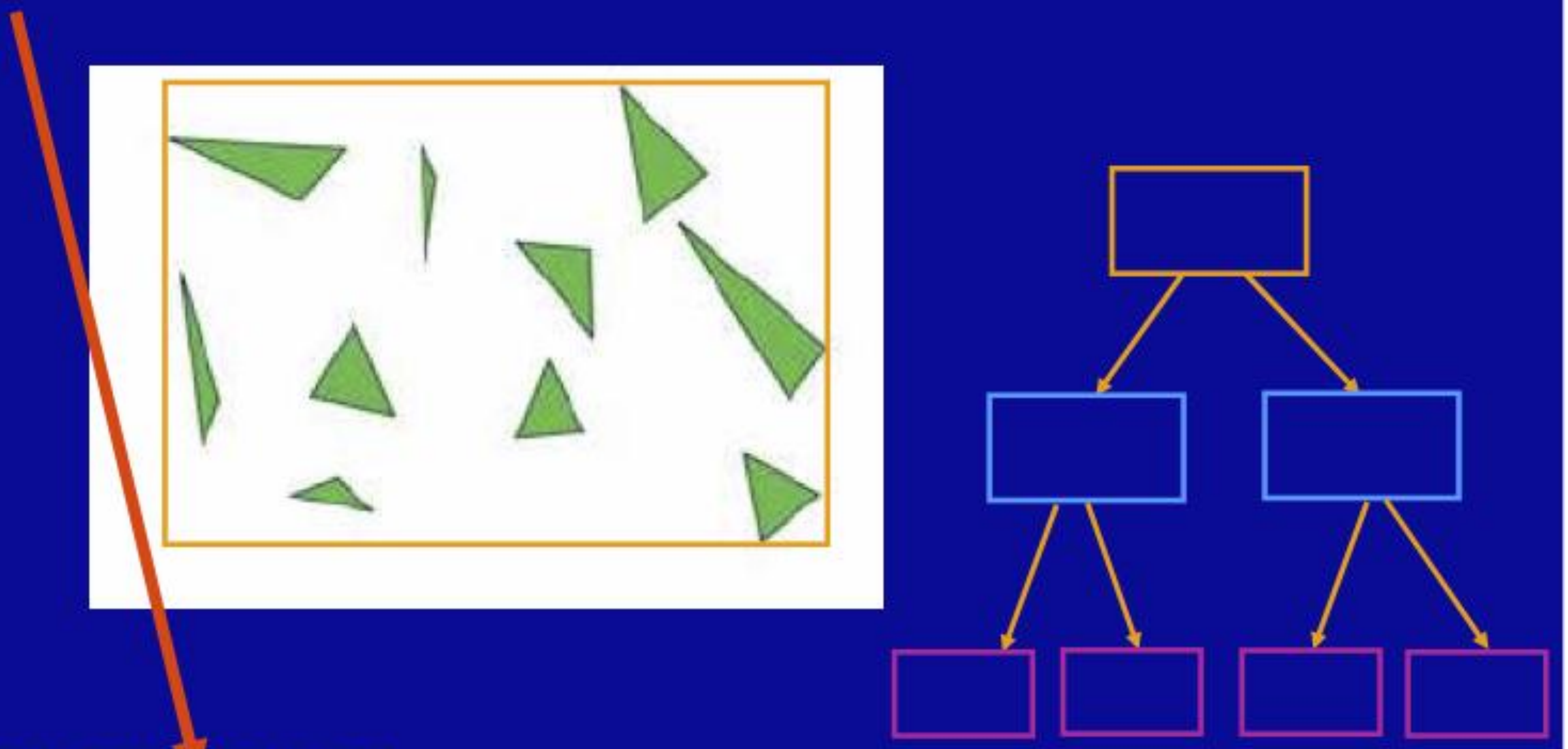

Check intersect root
If not return no intersections

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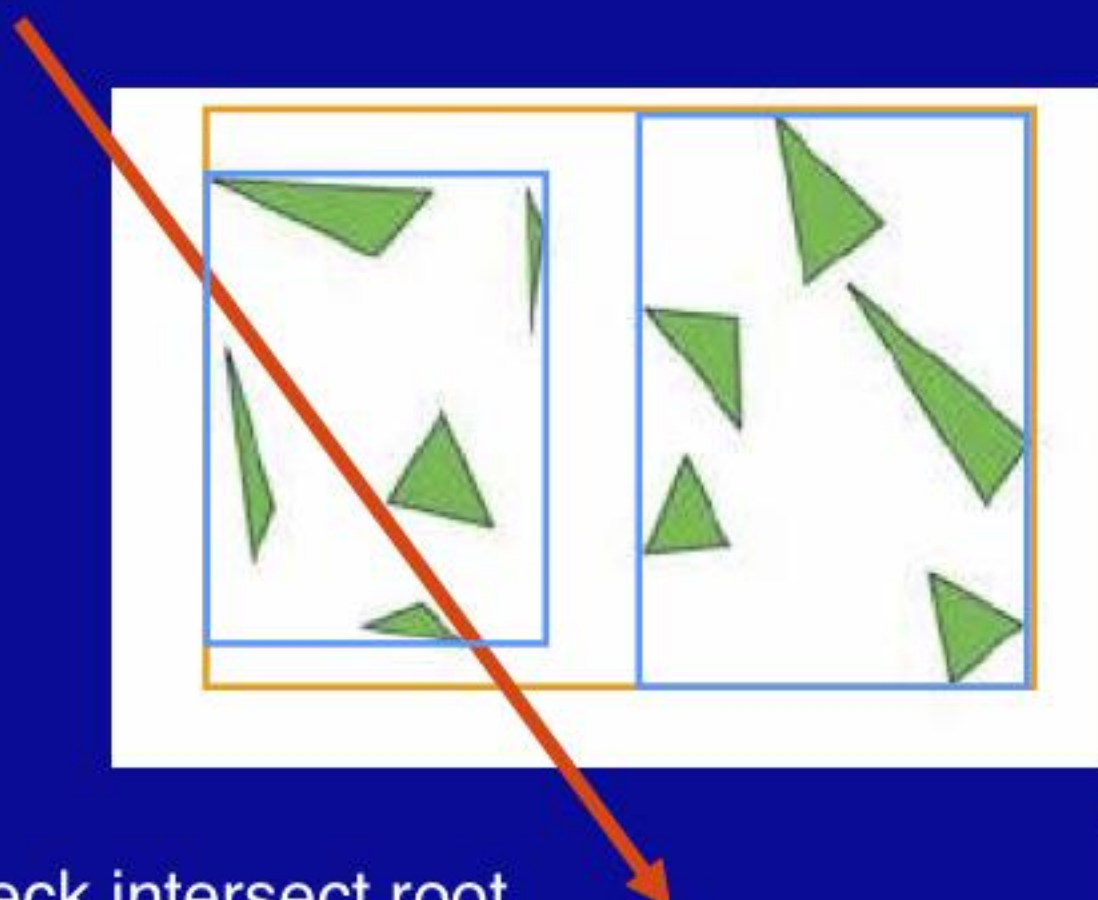

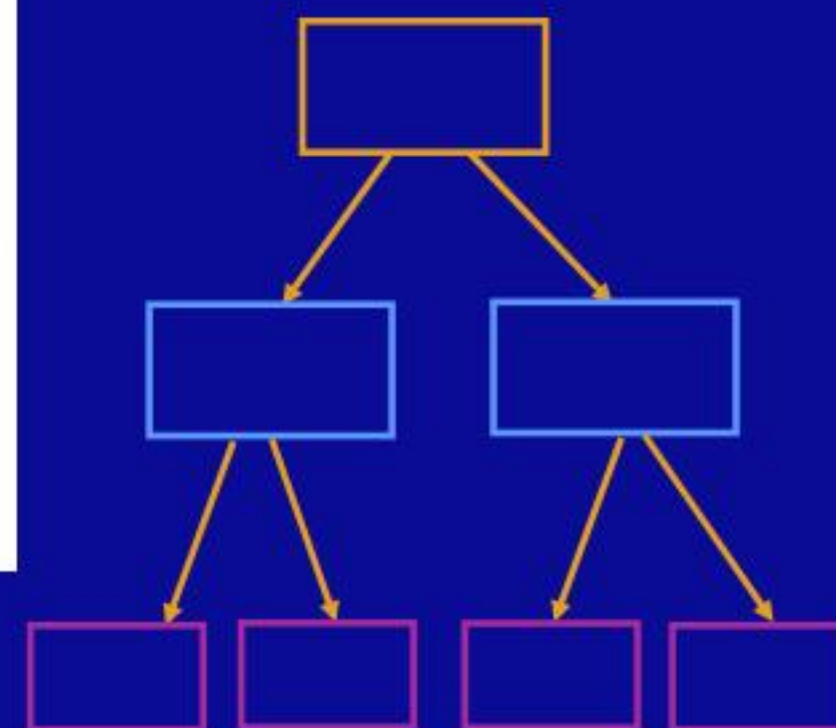

Check intersect root
If intersect
    check intersect left sub-tree
    check intersect right sub-tree

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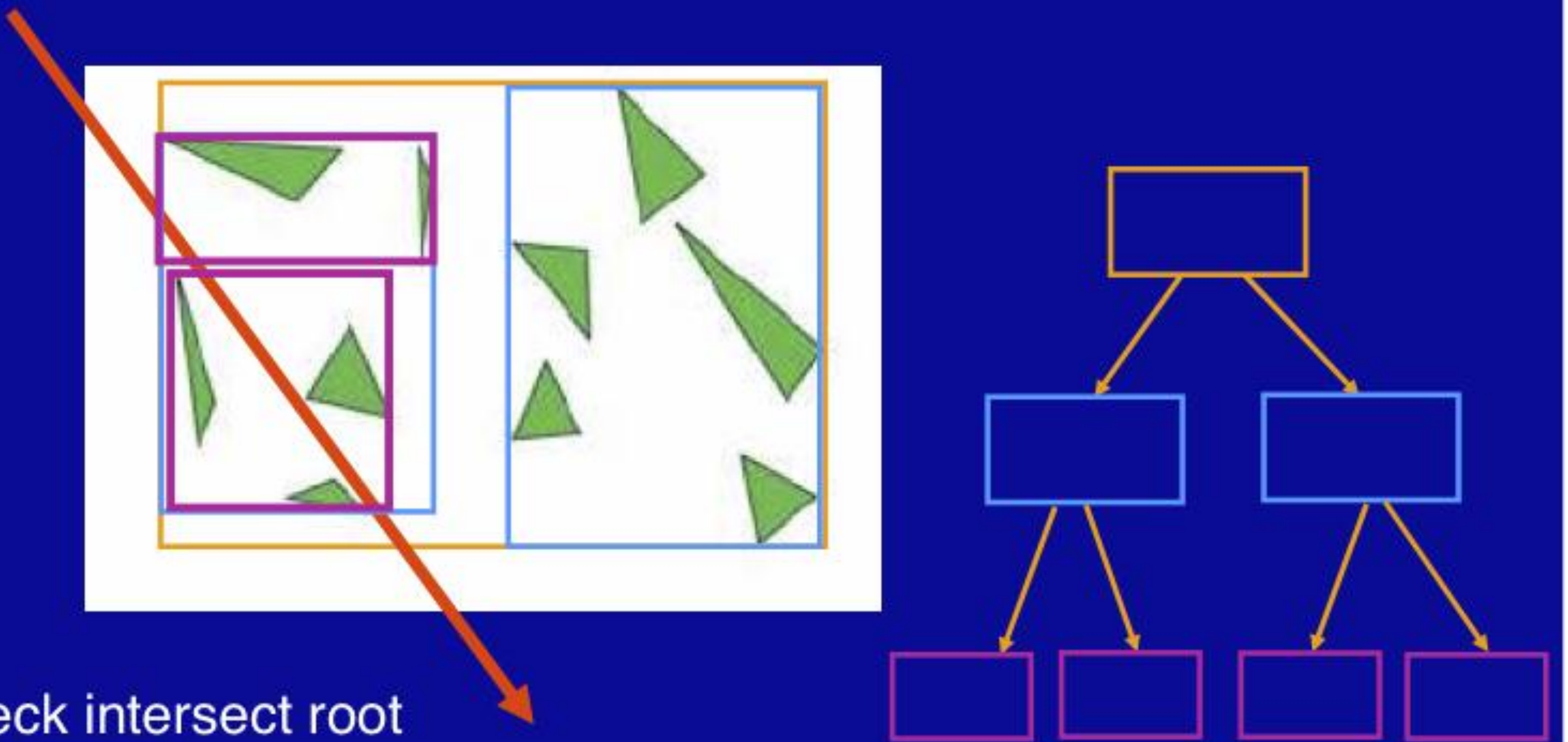

