15-462: Computer Graphics
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Subdivision and Project 2

Outline

- Subdivision
  - What is it?
  - What properties does it have / do we want?
  - What kinds of algorithms exist and what advantages do they have?
- Project 2
  - Texture mapping crash course
  - Loop subdivision algorithm

What is subdivision?

- Start with a given polygon mesh
- Apply refinement scheme to get an increasingly smooth surface by taking in the mesh and subdividing it to create new vertices and faces
- The limit of this subdivision is a smooth surface, though in practice we can’t apply it this many times
  - Caveat – provided we don’t define creases and boundaries

What properties might we want?

- Efficiency
  - use a small number of floating point operations
- Local definition
  - don’t look very far away from current point
- Simplicity
  - We probably do not want a ton of rules
- Continuity
  - What kind of properties can we prove about the resulting surface?

What properties might we want?

- Efficiency & Local Definition
  - Subdivision is efficient because only several neighboring points are used in the computation of new points
  - By contrast, rendering a surface defined by an implicit equation is expensive, requiring an algorithm such as marching cubes

Quick terminology

- Ordinary vertices
  - For triangular meshes, vertices of valence 6 on the interior and valence 4 on the boundaries
  - For quadrilateral meshes, vertices of valence 4 on the interior and valence 3 on the boundaries
- Extraordinary vertices
  - All other valences
- Odd and even vertices
  - Odd vertices are those that are added on the current step of the subdivision
  - Even vertices are those that are inherited from the previous level

Subdivisions Schemes

- In general, there is a fairly straightforward way to classify the subdivision schemes that exist
  - Type of refinement – face split or vertex split
  - Type of generated mesh – triangular or quadrilateral
  - Approximating vs. interpolating
  - Smoothness of the limit surface for regular meshes

Subdivision Schemes

- Face split vs. Vertex split

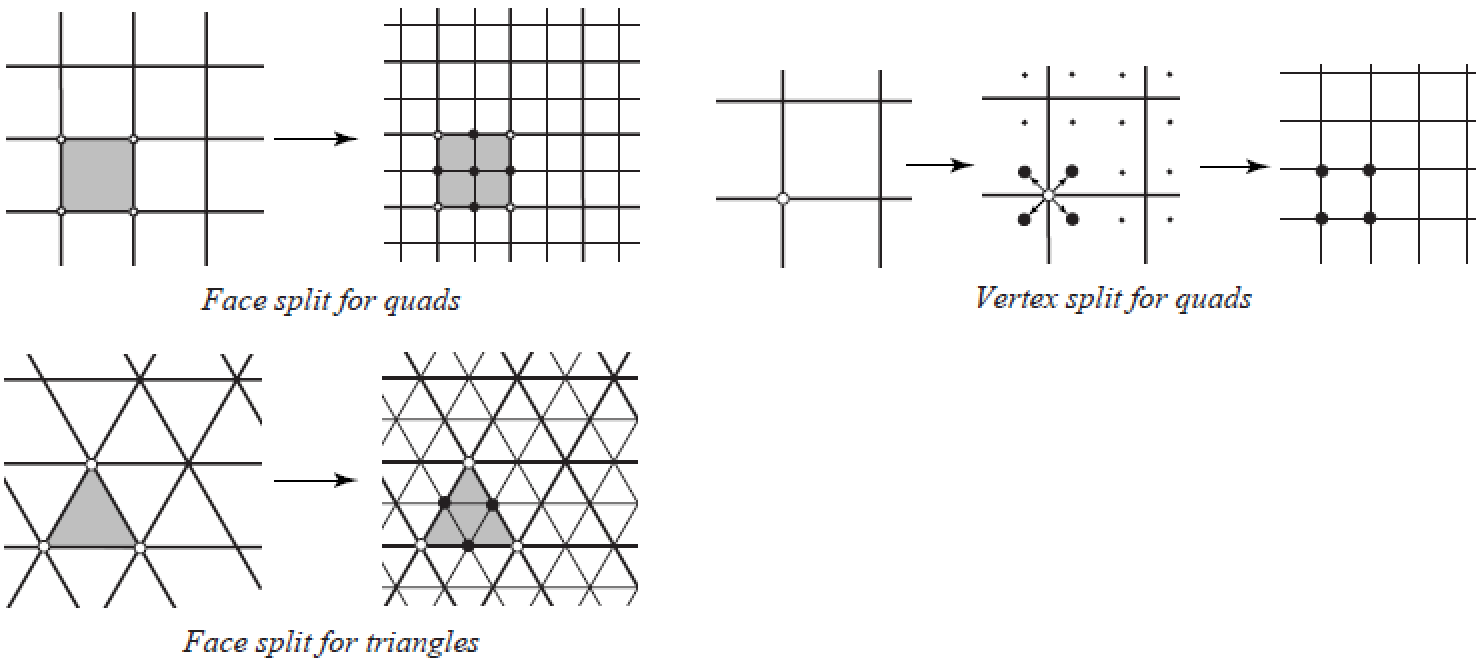


Figure 4.1: *Different refinement rules.*

Subdivision Schemes

- Approximation vs. Interpolation
  - Interpolation – original points remain the same
  - Approximation – original points not the same
  - Face splitting can be either since the vertices of the coarser tiling are also vertices in the refined tiling
  - Approximating generally produces smoother surfaces

