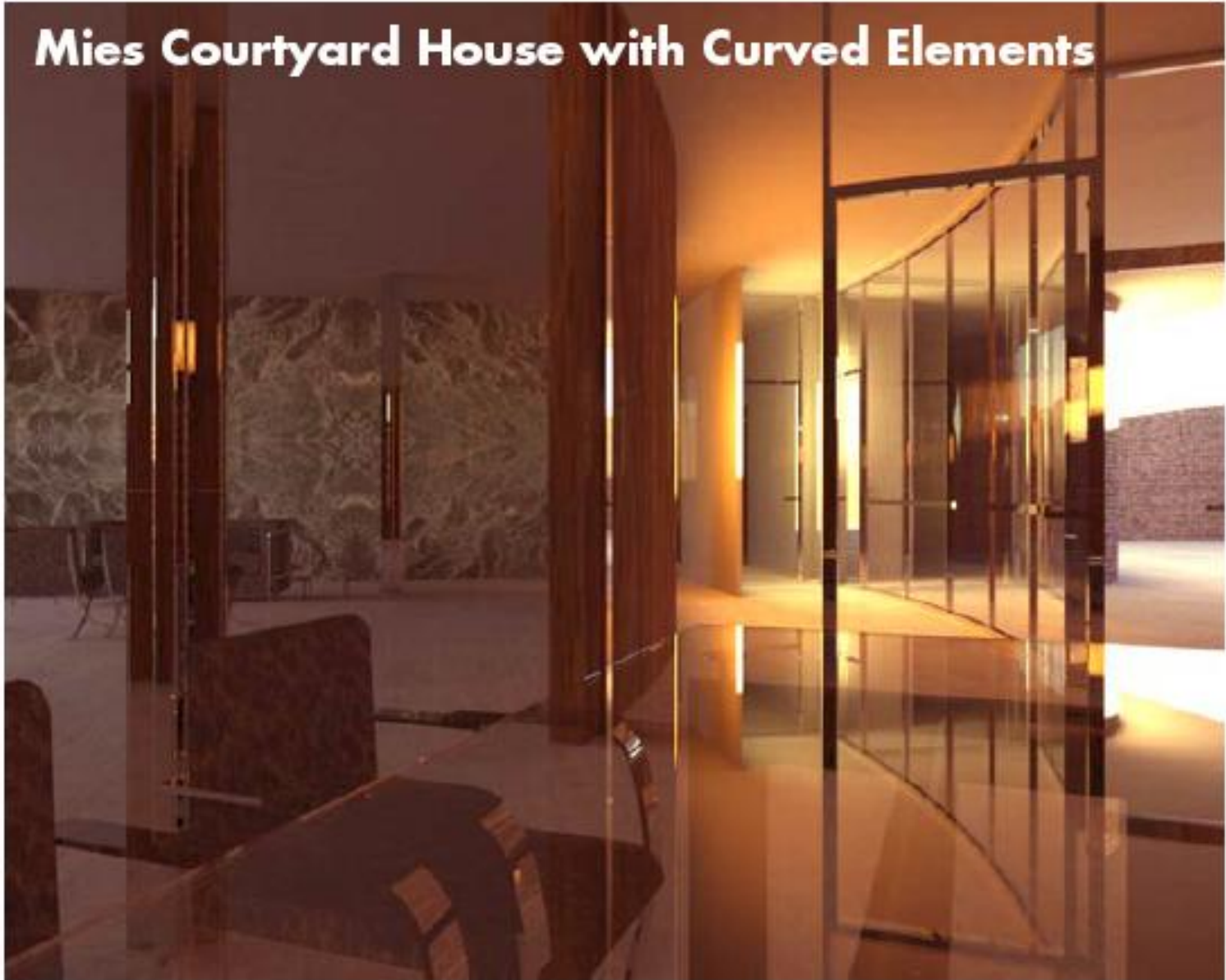


Radiosity

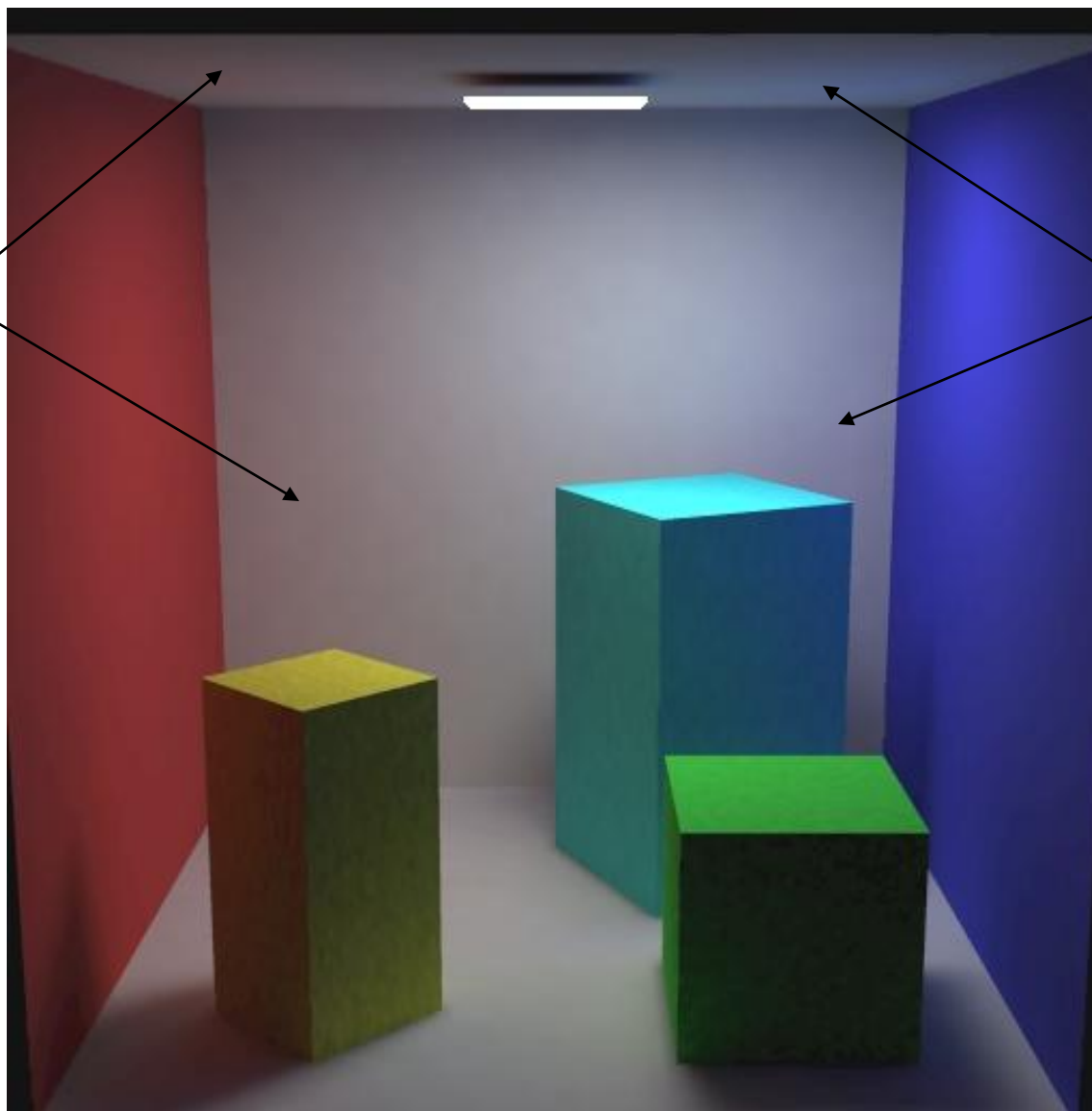
Thanks to Kavita Bala, Pat Hanrahan, Doug James, Ledah Casburn