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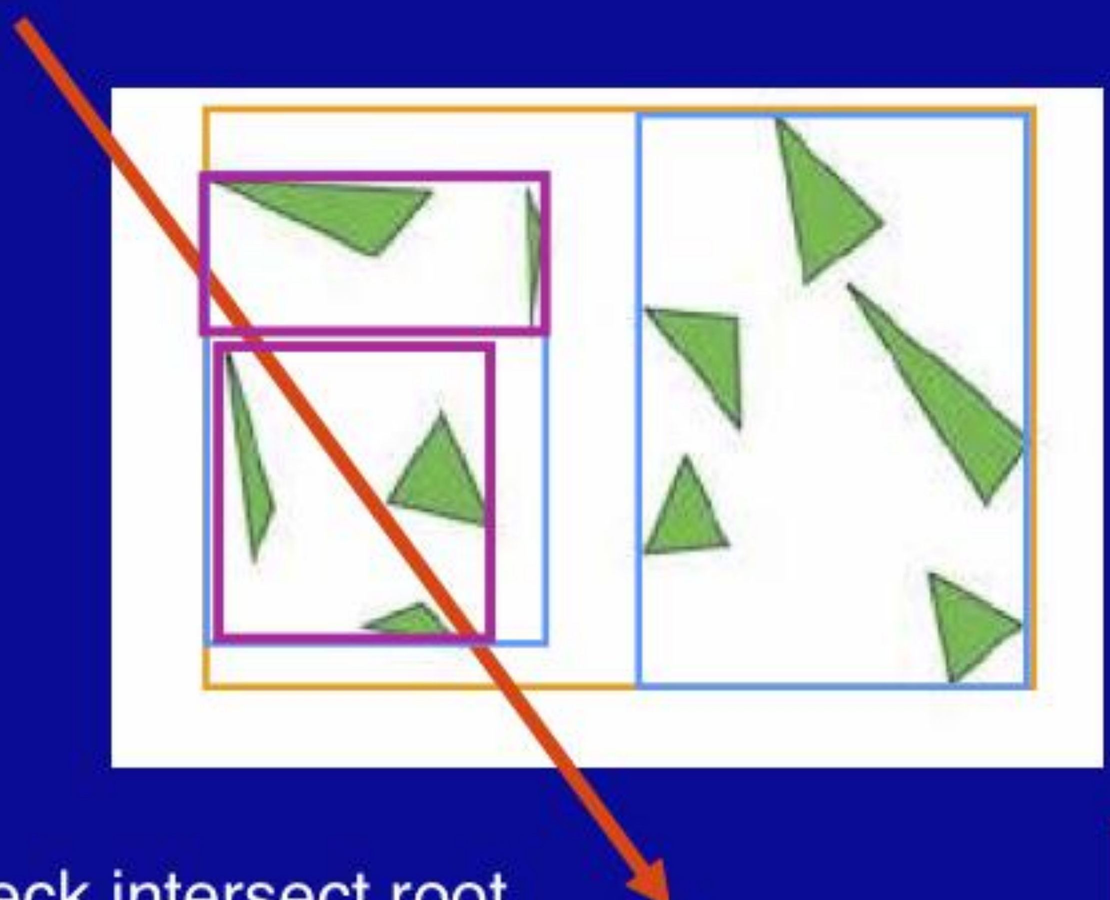

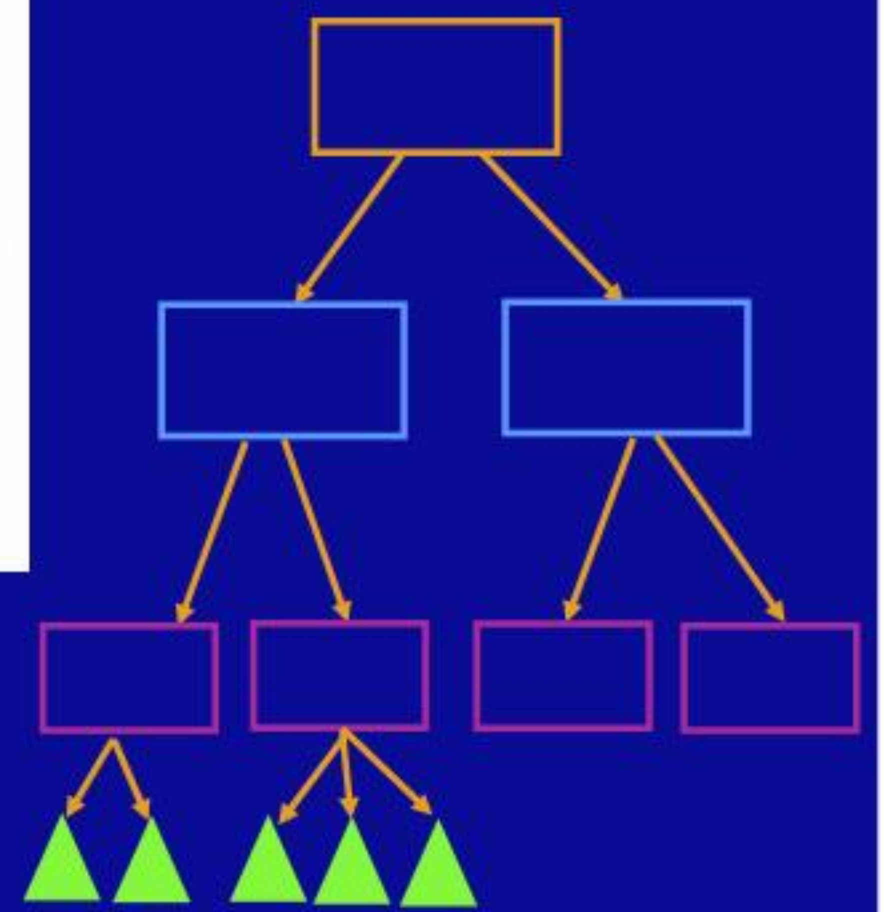

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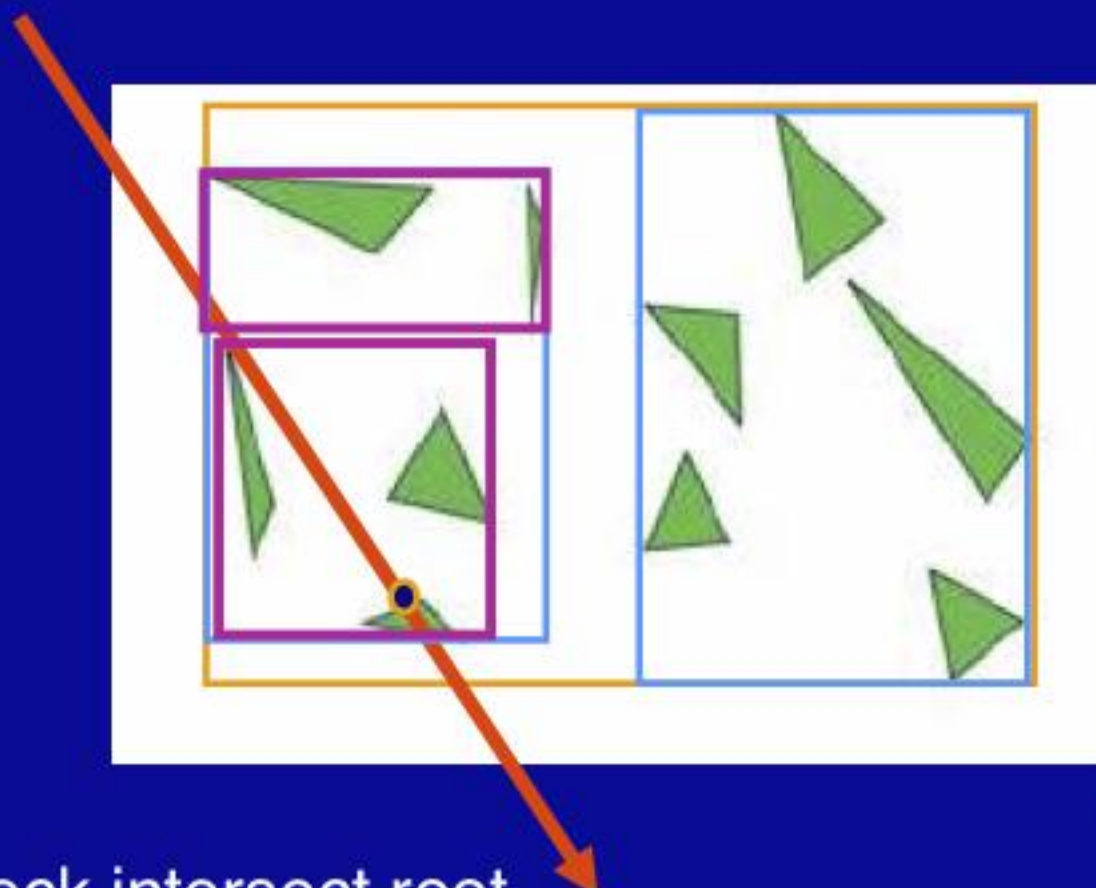