Subdivision Schemes

Face Split		
	Triangular meshes	**Quadrilateral Meshes**
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Modified Butterfly (C^1)	Kobbelt (C^1)

Vertex Split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

Loop Scheme

- Face splitting, approximating scheme for triangular meshes proposed by Charles Loop.
- C^1 continuity for all valences and C^2 continuity over regular meshes
- Can be applied to polygon meshes after triangulating the mesh

Loop Scheme

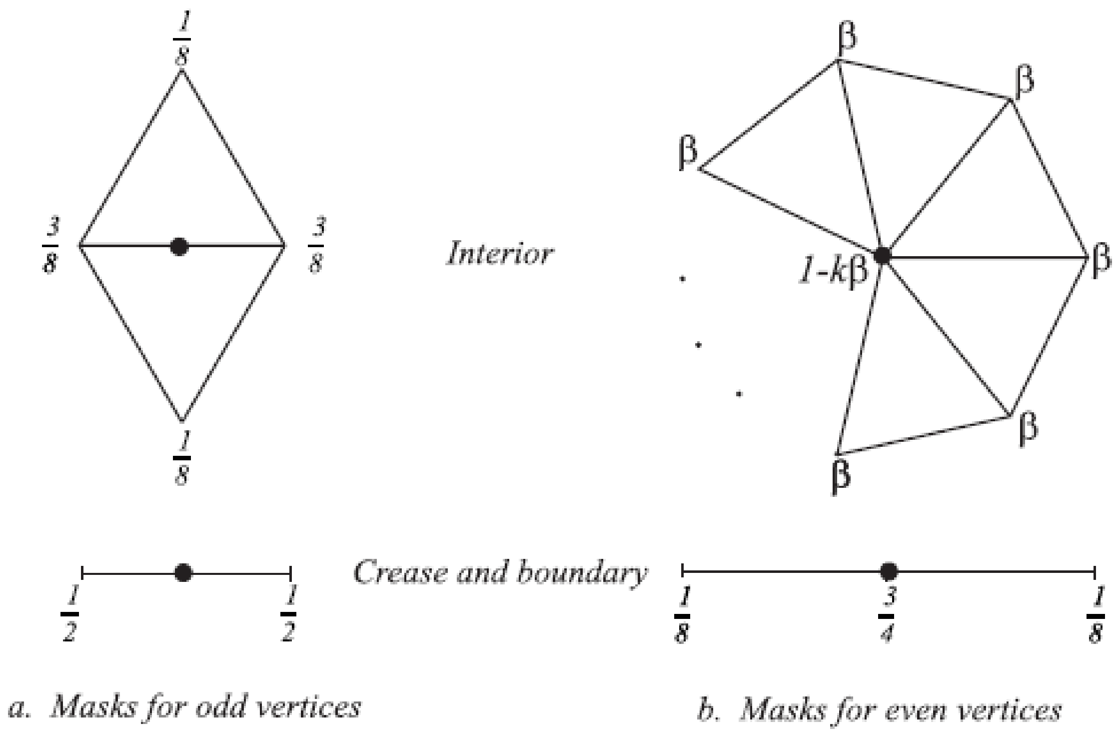


Figure 4.3: *Loop subdivision: in the picture above,* β *can be chosen to be either* $\frac{1}{n}(5/8-(\frac{3}{8}+\frac{1}{4}\cos\frac{2\pi}{n})^2)$ *(original choice of Loop [16]), or, for* $n>3$, $\beta=\frac{3}{8n}$ *as proposed by Warren [33]. For* $n=3$, $\beta=3/16$ *can be used.*

Loop Scheme

- Computing Tangents
  - Interior
    $$t_1 = \sum_{i=0}^{k-1} \cos \frac{2\pi i}{k} p_{i,1}$$
    $$t_2 = \sum_{i=0}^{k-1} \sin \frac{2\pi i}{k} p_{i,1}.$$
  - Boundary $t_{along} = p_{0,1} - p_{k-1,1}$

$$t_{across} = p_{0,1} + p_{1,1} - 2p_0 \quad \text{for } k = 2$$
$$t_{across} = p_{2,1} - p_0 \quad \text{for } k = 3$$
$$t_{across} = \sin\theta\,(p_{0,1} + p_{k-1,1}) + (2\cos\theta - 2) \sum_{i=1}^{k-2} \sin i\theta\, p_{i,1} \quad \text{for } k \geq 4 \qquad \theta = \pi/(k-1)$$

- Computing the normal at that point is then just $t_1 \times t_2$.

