Image Processing
Overview

Images

Pixel Filters

Neighborhood Filters

Dithering
We can think of an image as a function, $f$, 

$f: \mathbb{R}^2 \rightarrow \mathbb{R}$

- $f(x, y)$ gives the intensity at position $(x, y)$
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  
  $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$
Image as a Function
Image Processing

• Define a new image $g$ in terms of an existing image $f$
  – We can transform either the domain or the range of $f$
• Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?
Some operations preserve the range but change the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

Still other operations operate on both the domain and the range of $f$. 
Point Operations
Point Processing
Point Processing

**Original**

\[ x \]

**Darken**

\[ x - 128 \]

**Lower Contrast**

\[ \frac{x}{2} \]

**Nonlinear Lower Contrast**

\[ (\frac{x}{255.0}^{0.33}) \times 255.0 \]

**Invert**

\[ 255 - x \]

**Lighten**

\[ x + 128 \]

**Raise Contrast**

\[ x \times 2 \]

**Nonlinear Raise Contrast**

\[ (\frac{x}{255.0}^{2}) \times 255.0 \]
Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function

Send $v \to$ actually display a pixel which has intensity equal to $v^{2.5}$

\[ \phi = 1.0; f(v) = v \]

\[ \phi = 2.5; f(v) = v^{1/2.5} = v^{0.4} \]
Neighborhood Operations
Convolution

\[ F = \begin{bmatrix}
0.2 & 0.1 & -1.0 \\
0.3 & 0.0 & 0.9 \\
0.1 & 0.3 & -1.0
\end{bmatrix} \]

\[ I' = F \ast I \]
Linear Shift Invariant Systems (LSIS)

Linearity:

\[ f_1 \rightarrow g_1 \rightarrow f_2 \rightarrow g_2 \]

\[ \alpha f_1 + \beta f_2 \rightarrow \alpha g_1 + \beta g_2 \]

Shift invariance:

\[ f(x-a) \rightarrow g(x-a) \]
LSIS is doing convolution; convolution is linear and shift invariant

\[ g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau \quad g = f \ast h \]
Convolution - Example

\[ c = a \ast b \]
Properties of Convolution

• Commutative

\[ a * b = b * a \]

• Associative

\[ (a * b) * c = a * (b * c) \]

• Cascade system

\[ f \rightarrow h_1 \rightarrow h_2 \rightarrow g \]

\[ = f \rightarrow h_1 * h_2 \rightarrow g \]

\[ = f \rightarrow h_2 * h_1 \rightarrow g \]
Point Spread Function

- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.
- Point spread function of Human Eyes
Point Spread Function

normal vision  myopia  hyperopia

astigmatism

Images by Richmond Eye Associates
Blurred Image
Gaussian Smoothing

by Charles Allen Gillbert

by Harmon & Julesz

http://www.michaelbach.de/ot/cog_blureffects
Gaussian Smoothing

http://www.michaelbach.de/ot/cog_blureffects
Noise
Blurred Noise
Median Filter

- **Smoothing is averaging**
  - (a) Blurs edges
  - (b) Sensitive to outliers

- **Median filtering**
  - Sort $N^2-1$ values around the pixel
  - Select middle value (median)
  - Non-linear (Cannot be implemented with convolution)
Median Filter

Can this be described as a convolution?
Example: Noise Reduction

Image with noise

Median filter (5x5)
Example: Noise Reduction

Original image

Image with noise

Median filter (5x5)
X-Edge Detection
Y-Edge Detection
Edge detection filters

Roberts (2 x 2):

\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]

Sobel (3 x 3):

\[
\begin{pmatrix}
-1 & 0 & 1 \\
-1 & 0 & 1 \\
-1 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 1 & 1 \\
0 & 0 & 0 \\
-1 & -1 & 1
\end{pmatrix}
\]

Sobel (5 x 5):

\[
\begin{pmatrix}
-1 & -2 & 0 & 2 & 1 \\
-2 & -3 & 0 & 3 & 2 \\
-3 & -5 & 0 & 5 & 3 \\
-2 & -3 & 0 & 3 & 2 \\
-1 & -2 & 0 & 2 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 2 & 3 & 2 & 1 \\
2 & 3 & 5 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 \\
-2 & -3 & -5 & -3 & -2 \\
-1 & -2 & -3 & -2 & -1
\end{pmatrix}
\]
General Edge Detection

Can this be described as a convolution?
Some operations preserve the range but change the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

Still other operations operate on both the domain and the range of $f$. 
This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?
Image Sub-Sampling

Throw away every other row and column to create a 1/2 size image - called *image sub-sampling*
Image Sub-Sampling

1/2

1/4 (2x zoom)

1/8 (4x zoom)
Good and Bad Sampling

Good sampling:
• Sample often or,
• Sample wisely

Bad sampling:
• see aliasing in action!
Aliasing
Alias: n., an assumed name

Picket fence receding into the distance will produce aliasing...

WHY?

Input signal:

Matlab output:

x = 0:.05:5; imagesc(sin((2.^x).*x))

Alias!

Not enough samples
Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Sub-Sampling with Gaussian Pre-Filtering

Gaussian 1/2

- Solution: filter the image, *then* subsample
  - Filter size should double for each ½ size reduction. Why?
Sub-Sampling with Gaussian Pre-Filtering

Gaussian 1/2  G 1/4  G 1/8
Compare with...

1/2

1/4 (2x zoom)

1/8 (4x zoom)
1 Introduction

This manual corresponds to version 3.2 of the library.
The library’s fundamental purpose is to provide a consistent interface for users, similar to what shells do when looking up the built-in commands.

The following software, all of which we maintain:

- Dviljk (see the ‘dvljk’ man page)
- Dvipsk (see section “Introduction” in Dvipsk manual)
- GNU font utilities (see section “Introduction” in GNU font utilities manual)
- Web2c (see section “Introduction” in Web2c manual)
- Xdvik (see the ‘xdvi’ man page)

Other software that we do not maintain also use the same library:

- Xdvik (see the ‘xdvi’ man page)
- Web2c (see section “Introduction” in Web2c manual)
From Rick Matthews website, images by Dave Etchells
Warped Image
Warped Image

orig + vector field = warped

how?
Advection (just like a fluid)