Mies Courtyard House with Curved Elements



Modeling: Stephen Duck; Rendering: Henrik Wann Jensen

Cornell Box



red hue

blue hue

Lighting Effects

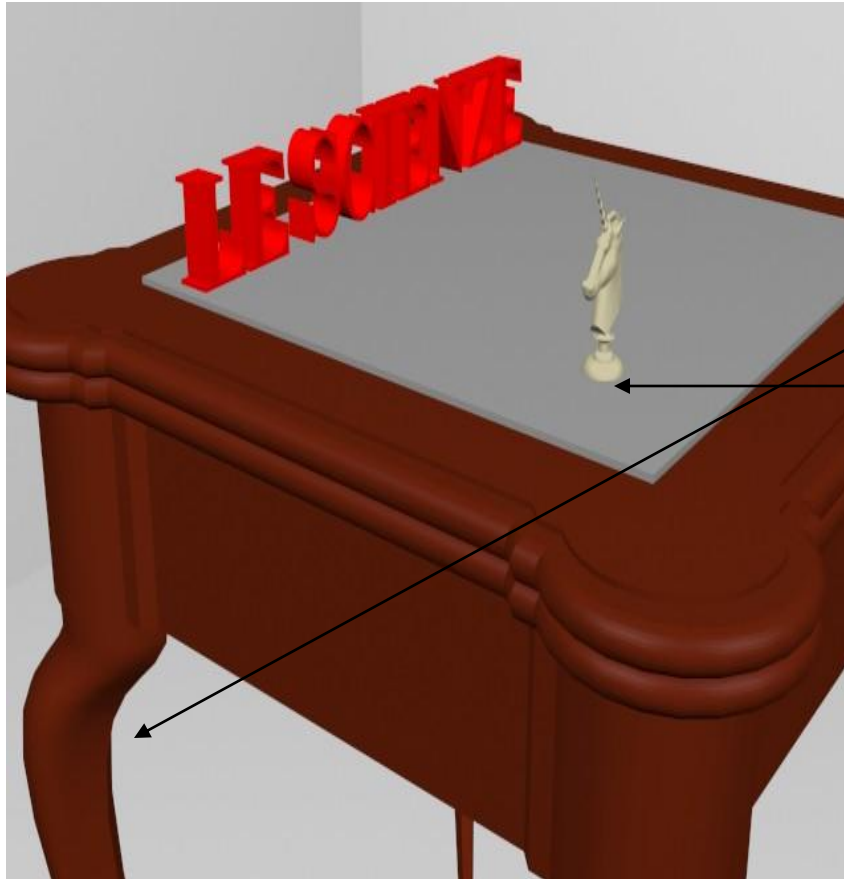


The ambient lighting in the upper-right image is approximated by a constant value. This is typical of most scanline algorithms. The middle and lower-left images were rendered with a ray tracing global illumination algorithm.



The middle image was rendered with no ambient light calculations. The lower-left image was rendered with several levels of diffuse re-reflection to give a better approximation of the ambient light in this scene.

Phong Shading



Plastic looking scene

- no object interactions
- no shadows

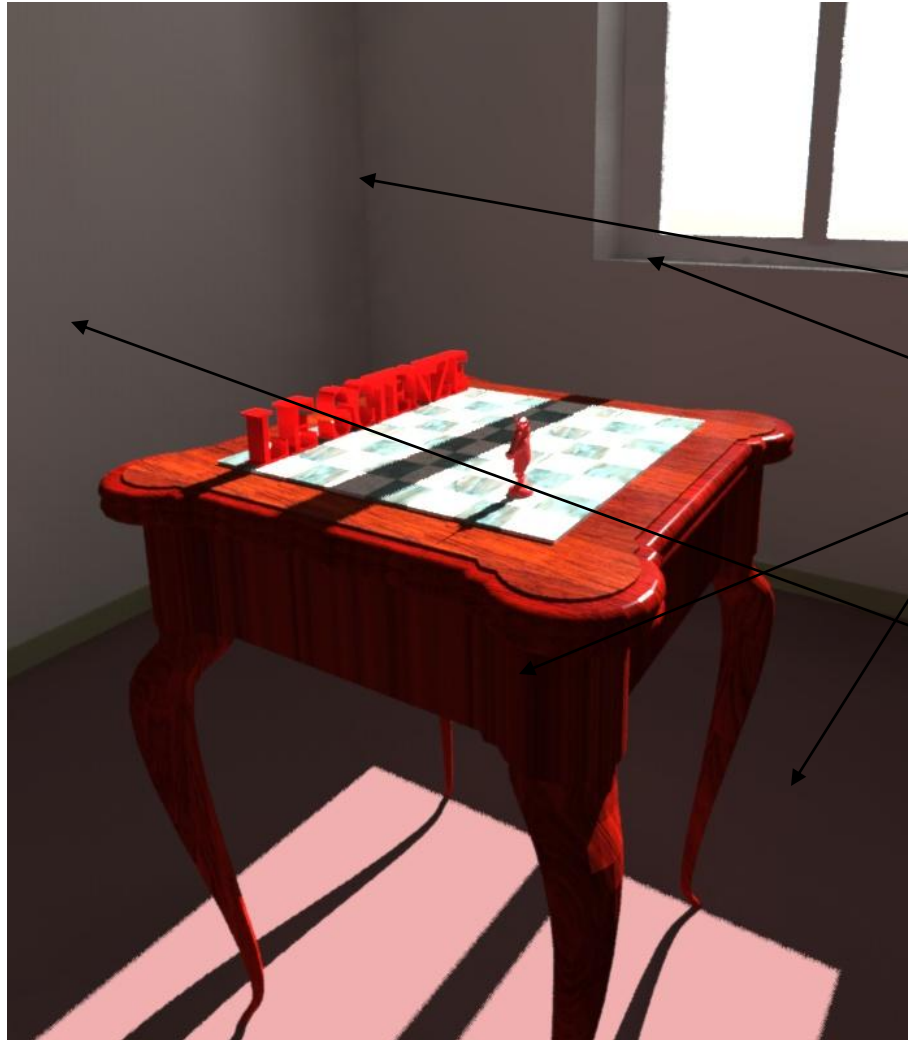
Ray Tracing



Scene doesn't look realistic enough.

- where is the corner of room?
- is window flush with wall?
- is the carpet and wood supposed to be this dark?

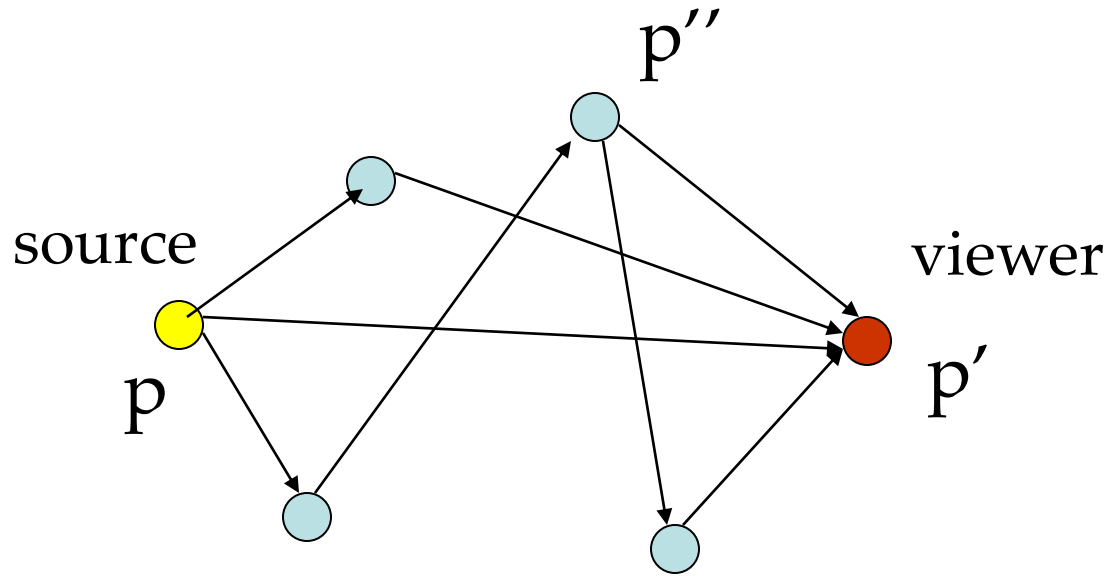
Radiosity – today's topic



Indirect lighting affects realism.