Modified Butterfly Scheme

- First proposed by Dyn, Gregory and Levin, but was not C^1 continuous
- A modified scheme was later proposed that produced C^1 continuous meshes for arbitrary surfaces
- Interpolating scheme applied to triangular meshes

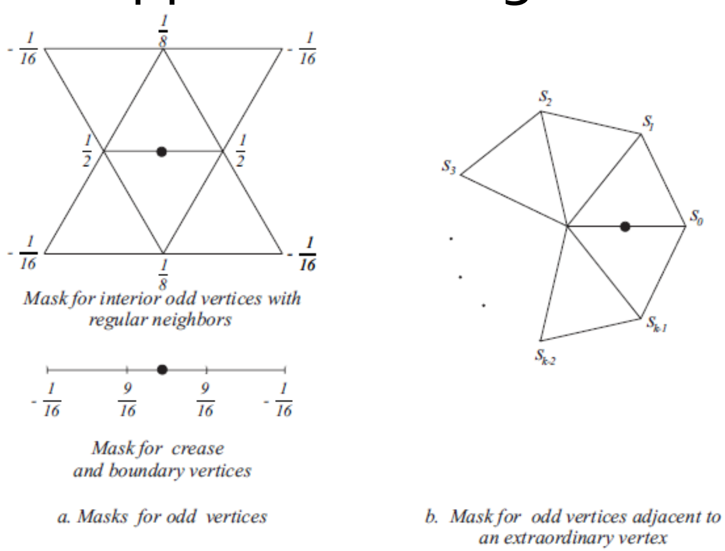


Figure 4.5: *Modified Butterfly subdivision. The coefficients* s_i *are* $\frac{1}{k}\left(\frac{1}{4}+\cos\frac{2i\pi}{k}+\frac{1}{2}\cos\frac{4i\pi}{k}\right)$ *for* $k>5$. *For* $k=3$, $s_0=\frac{5}{12}$, $s_{1,2}=-\frac{1}{12}$; *for* $k=4$, $s_0=\frac{3}{8}$, $s_2=-\frac{1}{8}$, $s_{1,3}=0$.

Modified Butterfly Scheme

- For a regular vertices, imagine arranging the control points into a vector
  $p = [p_0, p_{0,1}, p_{1,1}, \ldots, p_{5,1}, p_{0,2}, p_{1,2}, \ldots, p_{5,3}]$
  of length 19, then the tangents are given as follows

$$
\begin{aligned}
l_1 &= \left[0,16,8,-8,-16,-8,8,-4,0,4,4,0,-4,1,\frac{1}{2},-\frac{1}{2},-1,-\frac{1}{2},\frac{1}{2}\right] \\
l_2 &= \sqrt{3}\left[0,0,8,8,0,-8,-8,-\frac{4}{3},-\frac{8}{3},-\frac{4}{3},\frac{4}{3},\frac{8}{3},\frac{4}{3},0,\frac{1}{2},\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2}\right]
\end{aligned}
\tag{4.3}
$$

- Otherwise, the same tangent rules as the Loop scheme are applied.

Modified Butterfly Scheme

- Boundary rules are much more complicated in the butterfly scheme because the stencil is much bigger.
- We can break them into groups based on the two points on the edge where the point is being added.

Head	**Tail**	**Rule**
regular interior	regular interior	standard rule
regular interior	regular crease	regular interior-crease
regular crease	regular crease	regular crease-crease 1 or 2
extraordinary interior	extraordinary interior	average two extraordinary rules
extraordinary interior	extraordinary crease	same
extraordinary crease	extraordinary crease	same
regular interior	extraordinary interior	interior extraordinary
regular interior	extraordinary crease	crease extraordinary
extraordinary interior	regular crease	interior extraordinary
regular crease	extraordinary crease	crease extraordinary

