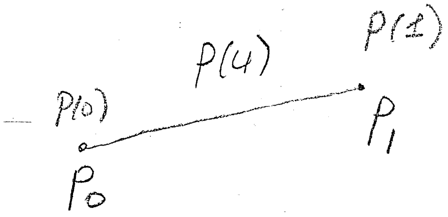
Linear Interpolation



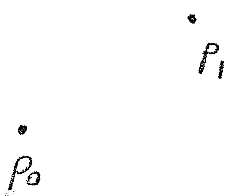
$$P(u) = a_1 u + a_0 \qquad \begin{cases} P_0 = P(u=0) \\ P_1 = P(u=1) \end{cases} \qquad \begin{cases} P_0 = a_0 \\ P_1 = a_1 + a_0 \end{cases} \qquad \begin{aligned} a_0 &= P_0 \\ a_1 &= P_1 - a_0 = P_1 - P_0 \end{aligned}$$

$$P(u) = (P_1 - P_0) \cdot u + P_0$$

$$P(u) = [u \;\; 1] \begin{bmatrix} P_1 - P_0 \\ P_0 \end{bmatrix}$$

$$= \underbrace{[u \;\; 1]}_{\text{variable}} \underbrace{\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}}_{\text{coeffic. (basis)}} \underbrace{\begin{bmatrix} P_0 \\ P_1 \end{bmatrix}}_{\text{control matrix}}$$

Cubic Curves



$$P(u) = au^3 + bu^2 + cu + d$$

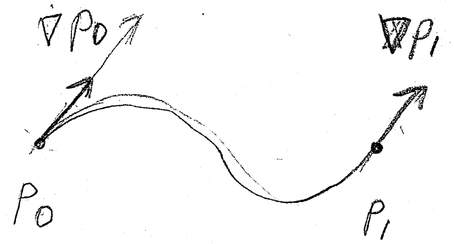
$$P(u) = [u^3 \;\; u^2 \;\; u \;\; 1] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Hermit Spline

$$P(u) = au^3 + bu^2 + cu + d$$

$$P(u) = [u^3 \quad u^2 \quad u \quad 1] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

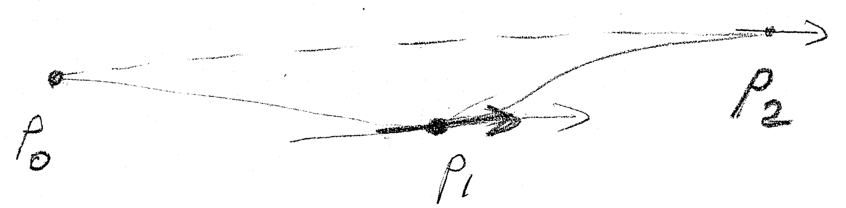
- need to solve for a, b, c, d
- 4 unknowns



$$\begin{cases} P_0 = P(u=0) & P_0 = d \\ P_1 = P(u=1) & P_1 = a+b+c+d \\ \nabla P_0 = P'(0) & \\ \nabla P_1 = P'(1) & \end{cases}$$

$$P'(u) = 3au^2 + 2bu + c$$

Catmull-Rom Spline



Tangent at $P_i = s \cdot (P_{i+1} - P_{i-1})$

$s \Rightarrow$ tension parameter
determines the magnitude (but not direction) of the tangent

curve between P_i & P_{i+1} is determined by

$P_{i-1} \quad P_i \quad P_{i+1} \quad P_{i+2}$