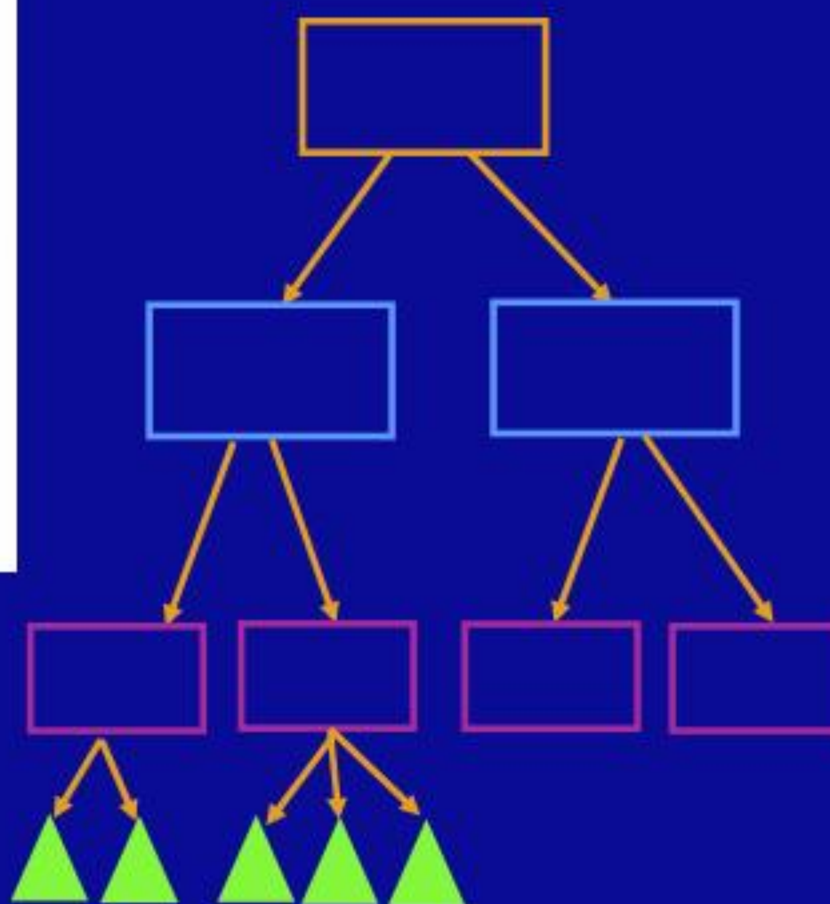

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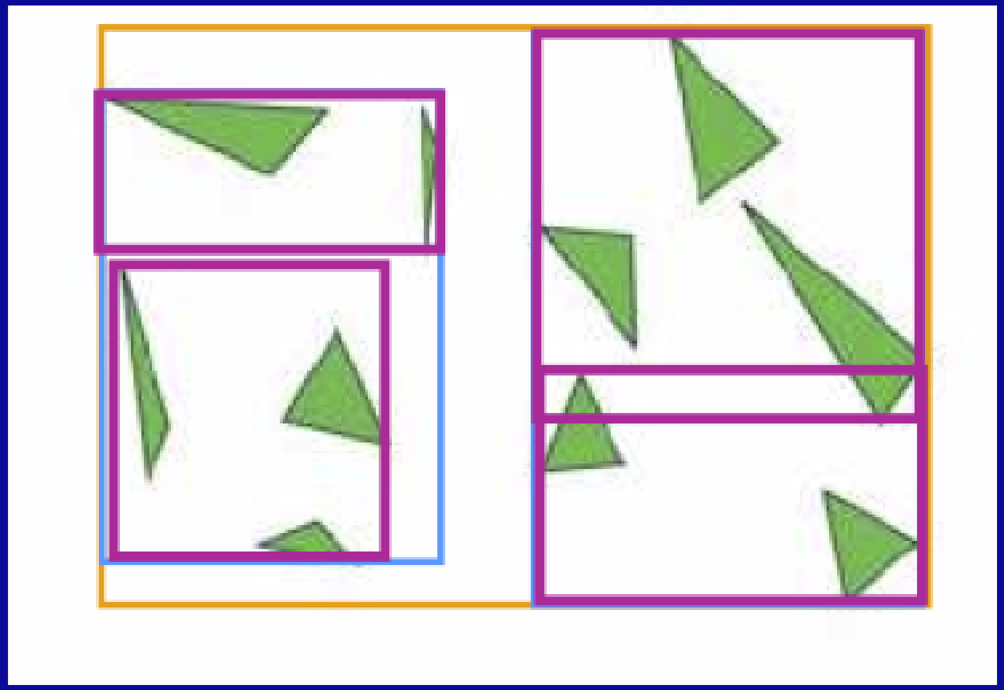
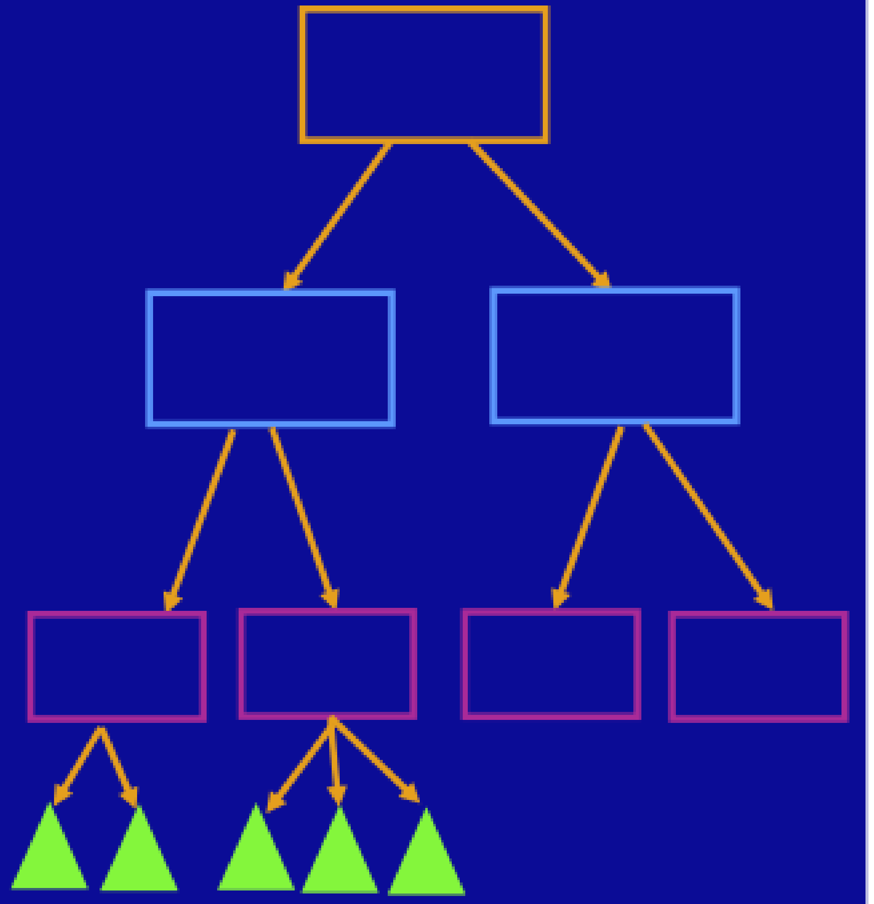
Hierarchical Bounding Volumes

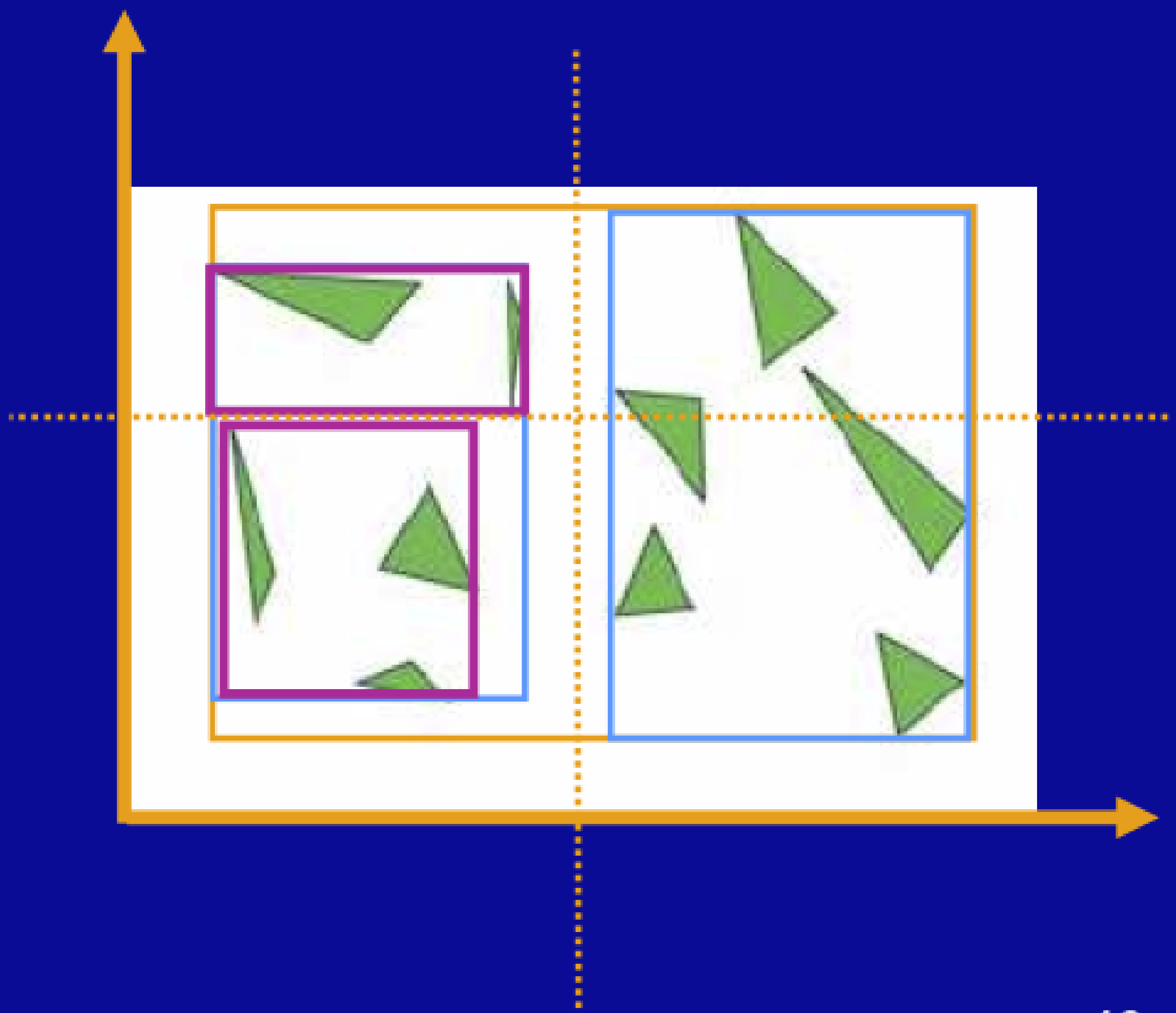
- Many ways to build a tree for the hierarchy
- Works well:
  - Binary
  - Roughly balanced
  - Boxes of sibling trees not overlap too much





Hierarchical Bounding Volumes

- Sort the surfaces along the axis before dividing into two boxes
- Carefully choose axis each time
- Choose axis that minimizes sum of volumes



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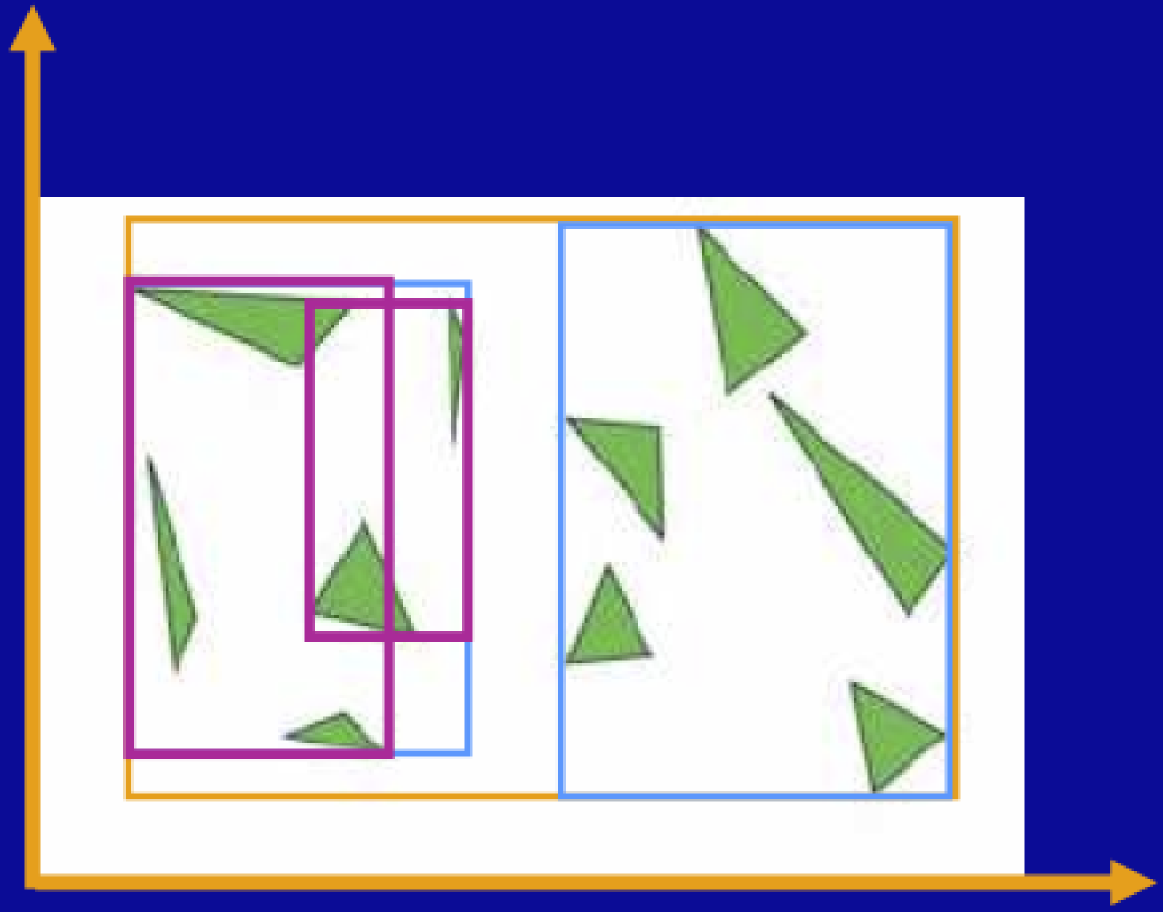