Modified Butterfly Scheme

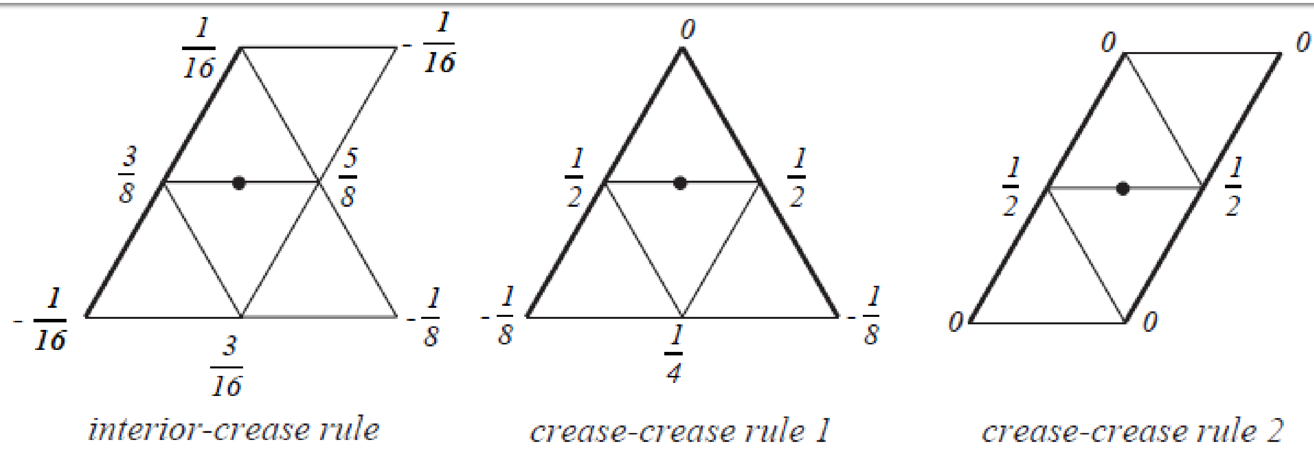


Figure 4.6: *Regular Modified Butterfly boundary/crease rules.*

$$c_0 = 1 - \big(1/(k-1)\big)\sin\theta_k \sin i\theta_k / (1-\cos\theta_k)$$
$$c_{i0} = c_{ik} = 1/4\cos i\theta_k - \big(1/4(k-1)\big)\sin 2\theta_k \sin 2\theta_k i / \big(\cos\theta_k - \cos 2\theta_k\big)$$
$$c_{ij} = (1/k)\big(\sin i\theta_k \sin j\theta_k + (1/2)\sin 2i\theta_k \sin 2j\theta_k\big)$$

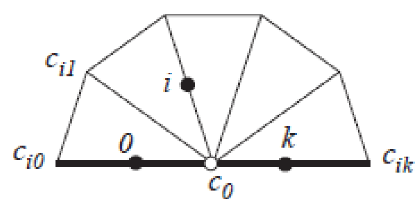
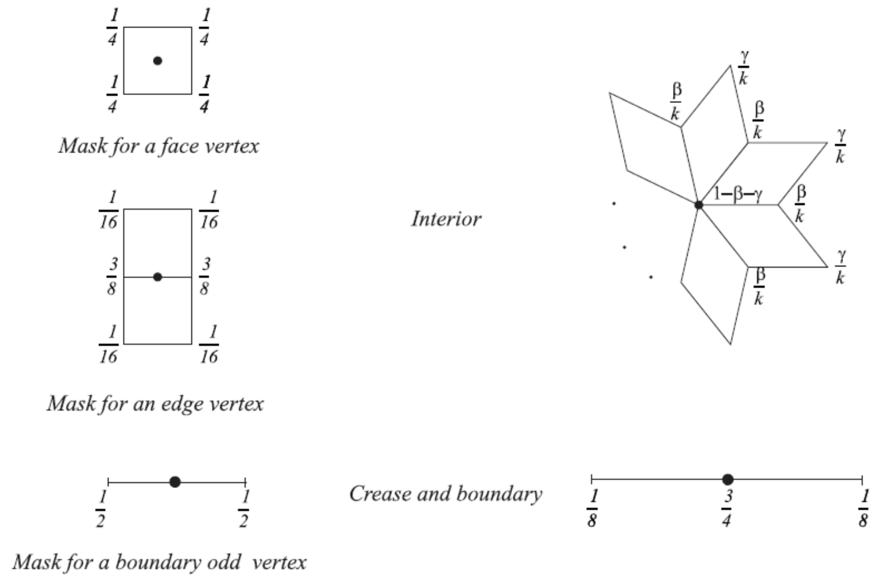


Figure 4.7: *Modified Butterfly rules for neighbors of a crease/boundary extraordinary vertex.*

Catmull-Clark Scheme

- Face splitting, approximating scheme on quadrilaterals
- Produces surfaces that are C^2 everywhere except extraordinary vertices where they are C^1



a. Masks for odd vertices *b. Mask for even vertices*

Figure 4.8: *Catmull-Clark subdivision. Catmull and Clark [4] suggest the following coefficients for rules at extraordinary vertices:* $\beta = \frac{3}{2k}$ *and* $\gamma = \frac{1}{4k}$

Kobbelt Scheme

- Face splitting, interpolating scheme on quadrilateral meshes
- C1 continuous for all valences