- room has a corner
- window has depth
- carpet and wood on table is lighter
- walls look more pink

The Rendering Equation – Graph Style



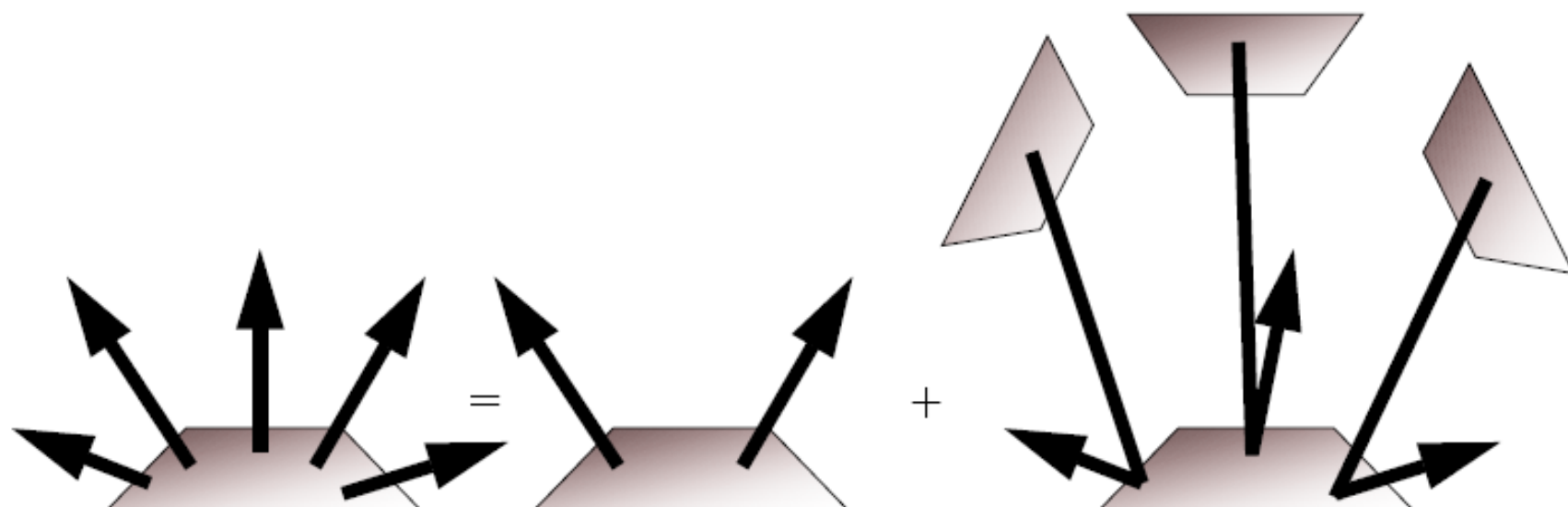
$$i(\mathbf{p}, \mathbf{p}') = v(\mathbf{p}, \mathbf{p}')(\epsilon(\mathbf{p}, \mathbf{p}') + \int \rho(\mathbf{p}, \mathbf{p}', \mathbf{p}'')i(\mathbf{p}', \mathbf{p}'')d\mathbf{p}'')$$

Visibility
(shadows)

Emission
(light source)

Reflectance from
Surfaces

Conservation of Energy



Emitted power = self-emitted power + received & reflected power

Diffuse Interreflections - Radiosity

- Consider lambertian surfaces and sources.
- Radiance independent of viewing direction.
- Consider total power leaving per unit area of a surface.
- Can simulate soft shadows and color bleeding from diffuse surfaces.
- Used abundantly in heat transfer literature

Irradiance, Radiosity

- Irradiance E is the power **received** per unit surface area
 - Units: W/m^2
- Radiosity
 - Power per unit area **leaving** the surface (like irradiance)

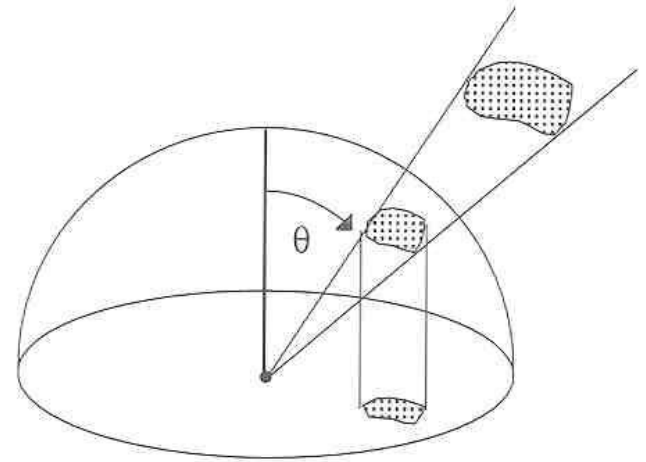


Figure 2.8: Projection of differential area.

Planar piecewise constancy assumption

- Subdivide scene into small “uniform” polygons

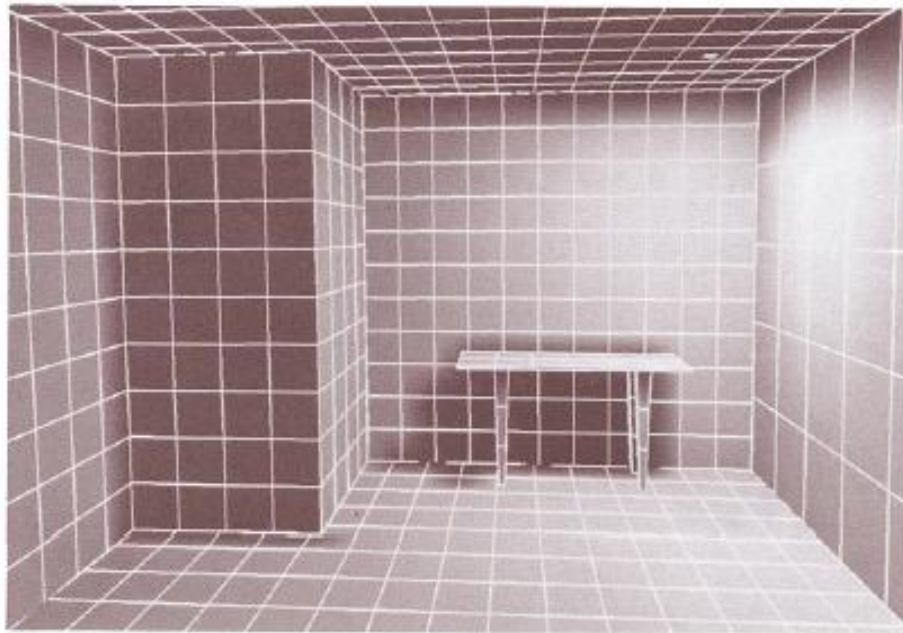
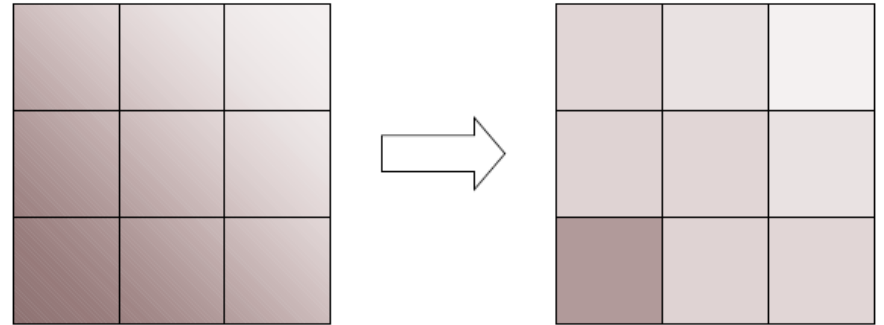


Table in room sequence from Cohen and Wallace

Power Equation

- Power from each polygon:

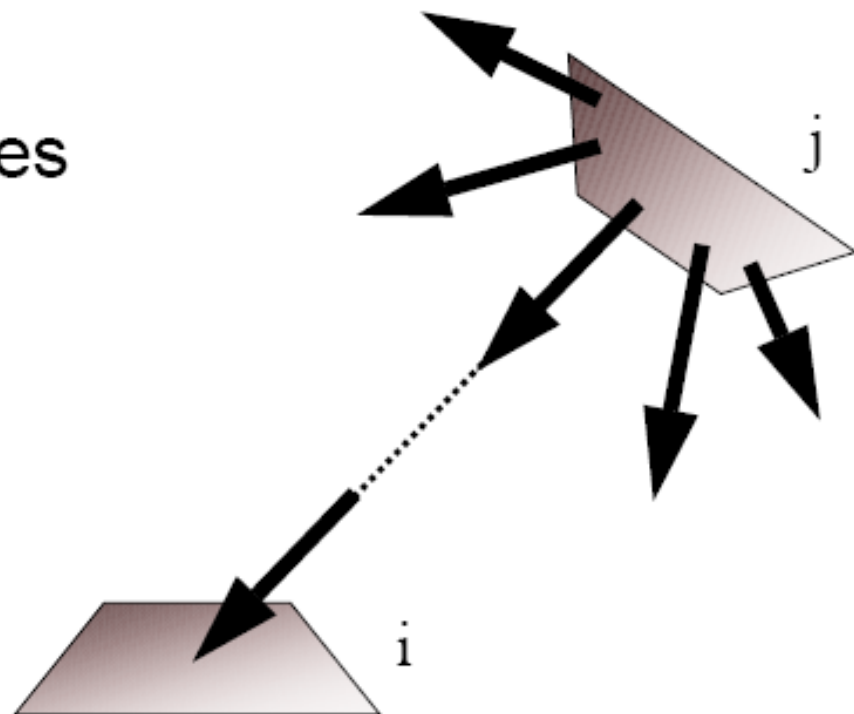
$$\forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^N \Phi_j F(i \rightarrow j)$$