Hierarchical Bounding Volumes

- Works well if you use good (appropriate) bounding volumes and hierarchy
- Should give O(log n) rather than O(n) complexity (n=# of objects)
- Can have multiple classes of bounding volumes and pick the best for each enclosed object

Hierarchical bounding volumes

Spatial Subdivision

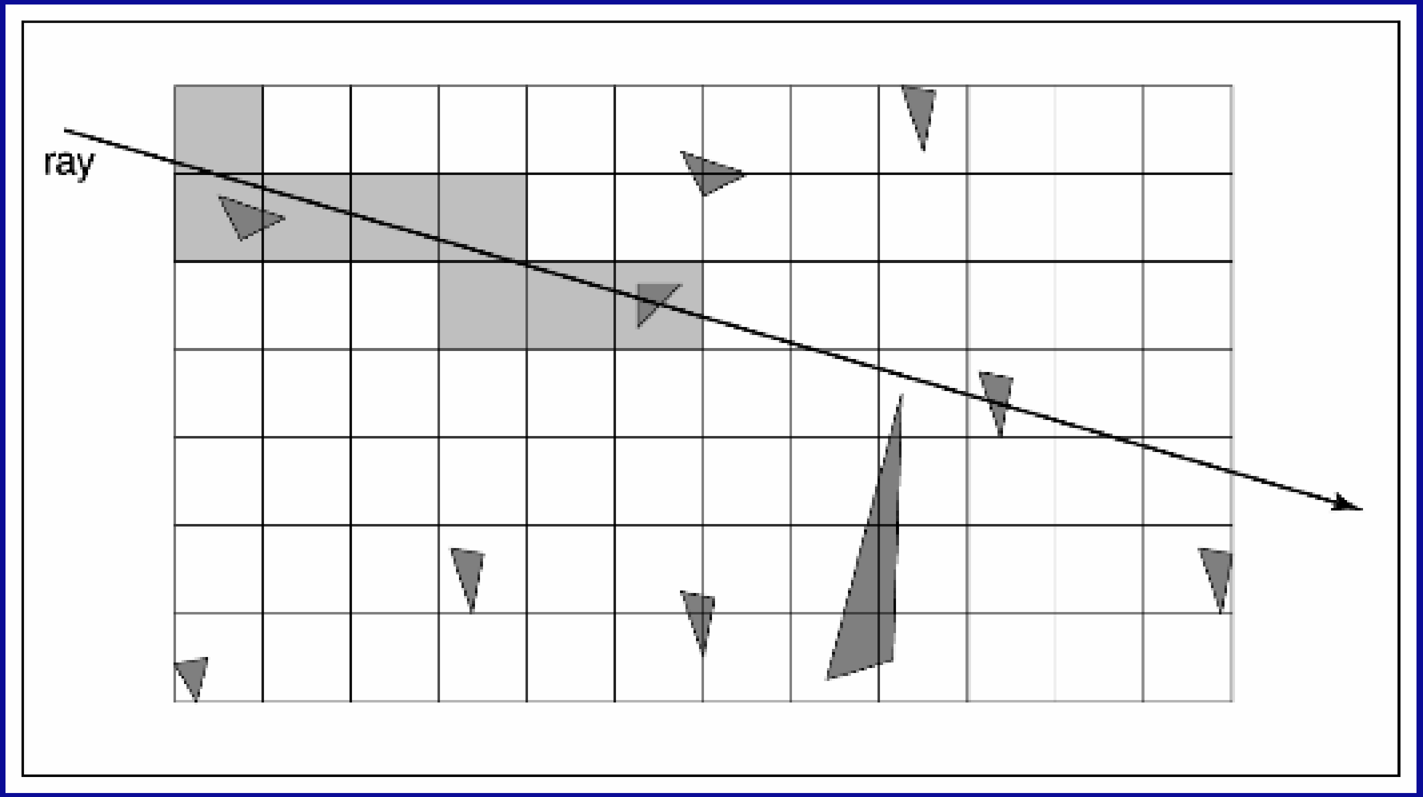
- Grids
- Octrees
- K-d trees and BSP trees

3D Spatial Subdivision

- Bounding volumes enclose the objects (object-centric)
- Instead could divide up the space—the further an object is from the ray the less time we want to spend checking it
  - Grids
  - Octrees
  - K-d trees and BSP trees

Grids

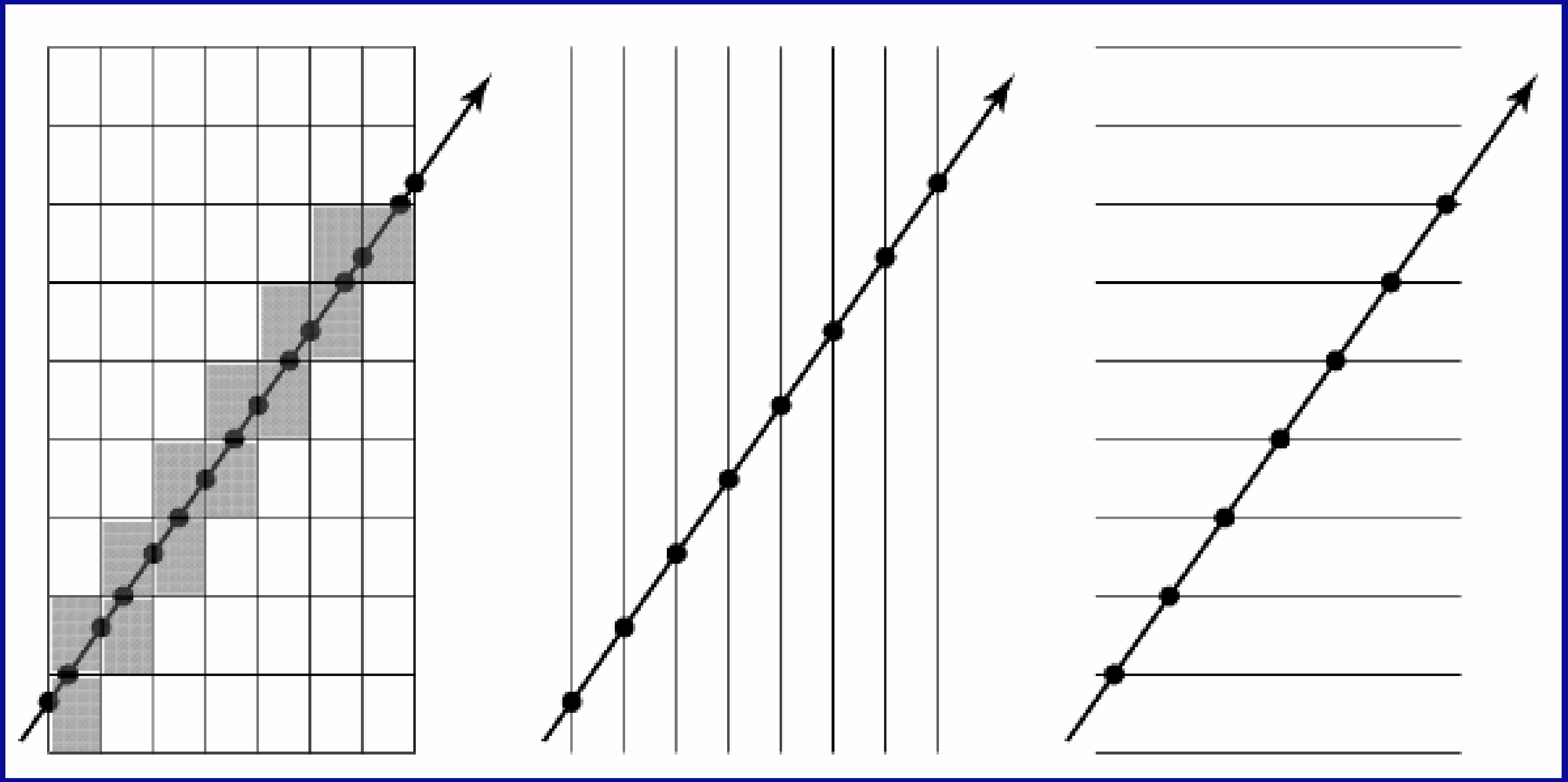
- Data structure: a 3-D array of cells (voxels) that tile space
  - Each cell points to list of all surfaces intersecting that cell



- Intersection testing:
  - Start tracing at cell where ray begins
  - Step from cell to cell, searching for the first intersection point
  - At each cell, test for intersection with all surfaces pointed to by that cell
  - If there is an intersection, return the closest one

Grids

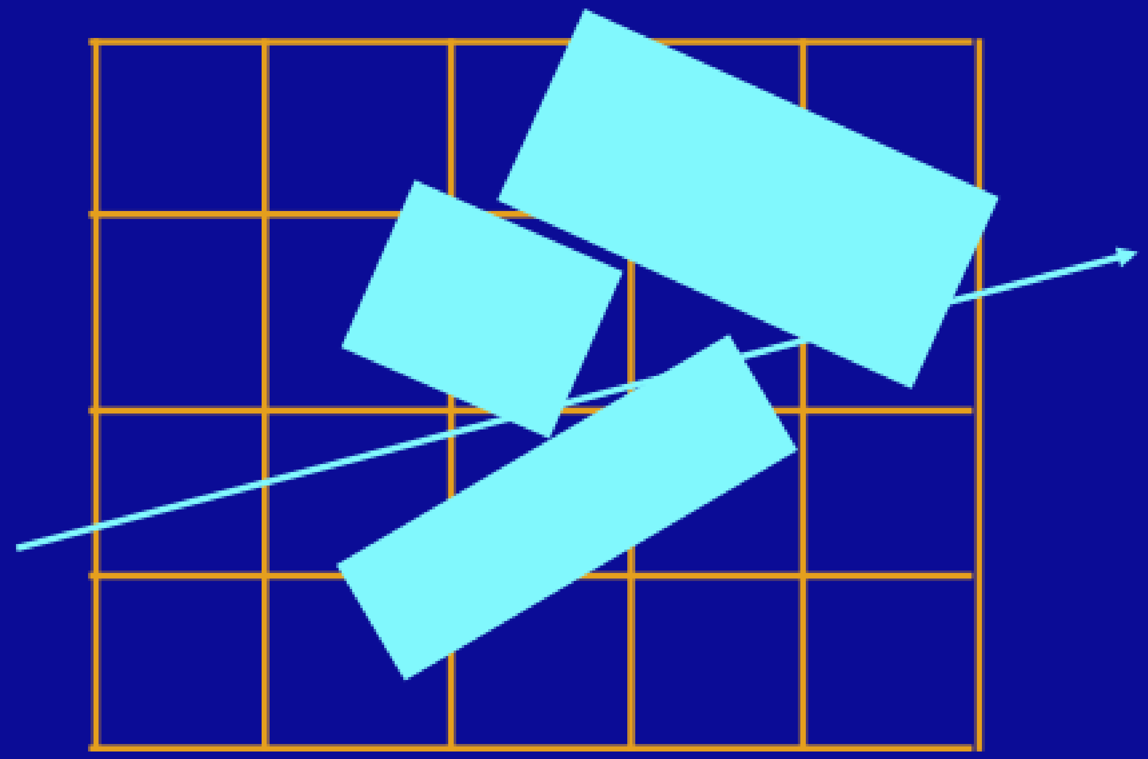
- Cells are traversed in an incremental fashion
- Hits of sets of parallel lines are very regular