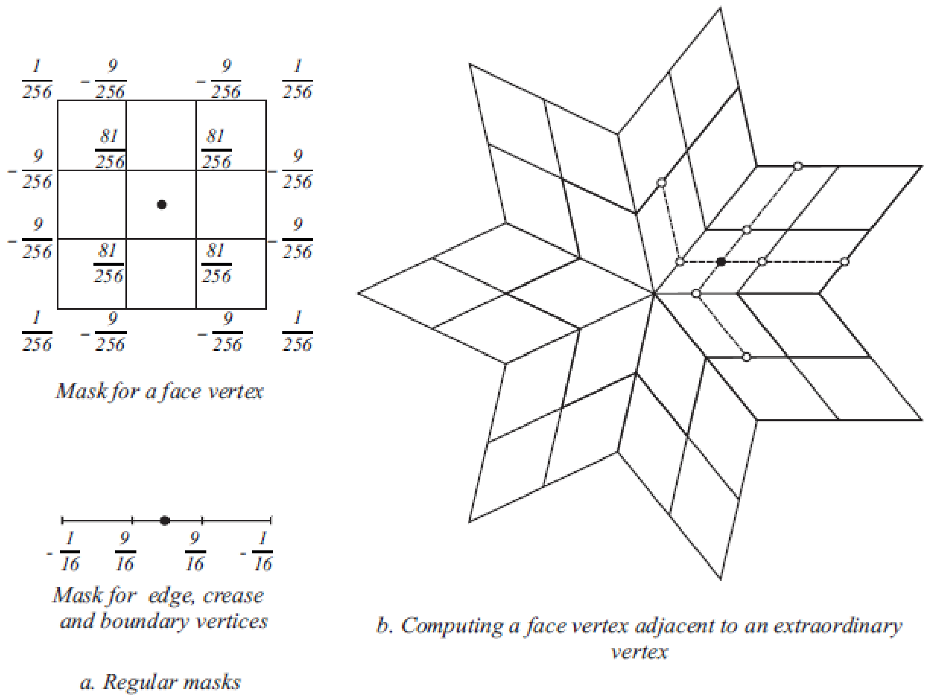
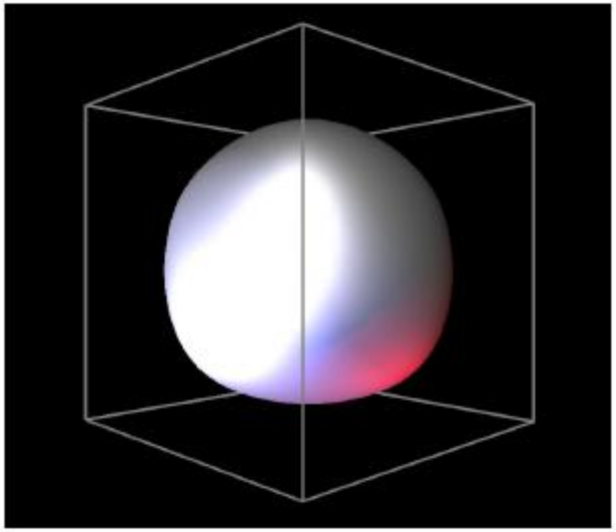