- Linear System of Equations:

- Φ_i : power of patch i (unknown)
- $\Phi_{e,i}$: emission of patch i (known)
- ρ_i : reflectivity of patch i (known)
- $F(j \rightarrow i)$: form-factor (coefficients of matrix)

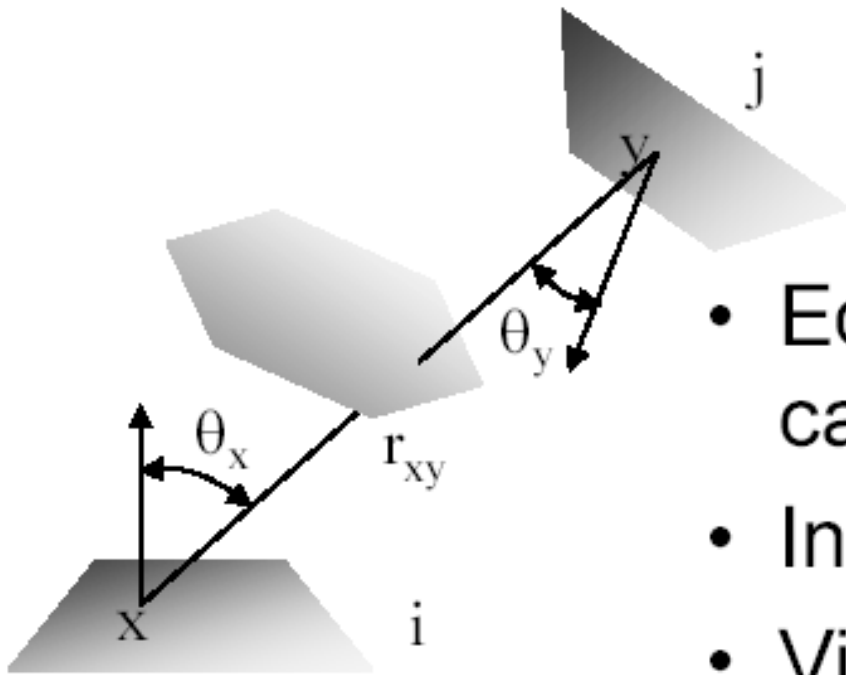
Form Factor

- $F_{j \rightarrow i}$ = the fraction of power emitted by j , which is received by i
- Area
 - if i is smaller, it receives less power
- Orientation
 - if i faces j , it receives more power
- Distance
 - if i is further away, it receives less power



Form Factor

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_x$$



- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder

Form Factors Invariant

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_x$$

$$F(i \rightarrow j) = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_x dA_y$$

$$F(i \rightarrow j)A_i = F(j \rightarrow i)A_j$$

Form Factor Computation

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_y$$

- Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.
- In general, computing double integral is hard.
- Use Monte Carlo Integration.

Being Smart about Form Factors

Form factors depend only on scene geometry. If geometry is constant, they only need to be calculated once.

Solution of the radiosity system is independent of viewing conditions, so if only the viewer position changes, it only needs to be solved once—can walk around the scene in real-time after it's initially generated

Being Smart about Form Factors

Form factors are complicated. Full numeric approximation of these is expensive—many special cases may be solved analytically.

Because we assume that radiosity is constant across a patch, two patches are typically assumed to be fully inter-visible or not at all inter-visible. That means that patches have to be small enough to resolve shadows and other complexities

How to perform visibility testing?

Two basic methods, both of which have aliasing problems:

- Raycasting (typically slow)

- Hemicube method (z-buffer exploit)

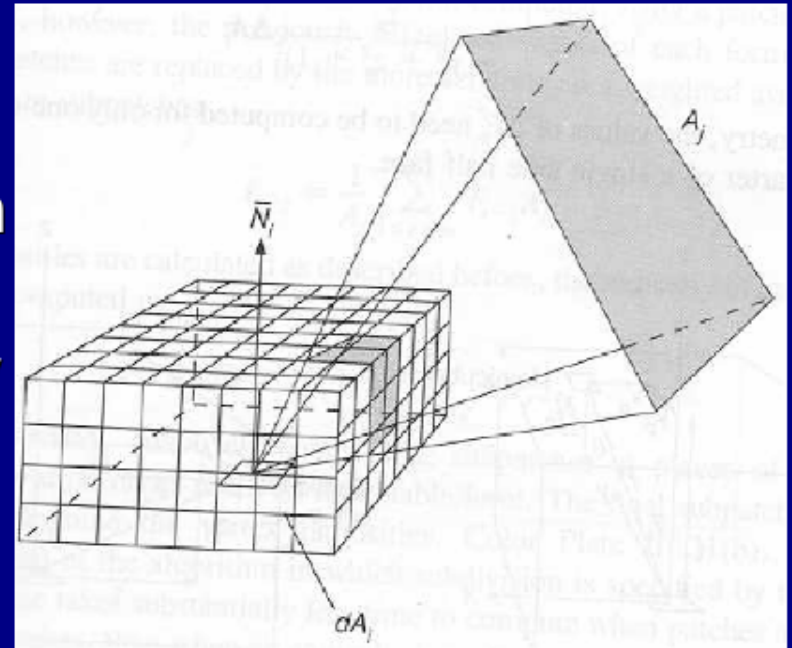
Anti-aliasing may be performed in both cases

Hemicube Visibility Testing

Render the entire scene from
the perspective of the
center of the current patch

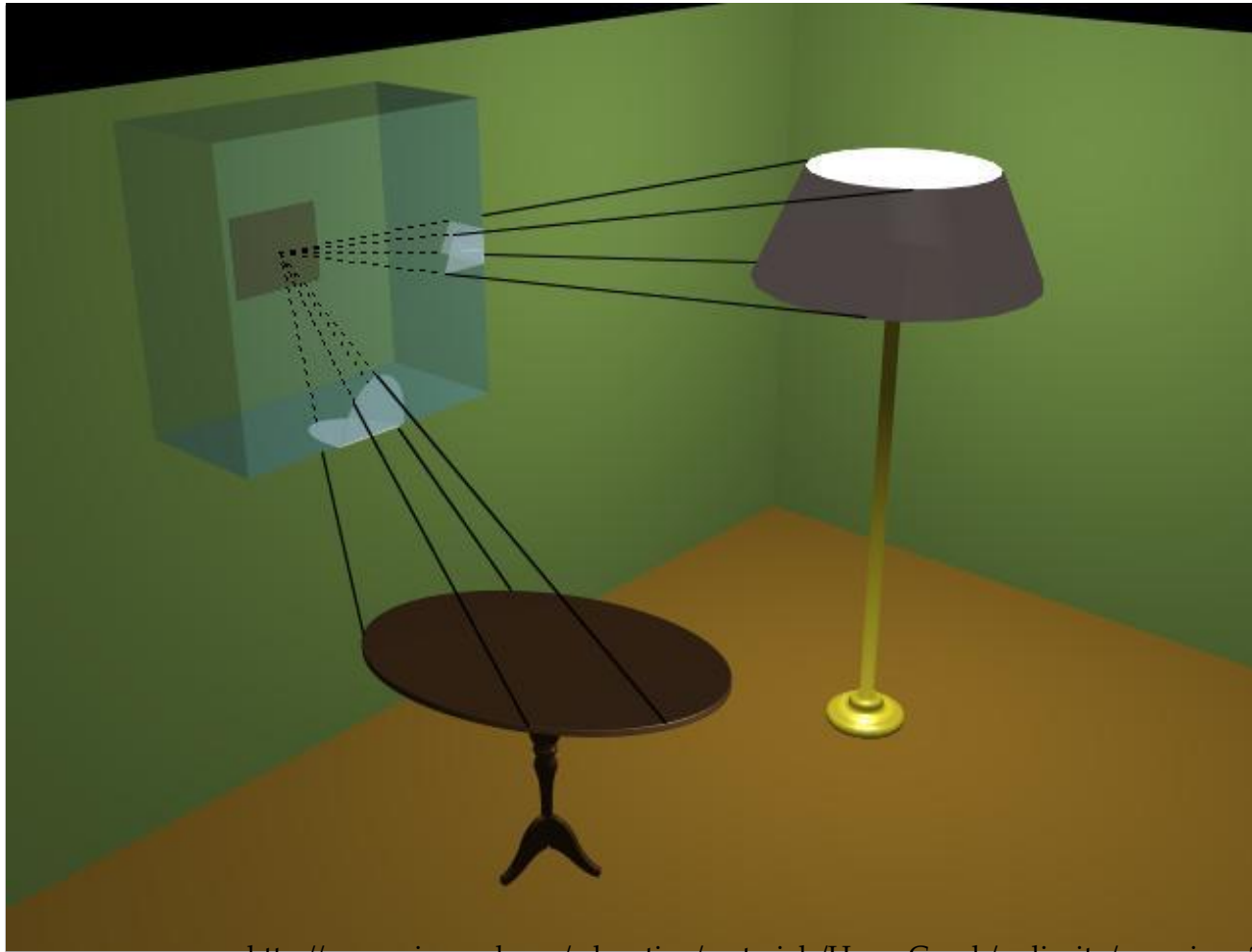
Rather than color, store patch
identifiers, using the z-
buffer to determine visibility