More on Grids

- Be Careful! The fact that a ray passes through a cell and hits an object doesn't mean the ray hit that object in *that* cell

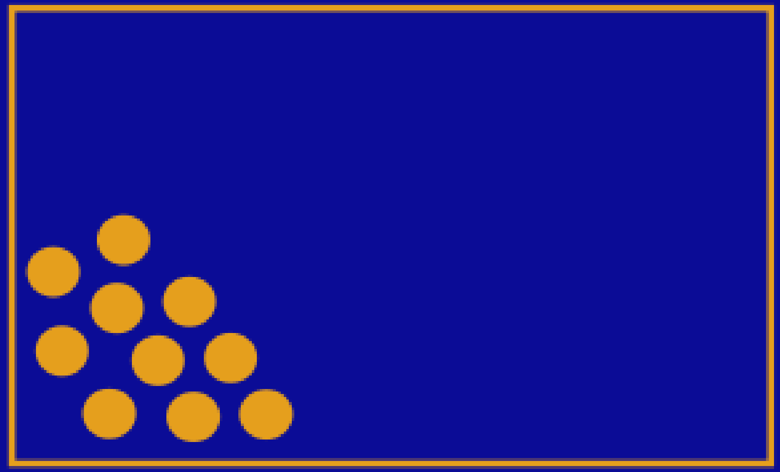
- Optimization: cache intersection point and ray id in "mailbox" associated with each object



- Step from cell to cell
- Get object intersecting cell
- Find closest intersection
- If found intersection --- done

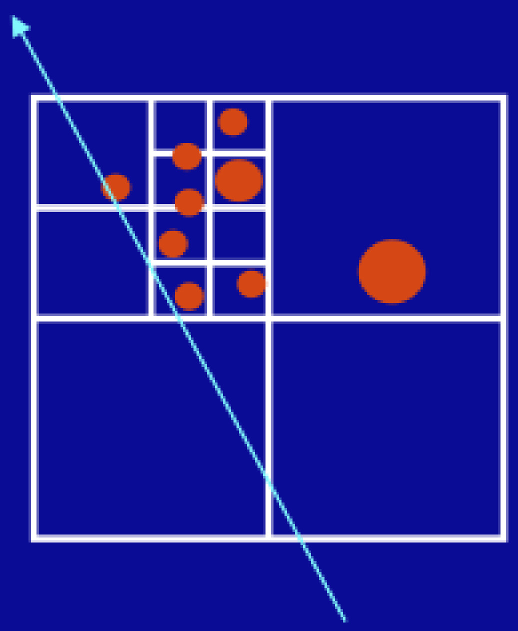
More on Grids

- Grids are a poor choice when the world is nonhomogeneous (clumpy)
  - many polygons clustered in a small space
- How many cells to use?
  - too few $\Rightarrow$ many objects per cell $\Rightarrow$ slow
  - too many $\Rightarrow$ many empty cells to step through $\Rightarrow$ slow
- Non-uniform spatial subdivision is better!



Octrees

- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”



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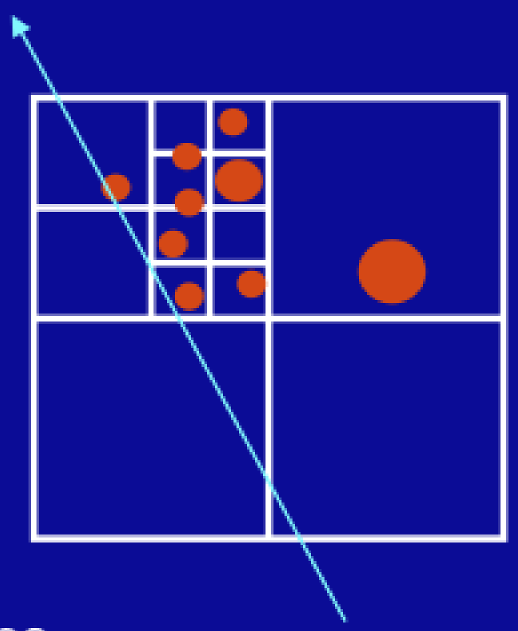


- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get "simple enough"
- Octree is the 3-D generalization of quadtree
  - node (cell) is a cube, recursively split into eight equal sub-cubes
  - for ray tracing:
    - stop subdivision based on number of objects
    - internal nodes store pointers to children, leaves store list of surfaces
  - more expensive to traverse than a grid
  - but an octree adapts to non-homogeneous scenes better

```
trace(cell, ray) {         // returns object hit or NONE
    if cell is leaf, return closest(objects_in_cell(cell))
    for child cells pierced by ray, in order     // 1 to 4 of these
        obj = trace(child, ray)
        if obj!=NONE return obj
    return NONE
}
```

Which Data Structure is Best for Ray Tracing?

Grids

- Easy to implement
- Require a lot of memory
- Poor results for inhomogeneous scenes

Octrees

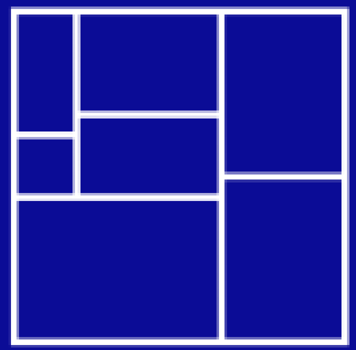
- Better on most scenes (more adaptive)

Spatial subdivision expensive for animations

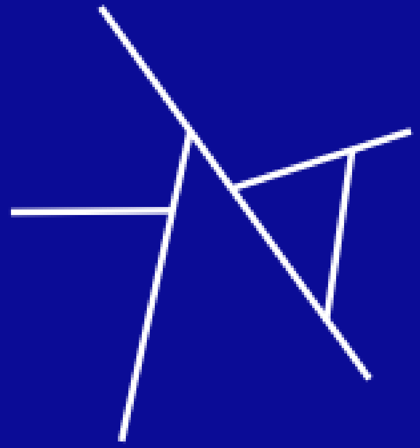
- Hierarchical bounding volumes
- Better for dynamic scenes
- Natural for hierarchical objects

k-d Trees and BSP Trees

- Relax the rules for quadtrees and octrees:



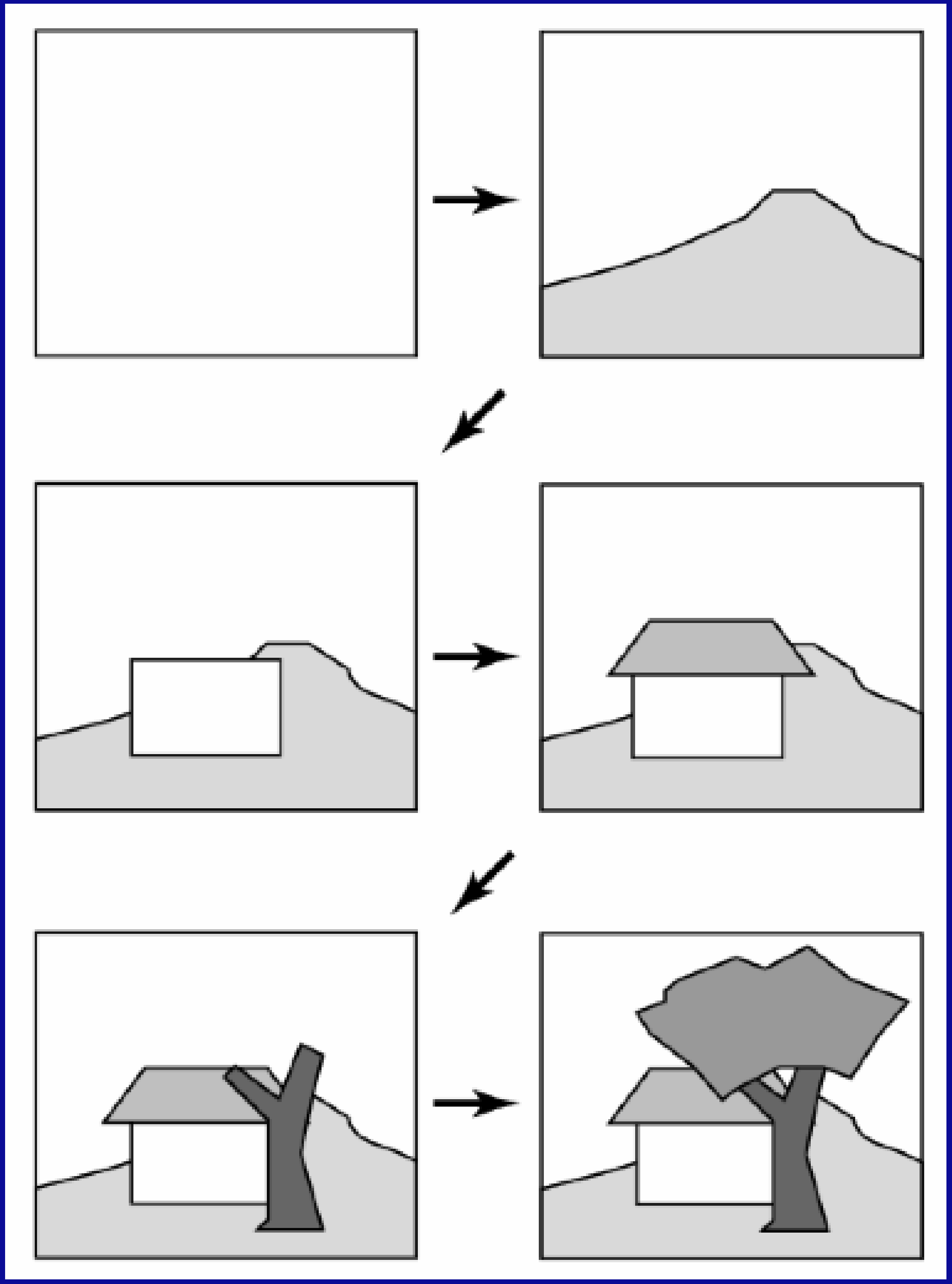
- k-dimensional (k-d) tree
  - don't always split at midpoint
  - split only one dimension at a time (i.e. *x* or *y* or *z*)
- binary space partitioning (BSP) tree
  - permit splits with any line
  - In 2-D space split with lines (most of our examples)
  - 3-D space split with planes
  - K-D space split with *k*-1 dimensional hyperplanes