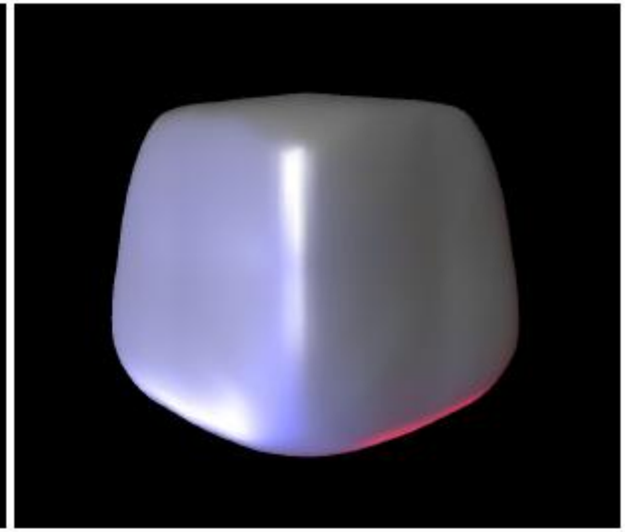
Figure 4.11: *Kobbelt subdivision.*

Other Subdivision Schemes

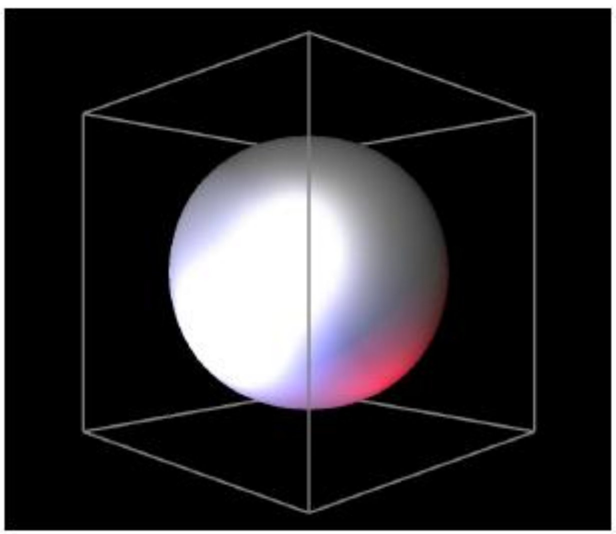
- Doo-Sabin, Midedge (C^1)
- Biquartic (C^2)
- Theses are vertex splitting algorithms.