Takes advantage of graphics
hardware



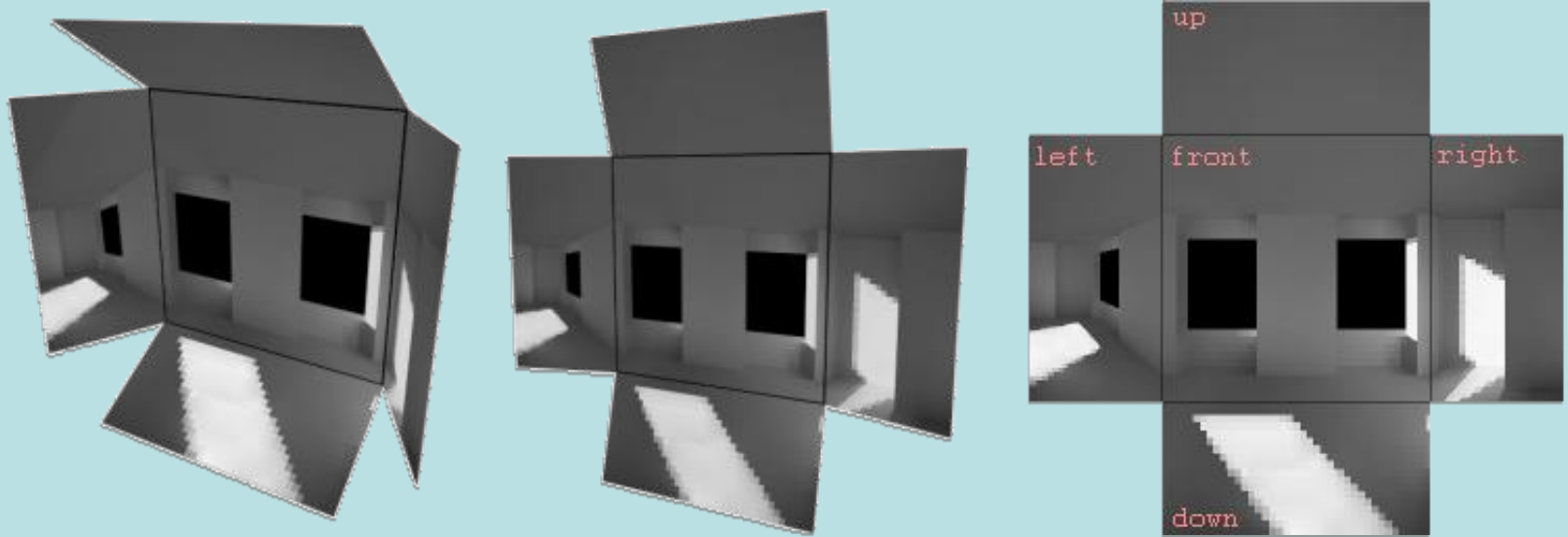
R. Ramamoorthi

Hemicube in Action



http://www.siggraph.org/education/materials/HyperGraph/radiosity/overview_2.htm

Hemicube in Action



http://en.wikipedia.org/wiki/File:Hemicube_Unfold.gif

Power → Radiosity

$$\Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^N \Phi_j F(j \rightarrow i)$$



Divide by A_i

$$\frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^N \frac{\Phi_j F(j \rightarrow i)}{A_i}$$
$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N \frac{\Phi_j \frac{F(i \rightarrow j) A_i}{A_j}}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N \frac{\Phi_j F(i \rightarrow j)}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

Linear System of Radiosity Equations

$$\forall \text{patches } i: \quad B_i = B_{ei} + \rho_i \sum_j F_{i \rightarrow j} B_j$$

$$\begin{bmatrix} 1 - \rho_1 F_{1 \rightarrow 1} & -\rho_1 F_{1 \rightarrow 2} & \cdots & -\rho_1 F_{1 \rightarrow n} \\ -\rho_2 F_{2 \rightarrow 1} & 1 - \rho_2 F_{2 \rightarrow 2} & \cdots & -\rho_2 F_{2 \rightarrow n} \\ \cdots & \cdots & \cdots & \cdots \\ -\rho_n F_{n \rightarrow 1} & -\rho_n F_{n \rightarrow 2} & \cdots & 1 - \rho_n F_{n \rightarrow n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_{e1} \\ B_{e2} \\ \cdots \\ B_{en} \end{bmatrix}$$

$\swarrow$
Known
 $\swarrow$
Unknown
 $\downarrow$
Known

- Matrix Inversion to Solve for Radiosities.

Iterative approaches

- Jacobi iteration
- Start with initial guess for energy distribution (light sources)
- Update radiosity/power of all patches based on the previous guess