- useful for Painter's algorithm (hidden surface removal)

Painters Algorithm

Hidden Surface Elimination

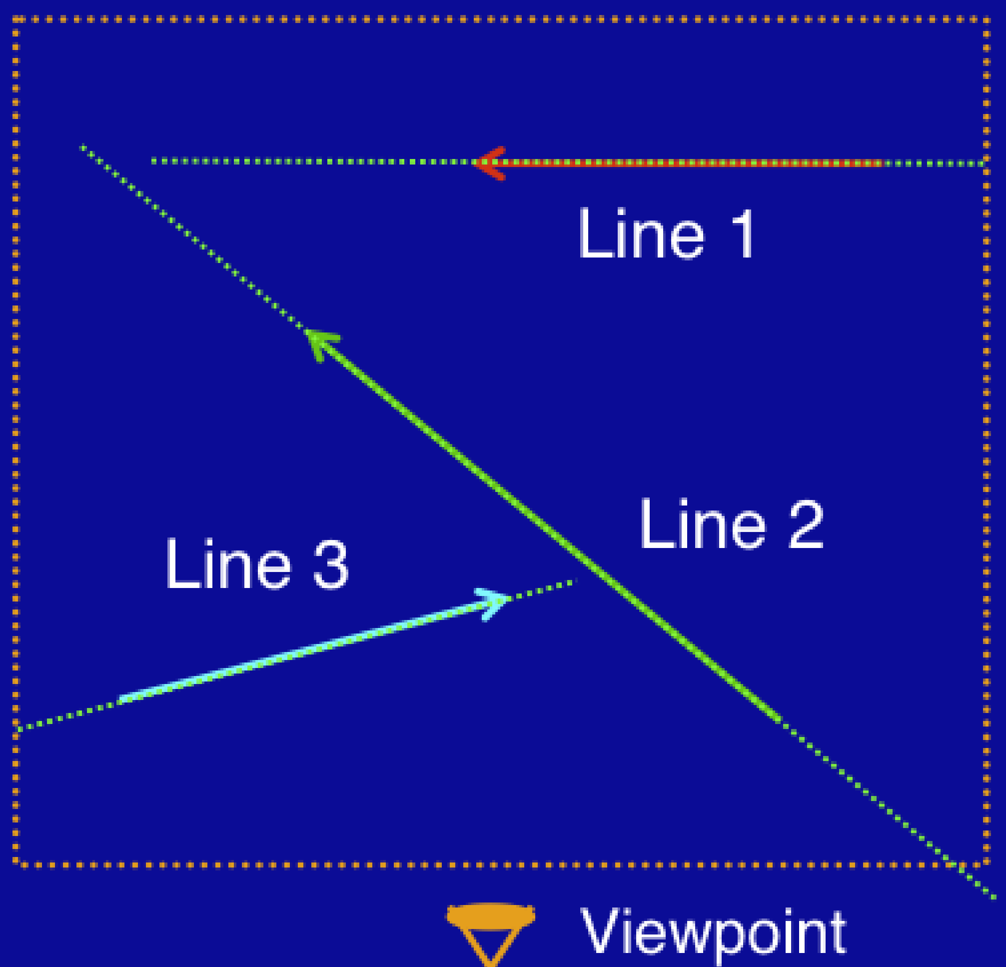


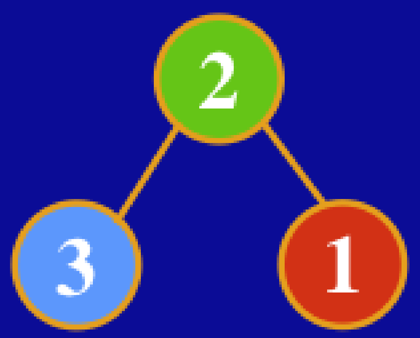
Painters Algorithm

- Need to sort objects back to front
- Order depends on the view point
- Partition objects using BSP tree
- View independent

Building a BSP Tree

- Let's look at simple example with 3 line segments
- Arrowheads are to show left and right sides of lines.
- Using line 1 or 2 as root is easy.
- (examples from http://www.geocities.com/SiliconValley/2151/bsp.html)

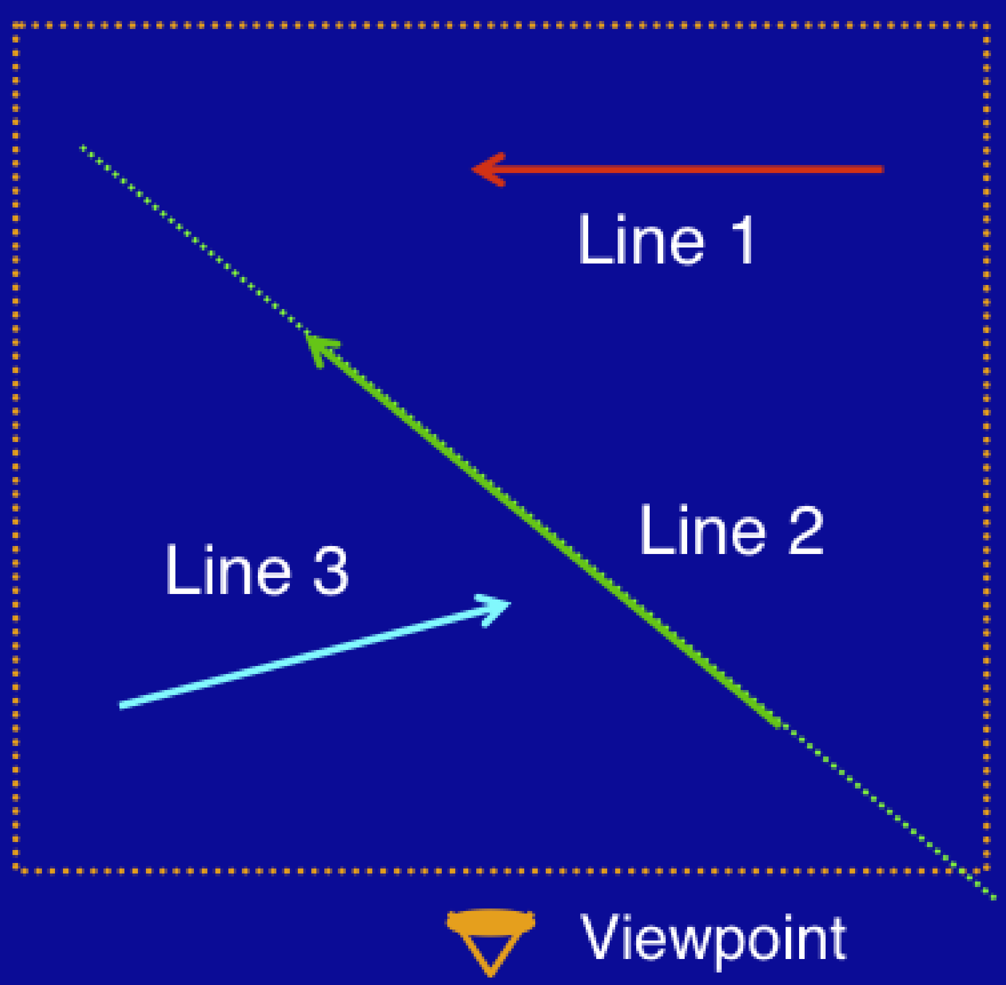


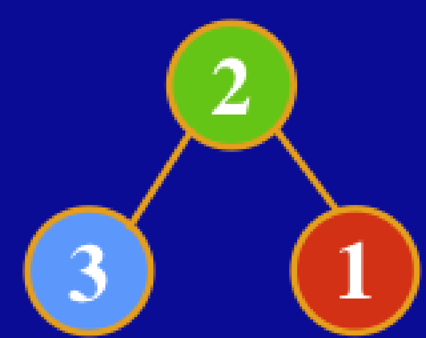


a BSP tree using 2 as root

Drawing Objects

- Traverse the tree from the root
- If view point is on the left of the line --- traverse right sub-tree first
- Draw the root
- Traverse left sub-tree





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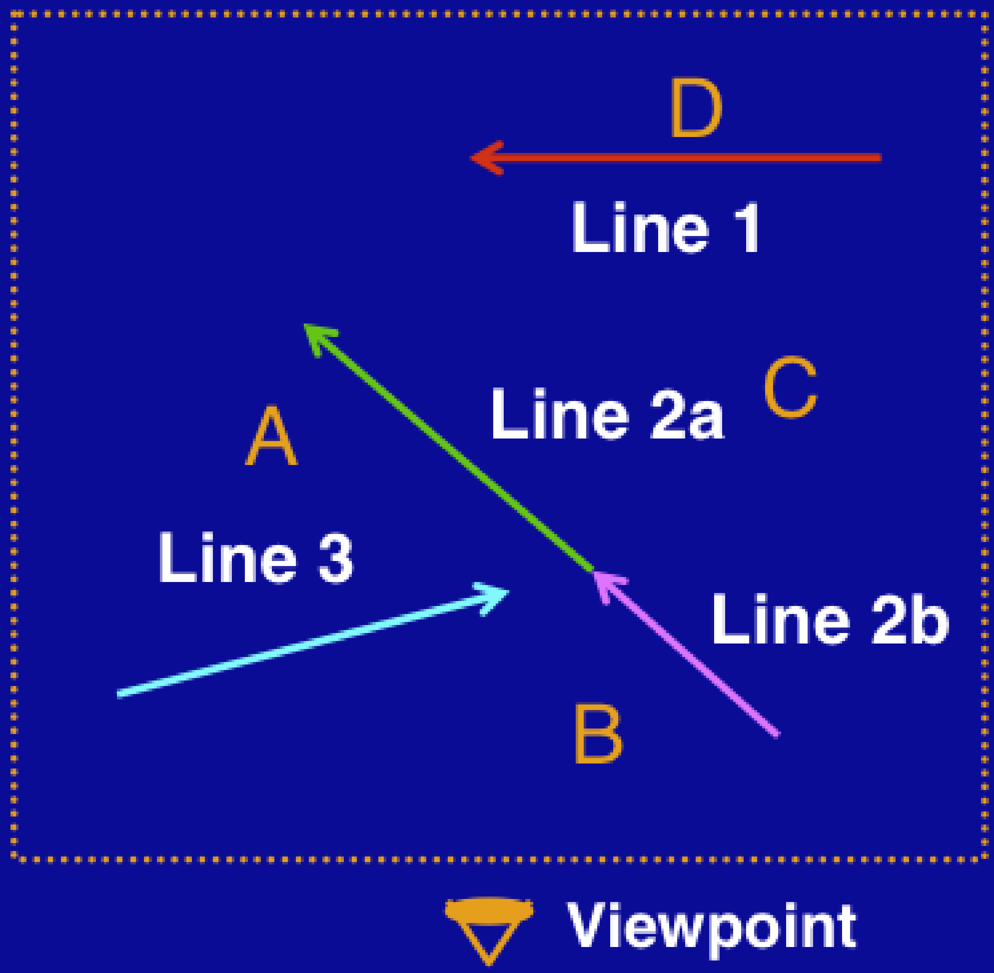
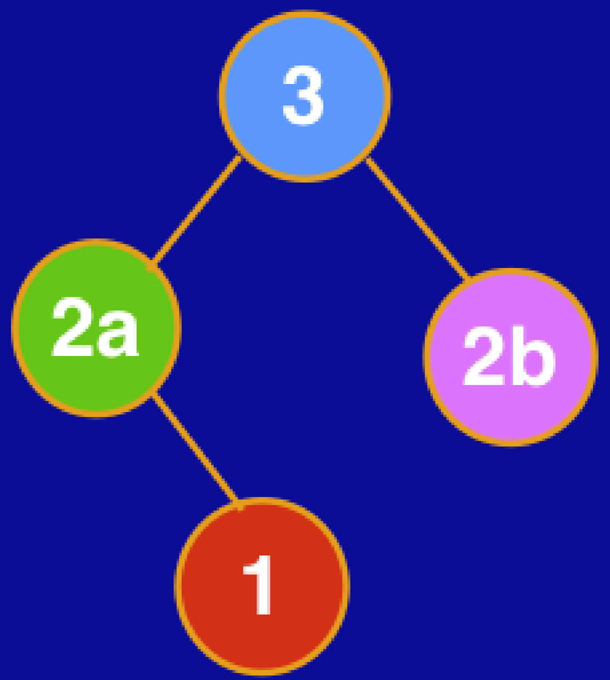
Building the Tree 2