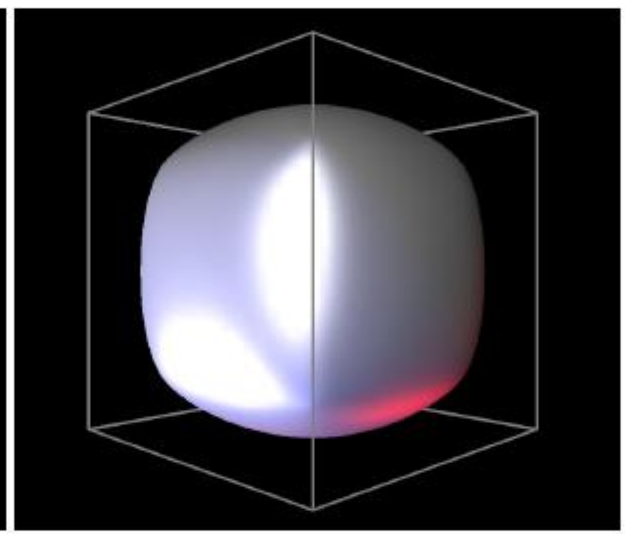
Loop



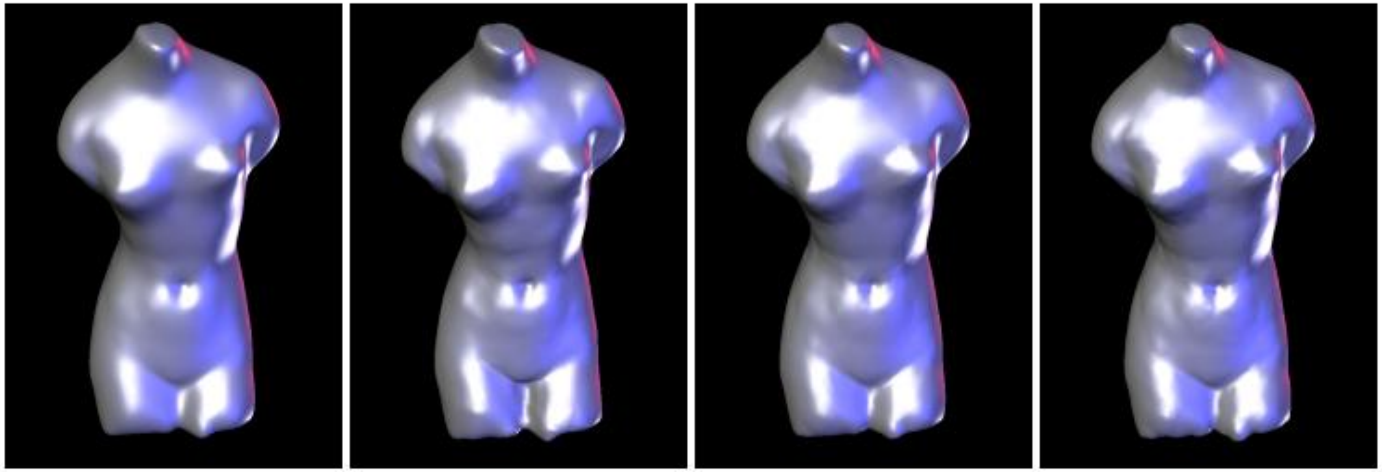
Butterfly



Catmull-Clark

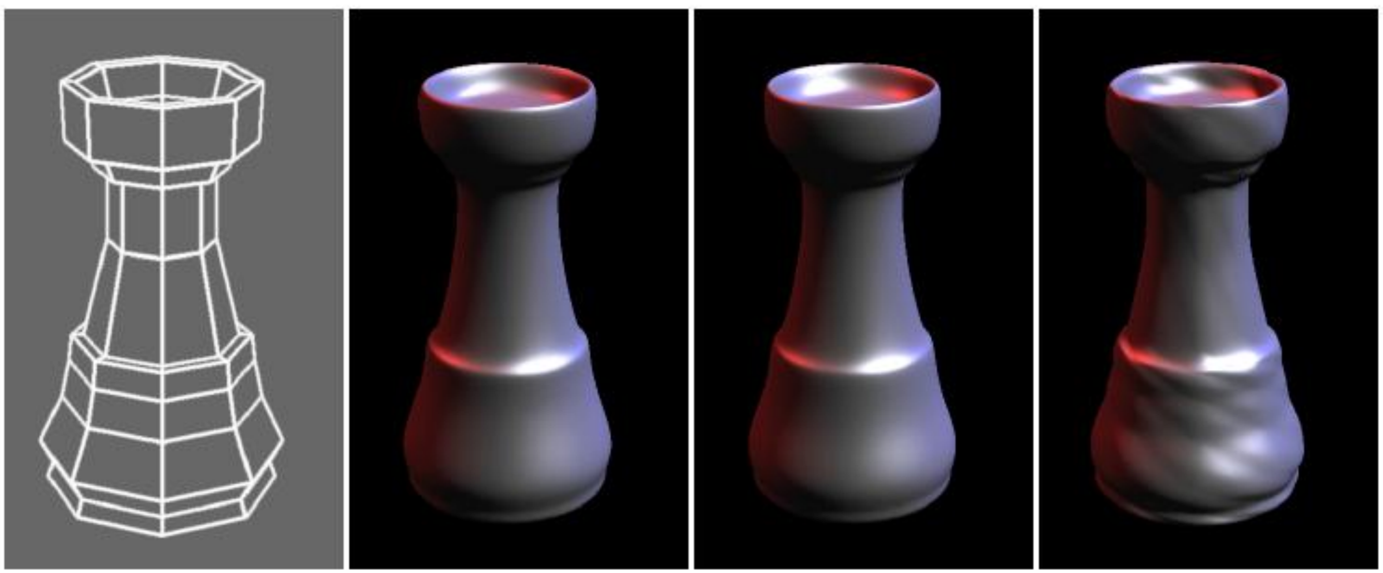


Doo-Sabin



Loop *Butterfly* *Catmull-Clark* *Doo-Sabin*

Figure 4.20: *Different subdivision schemes produce similar results for smooth meshes.*



Initial mesh *Loop* *Catmull-Clark* *Catmull-Clark,after triangulation*

Project 2

- You will have 2 tasks in project 2.
  - Texture map a mesh given the texture and the texture coordinates
  - Implement the loop subdivision algorithm
- All initial positions, normals, texture coordinates and whether or not this particular mesh needs to be texture are given to you.

What is a texture?

- A texture is just a bitmap image
- Our image is a 2D array: texture[height][width][4]
- Pixels of the texture are called *texels*
- Texel coordinates are in 2D, in the range [0,1]
  - OpenGL uses (s, t) as the coordinate parameters.
  - Commonly referred to as (u, v) coordinates by most graphics programs.

Texture Mapping

- In order to map a 2D image to a piece of geometry, we consider two functions:
- A mapping function which takes 3D points to (u, v) coordinates.
  - f(x, y, z) returns (u, v)
- A sampling/lookup function which takes (u, v) coordinates and returns a color.
  - g(u, v) returns (r, g, b, a)

Texture Mapping

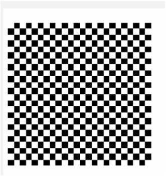
- The basic idea is that for some polygon (which may have arbitrary shape and size), we manually assign each of its vertices (u, v) coordinates in the range from [0, 1].
- We then use these (u, v) coordinates as rough indices into our texture array
  - These don't necessarily hit into the array so some sort of interpolation is generally used

OpenGL Texture Mapping

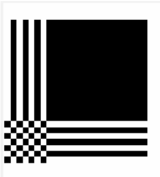
- Initialization
  - Enable GL texture mapping
  - Specify texture
  - Read image from file into array in memory or generate image using the program (procedural generation)
  - Specify any parameters
  - Define and activate the texture
- Draw
  - Draw objects and assign texture coordinates to vertices