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

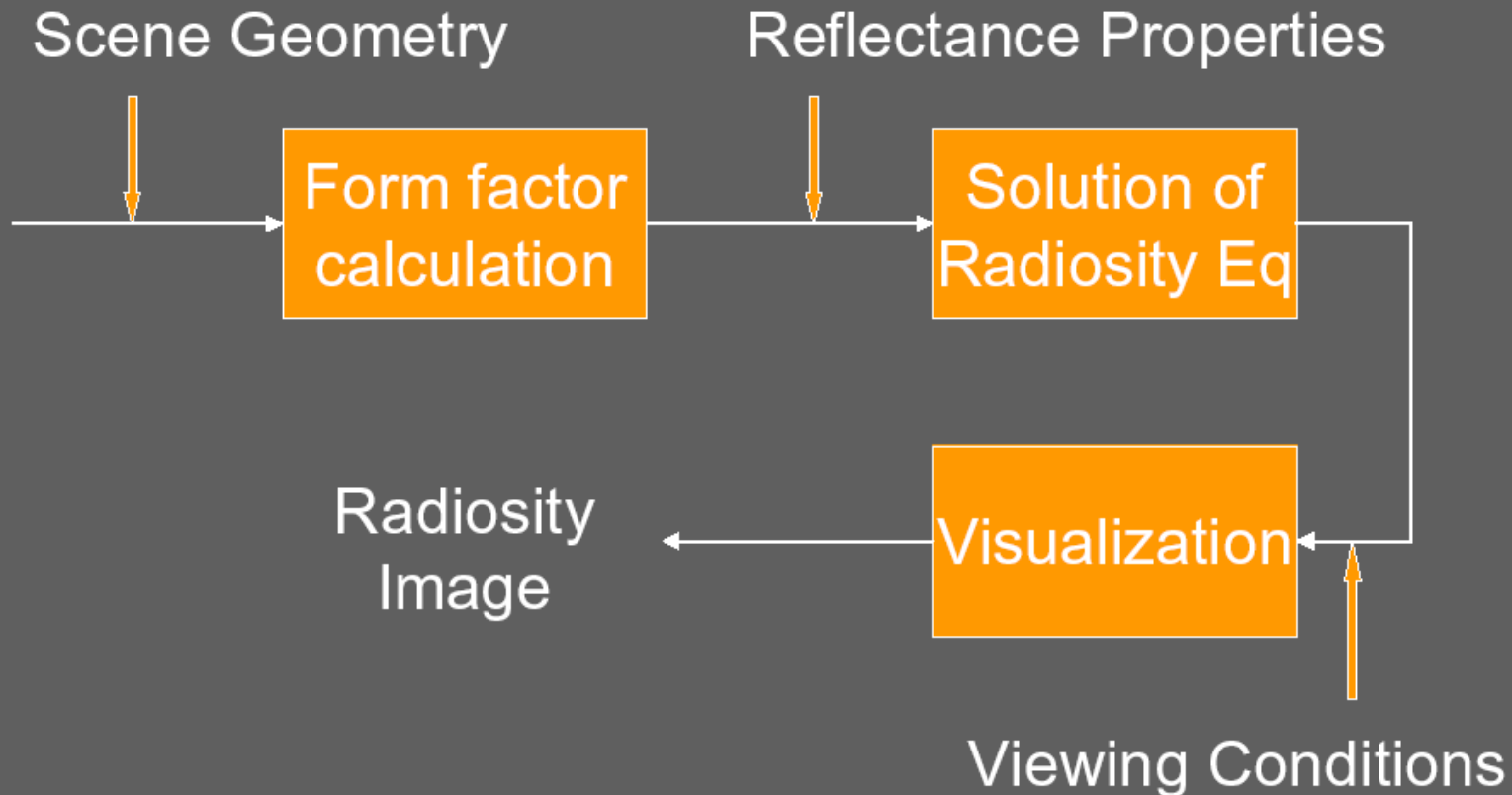


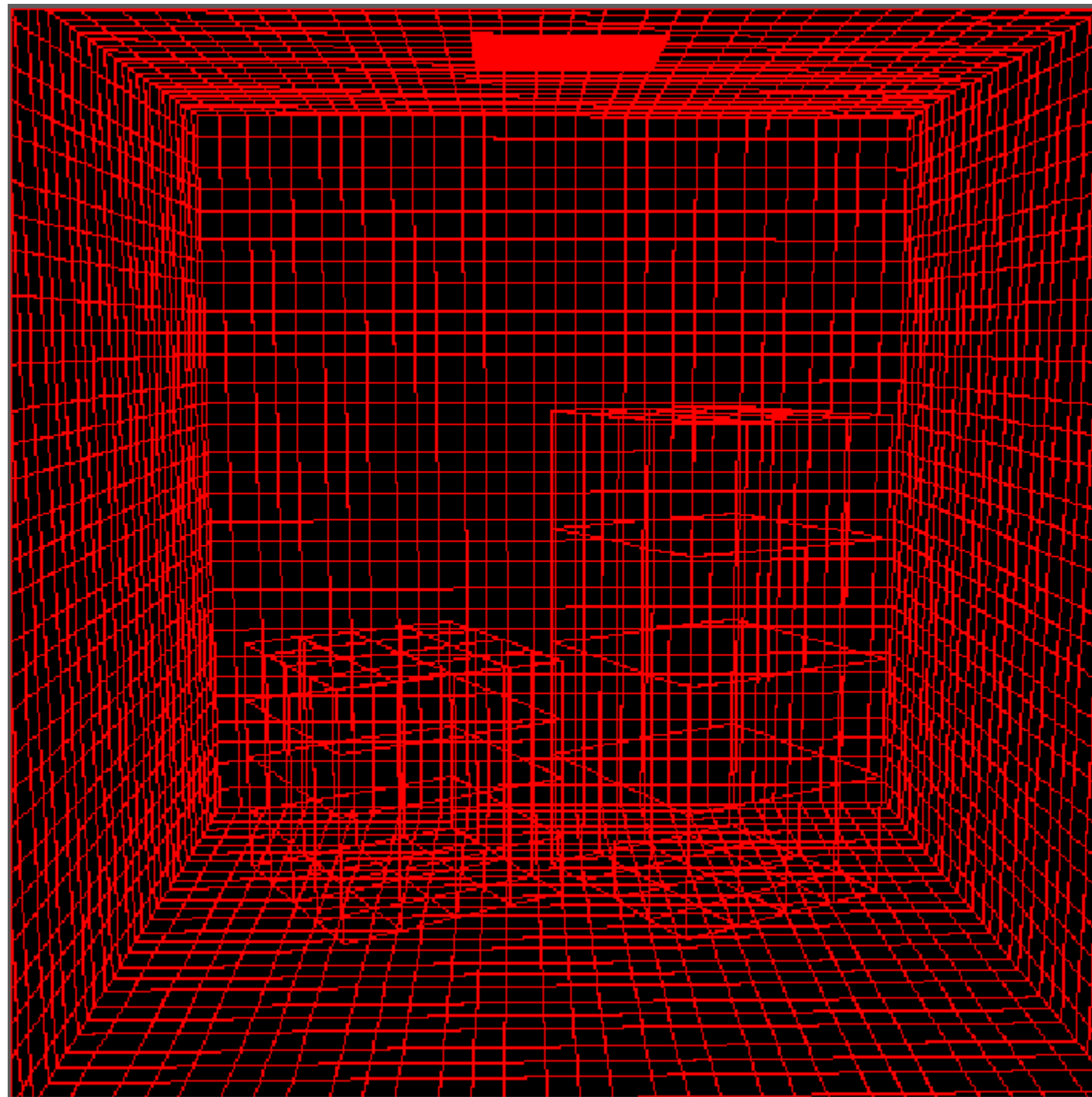
new value

old values

- Repeat until converged

Radiosity “Pipeline”