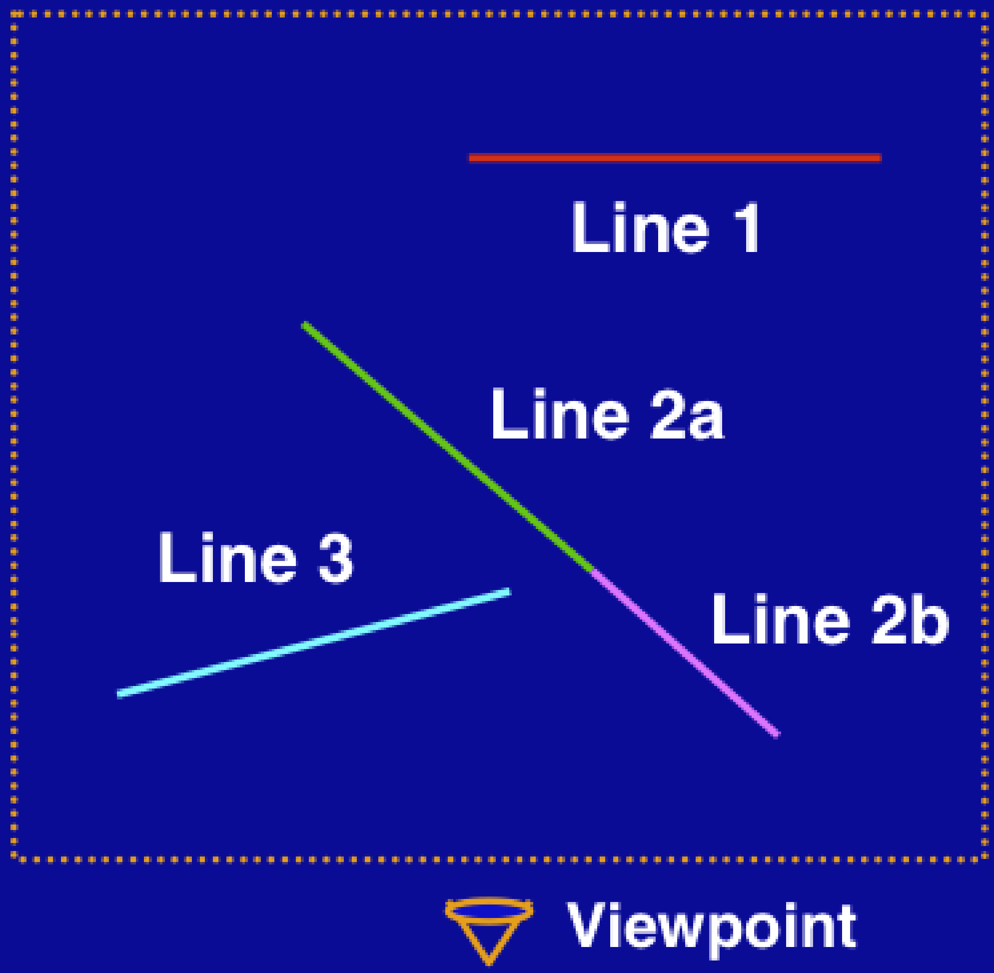
Using line 3 for the root requires a split





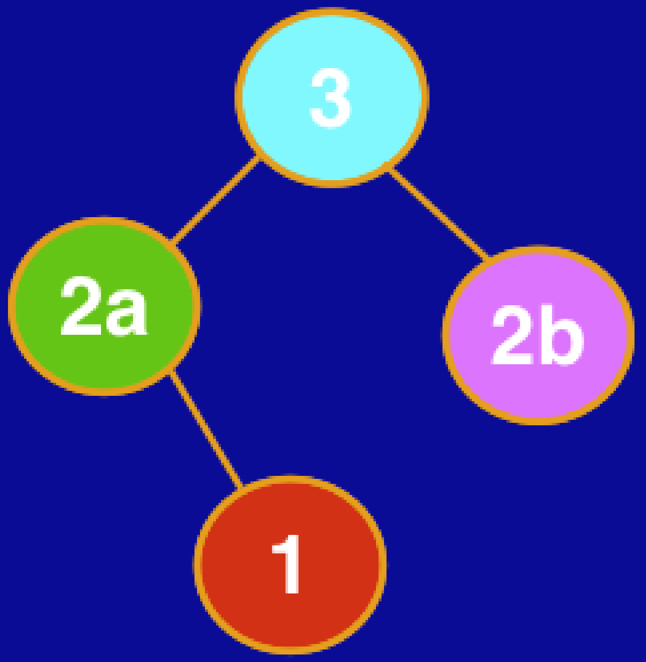
Drawing Back to Front

- Use Painter's Algorithm for hidden surface removal



Steps:

- *Draw objects on far side of line 3*
  - » *Draw objects on far side of line 2a*
    - Draw line 1
  - » Draw line 2a
- Draw line 3
- *Draw objects on near side of line 3*
  - » Draw line 2b