OpenGl Texture Mapping

- Color blending
  - How to determine the color of the final pixel?
    - GL_REPLACE – use texture color to replace object color
    - GL_BLEND – linear combination of texture and object color
    - GL_MODULATE – multiply texture and object color
  - Example:
    - `glTexEnvf(GL_TEXTURE_ENV,GL_TEXTURE_ENV_MODE,GL_REPLACE);`
- Texture Coordinates outside [0,1] → Two choices:
  - Repeat pattern (GL_REPEAT)
  - Clamp to maximum/minimum value (GL_CLAMP)
  - Example:
    - `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_S, GL_CLAMP)`
    - `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T, GL_CLAMP)`



repeat



clamp

OpenGL Texture Mapping

```
// somewhere else...
Gluinttexture_id;

void init(){
   // acquire load our texture into an array
   // the function we use this semester is in imageio.hpp
   char* pointer; // TODO: give me some values!

   // enable textures
   glEnable(GL_TEXTURE_2D);
   glGenTextures(1, &texture_id);
   glBindTexture(GL_TEXTURE_2D, texture_id);

   // sample: specify texture parameters
   glTexParameteri(GL_TEXUTRE_2D, GL_TEXTURE_WRAP_S, GL_REPEAT);
   glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T, GL_REPEAT);

   // set the active texture
   glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA, 256, 256, 0, GL_RGBA,
   GL_UNSIGNED_BYTE, pointer);
}
```

OpenGL Texture Mapping

- Use GLTexCoord2f(s,t) to specify texture coordinates
- Example:

```
glEnable(GL_TEXTURE_2D)
glBegin(GL_QUADS);
glTexCoord2f(0.0,0.0); glVertex3f(0.0,0.0,0.0);
glTexCoord2f(0.0,1.0); glVertex3f(2.0,10.0,0.0);
glTexCoord2f(1.0,0.0); glVertex3f(10.0,0.0,0.0);
glTexCoord2f(1.0,1.0); glVertex3f(12.0,10.0,0.0);
glEnd();
glDisable(GL_TEXTURE_2D)
```

- State machine: Texture coordinates remain valid until you change them or exit texture mode via glDisable (GL_TEXTURE_2D)

Subdivision

- We provide you with the initial positions, normals and texture coordinates in this lab.
- Your job is to implement the loop subdivision algorithm and output a subdivided mesh.
- You can use the same algorithm for the position, normals and the texture coordinates.

Subdivision

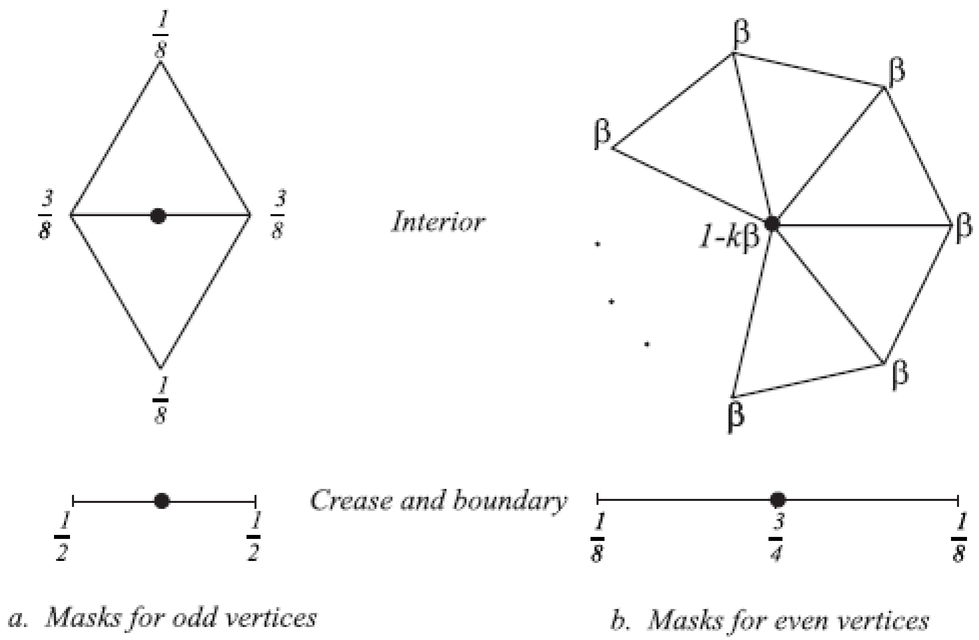


Figure 4.3: *Loop subdivision: in the picture above,* β *can be chosen to be either* $\frac{1}{n}(5/8-(\frac{3}{8}+\frac{1}{4}\cos\frac{2\pi}{n})^2)$ *(original choice of Loop [16]), or, for* $n>3$, $\beta=\frac{3}{8n}$ *as proposed by Warren [33]. For* $n=3$, $\beta=3/16$ *can be used.*

Subdivision

- Essentially requires 2 passes
  - First pass, handle creating odd vertices
  - Second pass, move even vertices
- Suggested path
  - Implement the interior cases first
    - This will allow you to test this on closed meshes before moving on to the ones with boundaries
  - Implement the boundary cases

Summary

- Subdivision
  - What is it?
  - What properties does it have / do we want?
  - What kinds of algorithms exist and what advantages do they have?
- Project 2
  - Texture mapping crash course
  - Loop subdivision algorithm

Image sources

http://www.mrl.nyu.edu/~dzorin/sigoocourse/