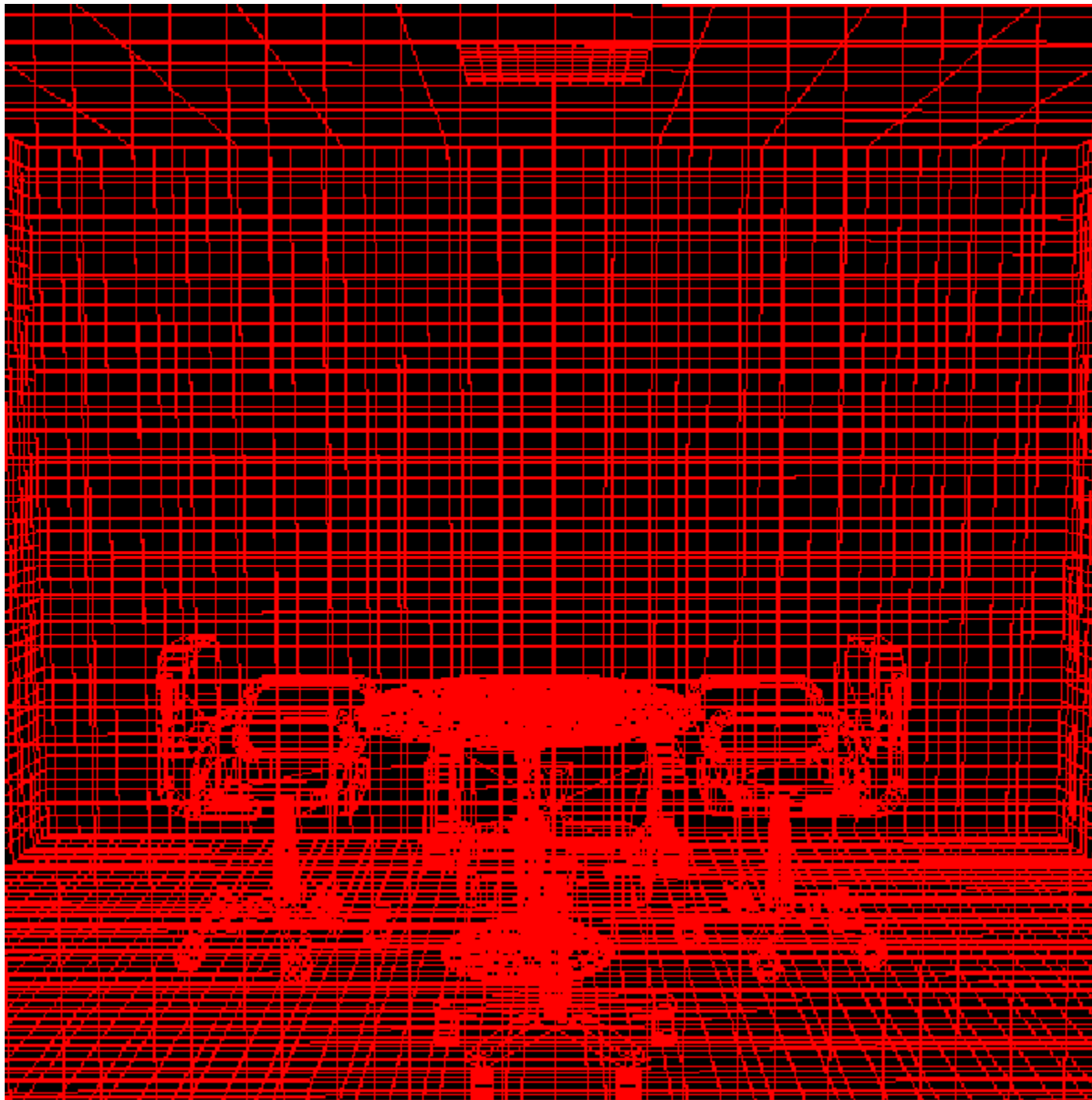




Wireframe



- Classical Approach
- No Interpolation



Wireframe



- Classical Approach
- Low Res

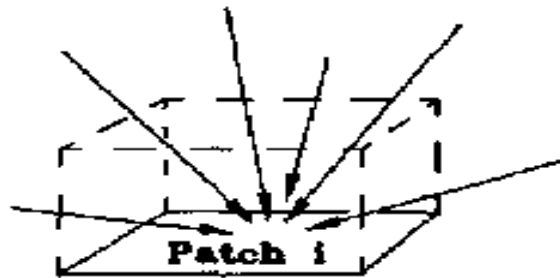


- Classical Approach
- High Res
- More accurate

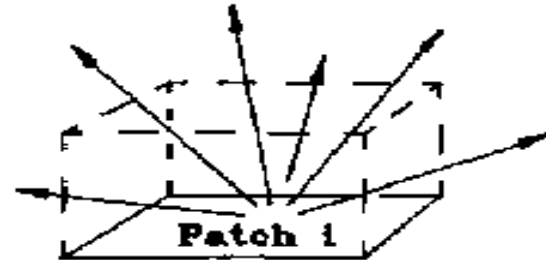


- Classical Approach
- High Res
- Interpolated

Progressive Solution



GATHERING



SHOOTING

vs.

$$\begin{bmatrix} \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{x} \end{bmatrix} + \begin{bmatrix} \text{XXXXXXXXXX} \end{bmatrix} \begin{bmatrix} \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \end{bmatrix}$$

$$\begin{bmatrix} \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \end{bmatrix} = \begin{bmatrix} \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \end{bmatrix} + \begin{bmatrix} \mathbf{x} \end{bmatrix} \begin{bmatrix} \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \\ \text{XXXXXX} \end{bmatrix}$$

$$B_1 = E_1 + \sum_{j=1}^N (\rho_1 R_{1j}) B_j$$

For all j:

$$B_j = B_j + B_1 (\rho_j R_{j1})$$

$$\text{where: } R_{j1} = R_{1j} A_1 / A_j$$

Figure 1: Gathering vs. Shooting



PROGRESSIVE SOLUTION

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.

Sample Scenes



Sample Scenes



From Cohen, Chen, Wallace and Greenberg 1988

Sample Scenes



Sample Scenes



Sample Scenes



Classic radiosity = finite element method

Assumptions

- **Diffuse reflectance**
- **Usually polygonal surfaces**

Advantages

- **Soft shadows and indirect lighting**
- **View independent solution**
- **Precompute for a set of light sources**
- **Useful for walkthroughs**

Review: Local vs. Global Illumination

- Global illumination: **Ray tracing**
 - **Realistic specular** reflection/transmission
 - Simplified diffuse reflection*
- Global illumination: **Radiosity**
 - **Realistic diffuse** reflection
 - Diffuse-only: No specular interaction*

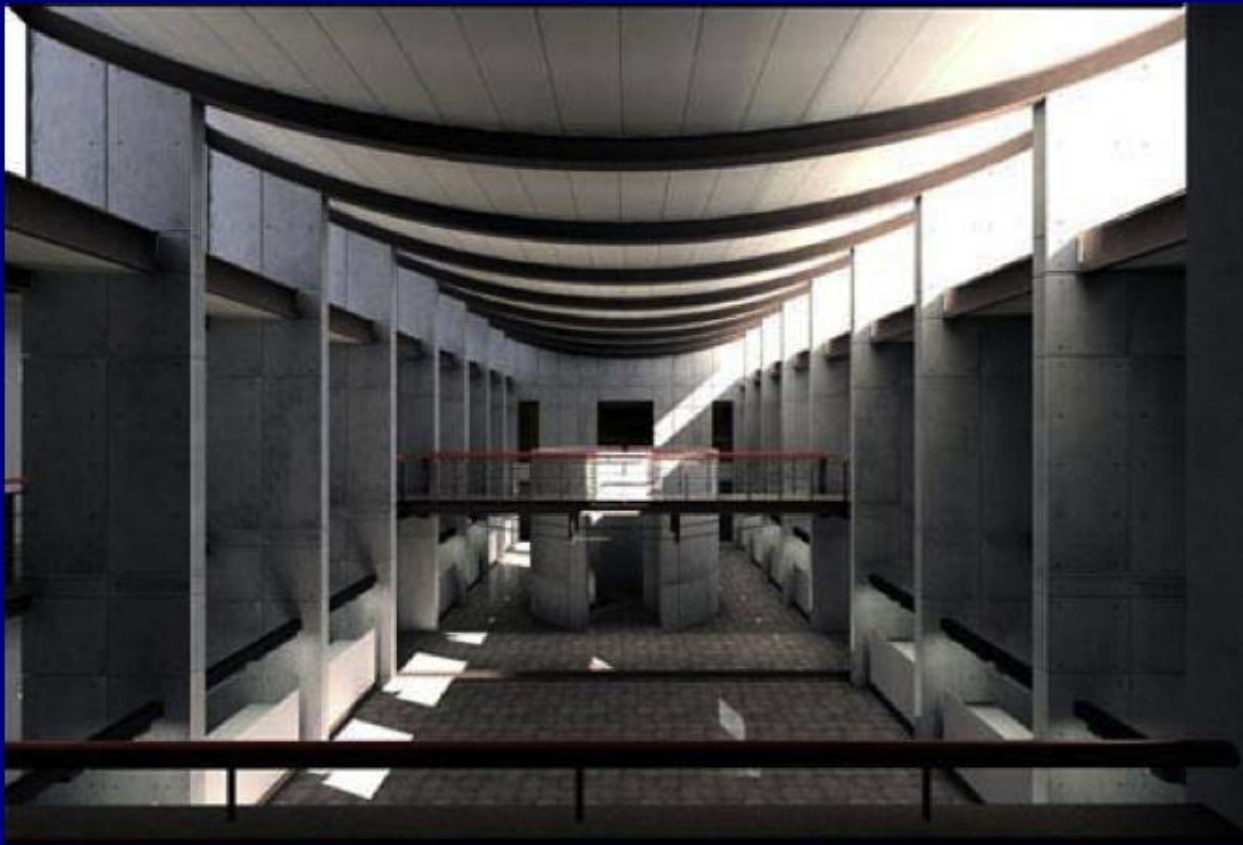


indirect

direct

both

Radiosity Examples



<http://www.autodesk.com/us/lightscape/examples/html/index.htm>

Raytracing Examples



<http://www.povray.org/>

Raytracing Examples



Radiosity Examples



<http://www.autodesk.com/us/lightscape/examples/html/index.htm>

Image vs. Object Space

- Image space: **Ray tracing**
 - Trace backwards from viewer
 - View-dependent calculation
 - Result: rasterized image (pixel by pixel)
- Object space: **Radiosity**
 - Assume only diffuse-diffuse interactions
 - View-independent calculation
 - Result: 3D model, color for each surface patch
 - Can render with OpenGL

A Better Idea: The Best of Both Worlds

Combine radiosity and raytracing

Goal: Represent four forms of light transport:

- Diffuse -> Diffuse
- Diffuse -> Specular
- Specular -> Diffuse
- Specular -> Specular

Two-pass approach, one for each method

First Pass: Enhanced Radiosity

Diffuse -> Diffuse

- Normal diffuse reflection model

- Diffuse transmission (translucent objects) – requires modified form factor

Specular -> Diffuse

- Specular transmission (transparent objects, e.g. windows) – involves extended form factor

- Specular reflection (reflective objects, e.g. mirrors) – create actual “mirror image” environment with copies of all patches. Expensive!