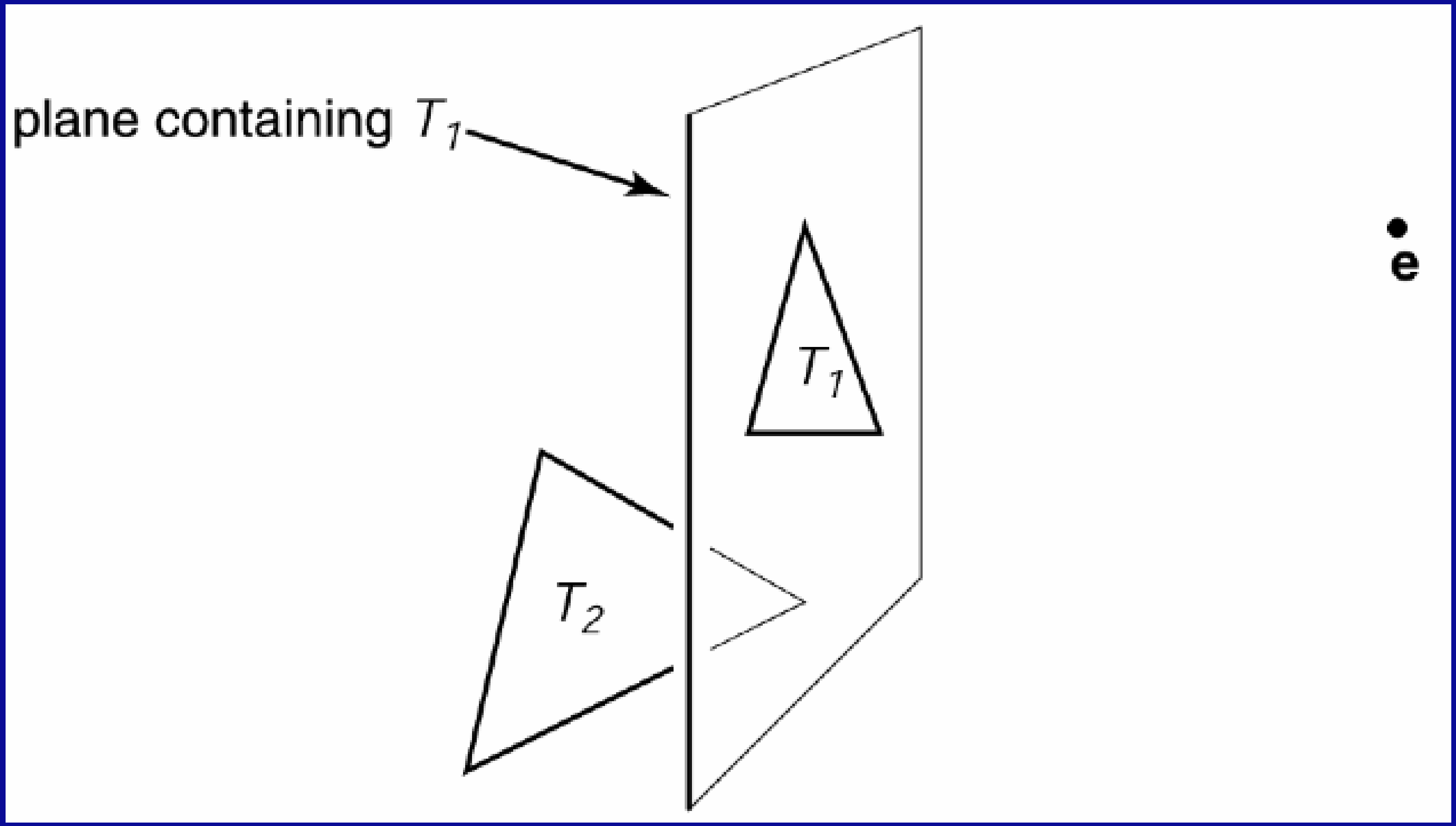


Triangles

Use plane containing triangle T_1 to split the space

If view point is on one side of the plane draw polygons on the other side first

T_2 does not intersect plane of T_1



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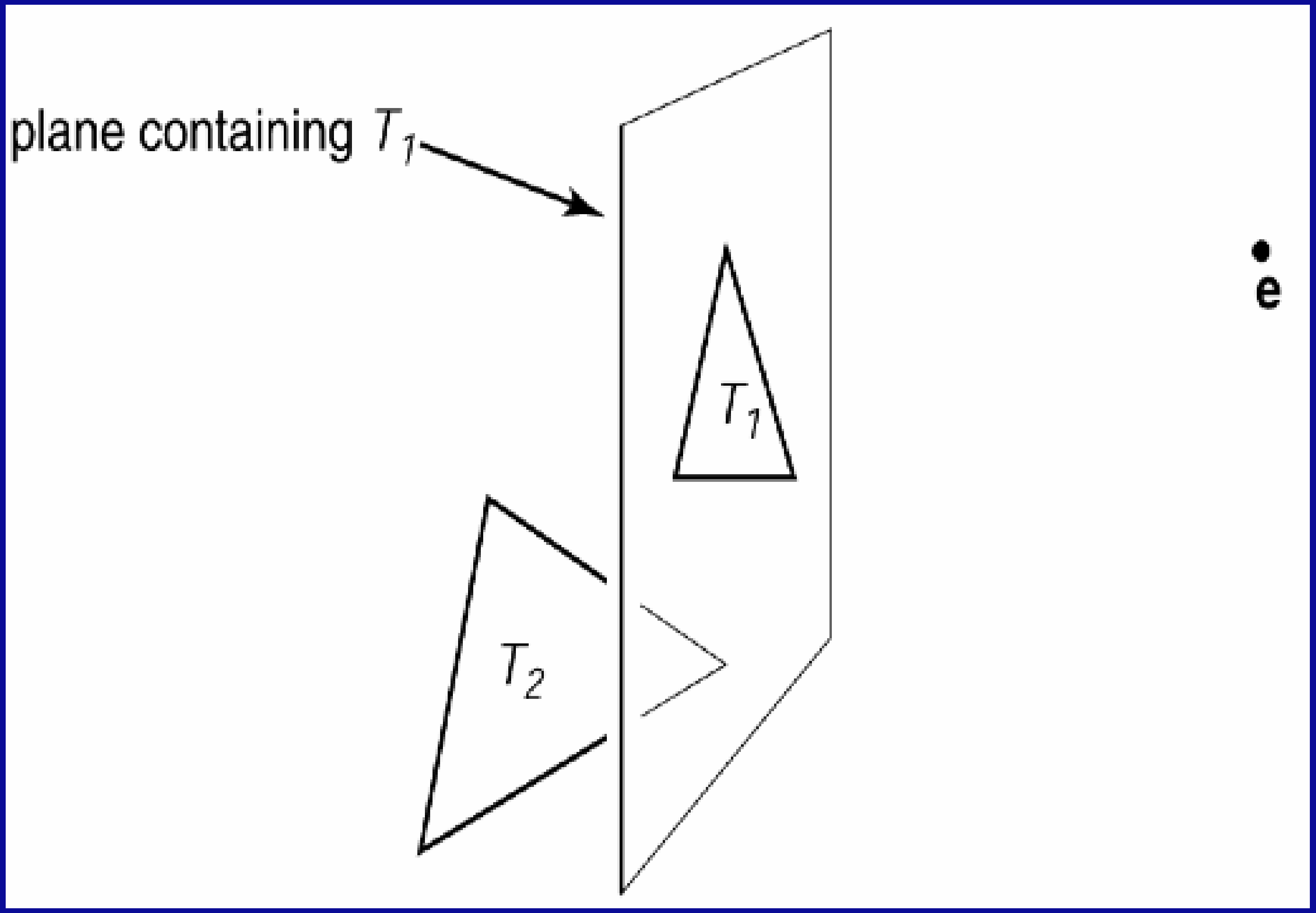
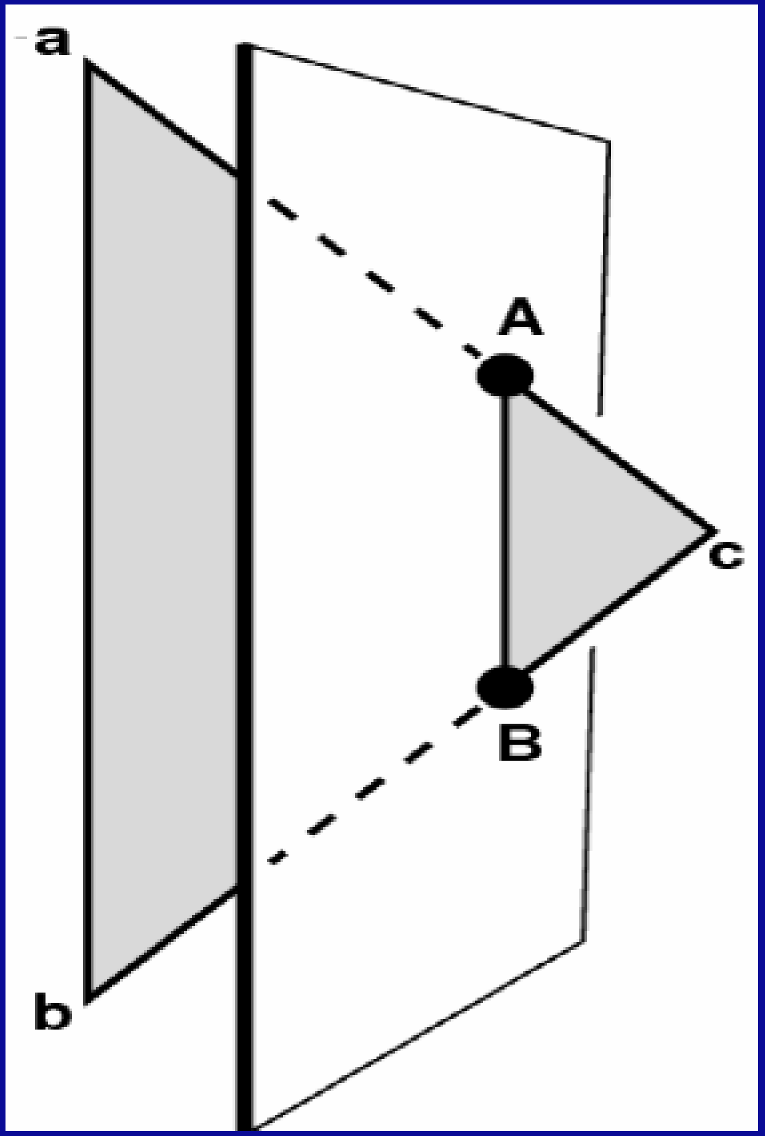
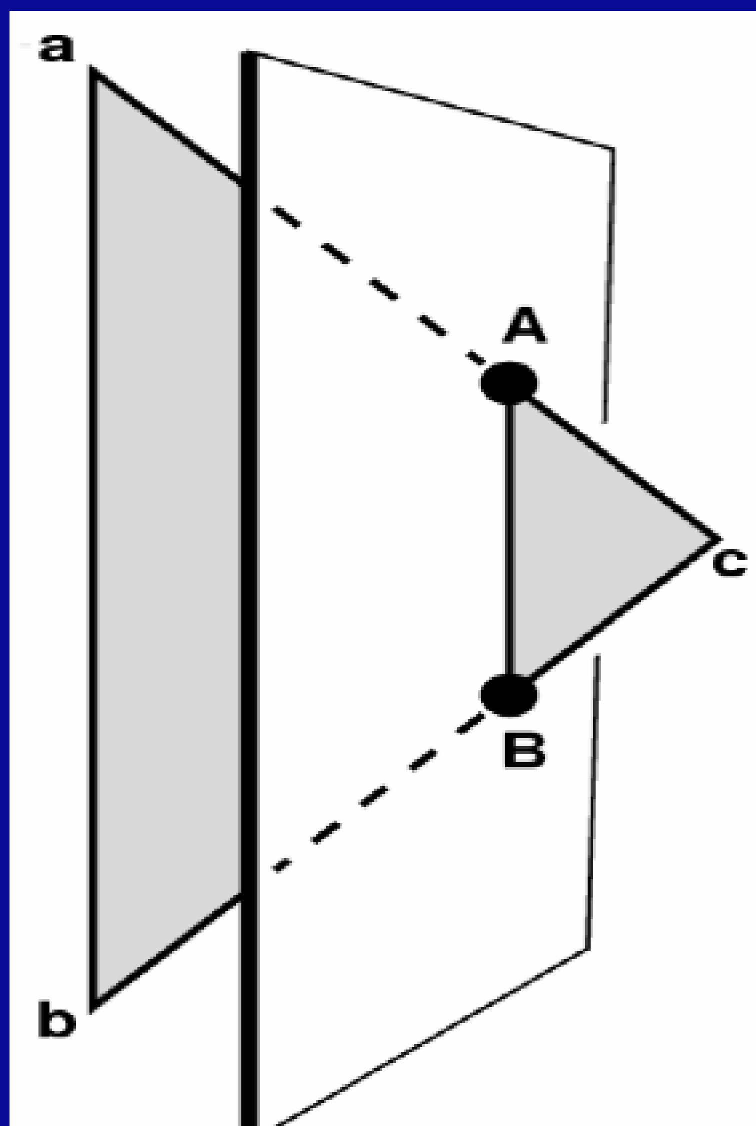
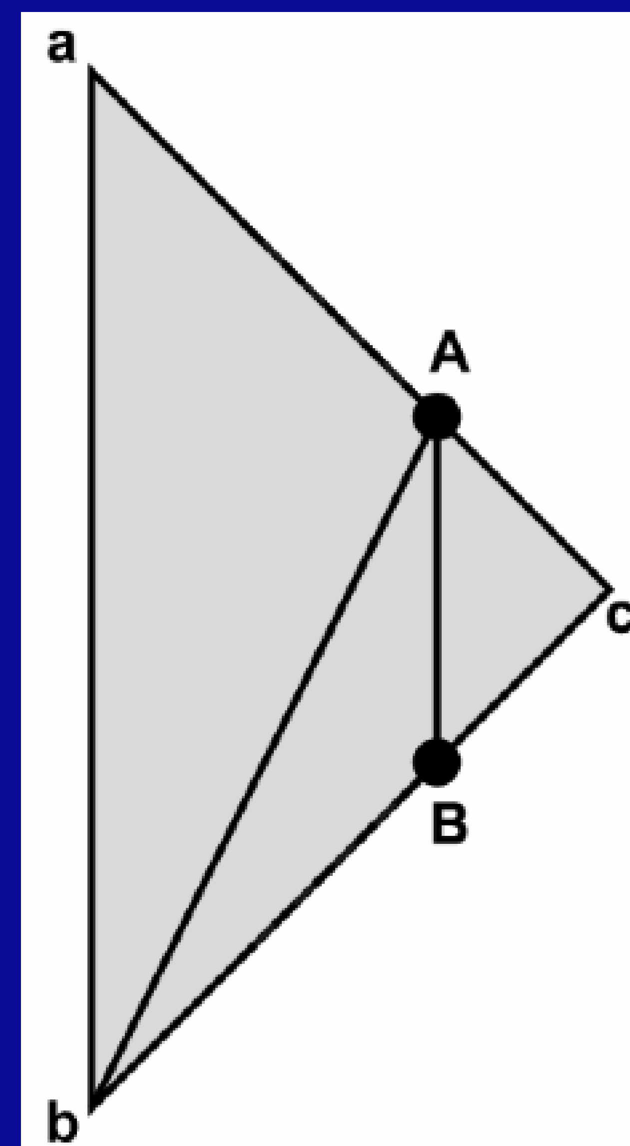




Triangles

Split Triangle





Painter’s Algorithm with BSP trees

- Build the tree
  - Involves splitting some polygons
  - Slow, but done only once for static scene
- Correct traversal lets you draw in back-to-front or front-to-back order for any viewpoint
  - Order is view-dependent
  - Pre-compute tree once
  - Do the “sort” on the fly
- Will not work for changing scenes

Drawing a BSP Tree

- Each polygon has a set of coefficients:

  $Ax + By + Cz + D$
- Plug the coordinates of the viewpoint in and see:

  >0 : front side

  <0 : back facing

  =0 : on plane of polygon
- Back-to-front draw: inorder traversal, do farther child first
- Front-to-back draw: inorder traversal, do near child first

```
front_to_back(tree, viewpt) {
   if (tree == null)  return;
   if (positive_side_of(root(tree), viewpt)) {
      front_to_back(positive_branch(tree, viewpt);
      display_polygon(root(tree));
      front_to_back(negative_branch(tree, viewpt);
   }
   else { …draw negative branch first…}
}
```

Building a Good Tree - the tricky part

- A naïve partitioning of *n* polygons will yield $O(n^3)$ polygons because of splitting!

- Algorithms exist to find partitionings that produce $O(n^2)$.
  - For example, try all remaining polygons and add the one which causes the fewest splits
  - Fewer splits -> larger polygons -> better polygon fill efficiency

- Also, we want a balanced tree.

Demos

BSP Tree construction

http://symbolcraft.com/graphics/bsp/index.html

- KD Tree construction

http://www.cs.umd.edu/~brabec/quadtree/index.html

Real-time and Interactive Ray Tracing

The OpenRT Real-Time Ray-Tracing Project
http://www.openrt.de/index.php

- Interactive ray tracing via space subdivision
http://www.cs.utah.edu/~reinhard/egwr/

- Interactive ray tracing with good hardware
http://www.cs.utah.edu/vissim/projects/raytracing/