Image Processing
Overview

Images

Pixel Filters

Neighborhood Filters

Dithering
We can think of an **image** as a function, \( f \),
- \( f: \mathbb{R}^2 \rightarrow \mathbb{R} \)
  - \( f(x, y) \) gives the **intensity** at position \((x, y)\)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - \( f: [a, b] \times [c, d] \rightarrow [0, 1] \)

A color image is just three functions pasted together. We can write this as a “vector-valued” function:

\[
    f(x, y) = \begin{bmatrix}
        r(x, y) \\
        g(x, y) \\
        b(x, y)
    \end{bmatrix}
\]
Image as a Function
Image Processing

• Define a new image $g$ in terms of an existing image $f$
  – We can transform either the domain or the range of $f$

• Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?
Some operations preserve the range but change the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

Still other operations operate on both the domain and the range of $f$.
Point Operations
Point Processing

- Original
- Darken
- Lower Contrast
- Nonlinear Lower Contrast
- Invert
- Lighten
- Raise Contrast
- Nonlinear Raise Contrast
Point Processing

Original: $x$

Darken: $x - 128$

Lower Contrast: $x / 2$

Nonlinear Lower Contrast: $(x / 255.0)^{0.33} * 255.0$

Invert: $255 - x$

Lighten: $x + 128$

Raise Contrast: $x * 2$

Nonlinear Raise Contrast: $(x / 255.0)^2 * 255.0$
Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function. Send $v \rightarrow$ actually display a pixel which has intensity equal to $v^{2.5}$.

$\gamma = 1.0; f(v) = v$

$\gamma = 2.5; f(v) = v^{1/2.5} = v^{0.4}$
Neighborhood Operations
Convolution

\[ F = \begin{bmatrix}
0.2 & 0.1 & -1.0 \\
0.3 & 0.0 & 0.9 \\
0.1 & 0.3 & -1.0
\end{bmatrix} \quad I' = F \ast I \]
Linear Shift Invariant Systems (LSIS)

Linearity:

\[ f_1 \rightarrow g_1 \quad f_2 \rightarrow g_2 \]

\[ \alpha f_1 + \beta f_2 \rightarrow \alpha g_1 + \beta g_2 \]

Shift invariance:

\[ f(x - a) \rightarrow g(x - a) \]
Convoluition

LSIS is doing convolution; convolution is linear and shift invariant

\[ g(x) = \int_{-\infty}^{\infty} f(\tau)h(x - \tau)\,d\tau \quad g = f \ast h \]
Convolution - Example
Convolution - Example

\[ c = a * b \]
Properties of Convolution

• Commutative

\[ a \ast b = b \ast a \]

• Associative

\[ (a \ast b) \ast c = a \ast (b \ast c) \]

• Cascade system

\[
\begin{align*}
  f & \rightarrow h_1 & \rightarrow h_2 & \rightarrow g \\
  = f & \rightarrow h_1 \ast h_2 & \rightarrow g \\
  = f & \rightarrow h_2 \ast h_1 & \rightarrow g
\end{align*}
\]
Point Spread Function

- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.

\[ \delta(x) \rightarrow \text{Optical System} \rightarrow \text{PSF}(x) \]

- Point spread function of Human Eyes
Point Spread Function

normal vision  myopia  hyperopia

astigmatism

Images by Richmond Eye Associates
Gaussian Smoothing

by Charles Allen Gillbert

by Harmon & Julesz

http://www.michaelbach.de/ot/cog_blureffects
Gaussian Smoothing

http://www.michaelbach.de/ot/cog_blureffects
Sharpened Image
Noise
Blurred Noise
Median Filter

- **Smoothing is averaging**
  
  (a) Blurs edges
  
  (b) Sensitive to outliers

- **Median filtering**
  
  - Sort $N^2 - 1$ values around the pixel
  
  - Select middle value (median)

  - Non-linear (Cannot be implemented with convolution)
Median Filter

Can this be described as a convolution?
Original Image
Example: Noise Reduction

Image with noise  Median filter (5x5)
Salt and pepper noise

Gaussian

3x3

5x5

7x7

Gaussian

Median

Gaussian

Median
Example: Noise Reduction

Original image

Image with noise

Median filter (5x5)
X-Edge Detection
Y-Edge Detection
Edge detection filters

Roberts (2 x 2):

Sobel (3 x 3):

Sobel (5 x 5):
General Edge Detection

Can this be described as a convolution?
• Some operations preserve the range but change the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

• Still other operations operate on both the domain and the range of $f$. 
Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?
Image Sub-Sampling

Throw away every other row and column to create a 1/2 size image - called image sub-sampling
Image Sub-Sampling

1/2  1/4 (2x zoom)  1/8 (4x zoom)
Good and Bad Sampling

Good sampling:
• Sample often or,
• Sample wisely

Bad sampling:
• see aliasing in action!
Aliasing
Alias: n., an assumed name

Picket fence receding into the distance will produce aliasing...

WHY?

Input signal:

Matlab output:

\[ x = 0:.05:5; \text{ imagesc}(\sin((2.^x).*x)) \]

Not enough samples
Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Sub-Sampling with Gaussian Pre-Filtering

• Solution: filter the image, *then* subsample
  – Filter size should double for each $\frac{1}{2}$ size reduction. Why?
Sub-Sampling with Gaussian Pre-Filtering

Gaussian 1/2  G 1/4  G 1/8
Chapter 1: Introduction

1 Introduction

This manual corresponds to version 3.2 of the library.

The library’s fundamental purpose is to provide a user-friendly interface, similar to what shells do when looking up commands.

The following software, all of which we maintain:

- Dviljk (see the ‘dvilj’ man page)
- Dvipsk (see section “Introduction” in Dvipsk’s man page)
- GNU font utilities (see section “Introduction” in GNU’s documentation)
- Web2c (see section “Introduction” in Web2c’s documentation)
- Xdvik (see the ‘xdvi’ man page)

Other software that we do not maintain also use aliasing:

- Dvips (see section “Introduction” in Dvips’s documentation)
- gv (see section “Introduction” in gv’s documentation)
- Xpdf (see section “Introduction” in Xpdf’s documentation)

Other software that we do not maintain also use aliasing:

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- Xpdf (see section “Introduction” in Xpdf’s documentation)
Canon D60 (w/ anti-alias filter)

Sigma SD9 (w/o anti-alias filter)

From Rick Matthews website, images by Dave Etchells
Warped Image

orig + vector field = warped

how?
Advection (just like a fluid)