Enhanced Radiosity - Evaluation

- Only accounts for a single specular reflection (try creating “mirror image” environments for two mirrors facing each other)
- Accurate diffuse model
- Equations solved as in the classical method
- Still viewer-independent

Second Pass: Enhanced Raytracing

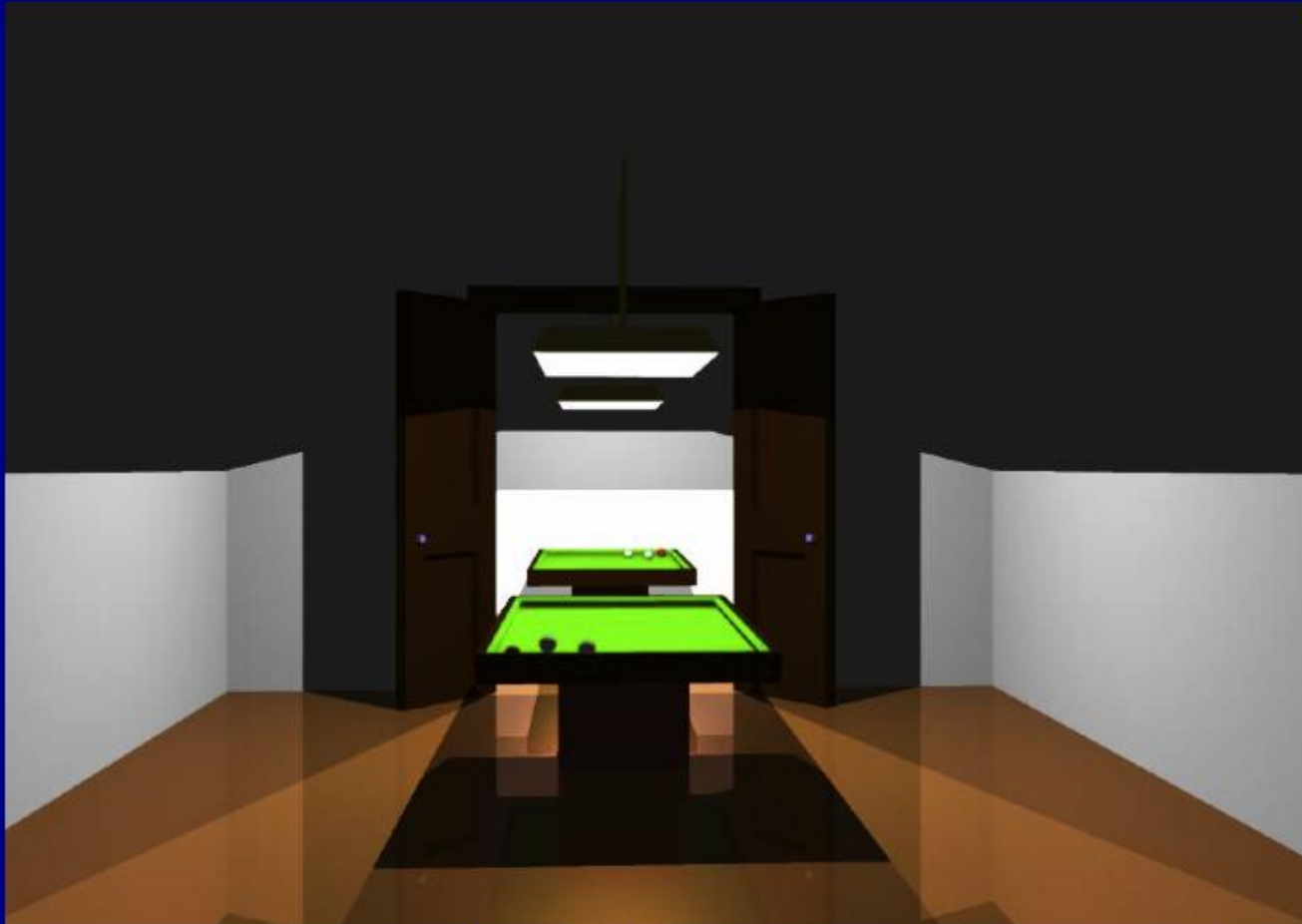
- Specular -> Specular
 - Reflection and transmission as in classical method
- Diffuse -> Specular
 - Use the radiosity calculated in the first pass
 - Integrate incoming light over a hemisphere (or hemicube), or approximate with a tiny frustum in the direction of reflection
 - Recurse if visible surface is specular

First Pass Result

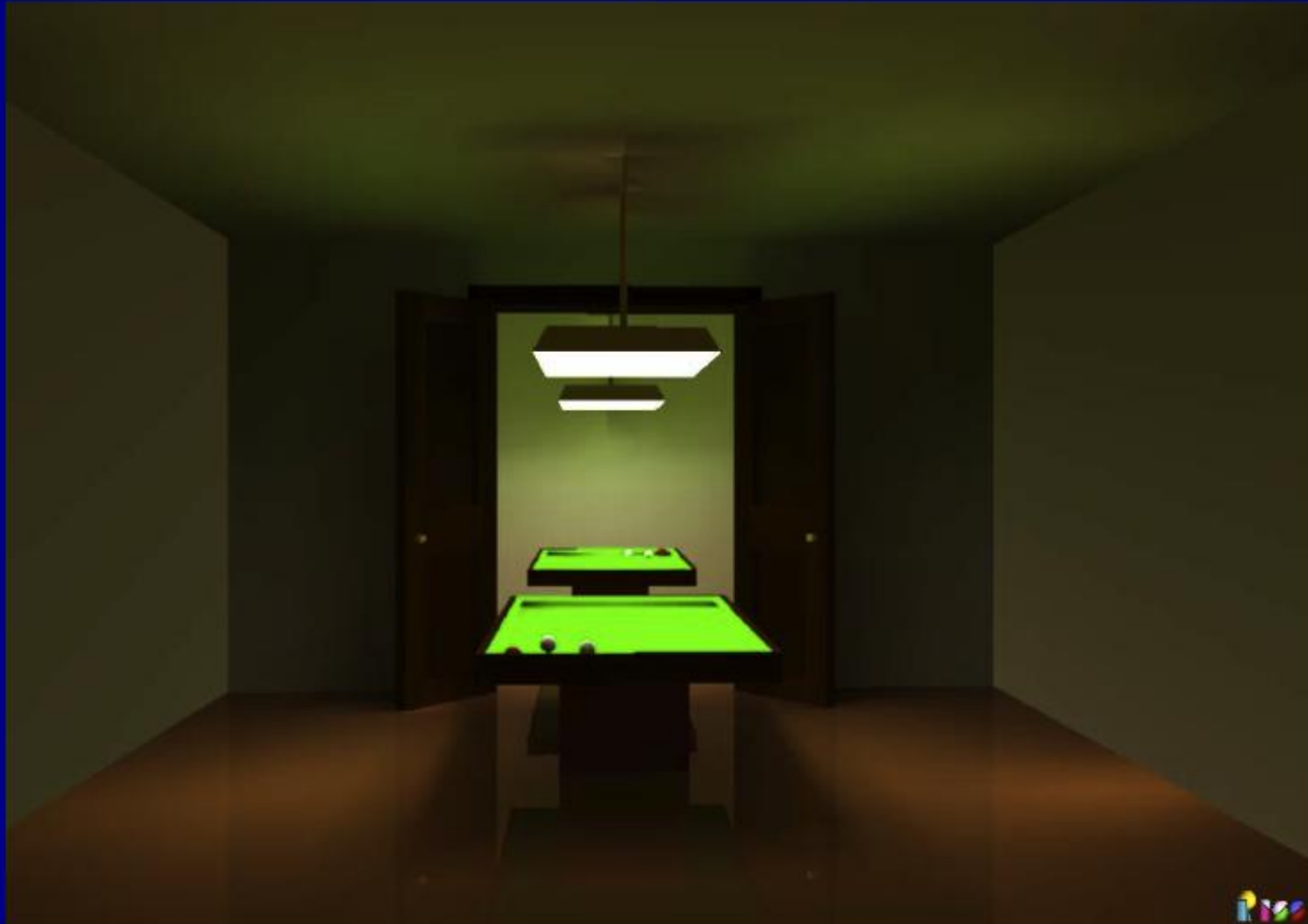


Second Pass Result

(radiosity info. not yet used, just raytracing)



Combined (Final) Result



Two-Pass Global Illumination: Evaluation

Very expensive. Takes the cost of radiosity added to the cost of raytracing and then throws even more calculations into the mix

Many approximations remain, particularly in specular $\rightarrow$ diffuse and diffuse $\rightarrow$ specular transport