Radiosity

Thanks to Kavita Bala, Pat Hanrahan, Doug James, Ledah Casburn
Mies Courtyard House with Curved Elements

Modeling: Stephen Duck; Rendering: Henrik Wann Jensen
Lighting Effects

Hard Shadows

Soft Shadows

Caustics

Indirect Illumination

Pat Hanrahan, Spring 2002
The ambient lighting in the upper-right image is approximated by a constant value. This is typical of most scanline algorithms. The middle and lower-left images were rendered with a ray tracing global illumination algorithm.

The middle image was rendered with no ambient light calculations. The lower-left image was rendered with several levels of diffuse re-reflection to give a better approximation of the ambient light in this scene.
Phong Shading

- Plastic looking scene
  - no object interactions
  - no shadows
Ray Tracing

Scene doesn’t look realistic enough.

- where is the corner of room?
- is window flush with wall?
- is the carpet and wood supposed to be this dark?
Radiosity – today’s topic

Indirect lighting affects realism.

- room has a corner
- window has depth
- carpet and wood on table is lighter
- walls look more pink
The Rendering Equation – Graph Style

\[ i(p, p') = v(p, p')(\epsilon(p, p') + \int \rho(p, p', p'')i(p', p'') dp'') \]

Visibility (shadows) \quad Emission (light source) \quad Reflectance from Surfaces
Conservation of Energy

Emitted power = self-emitted power + received & reflected power

© Kavita Bala, Computer Science, Cornell University
Diffuse Interreflections - Radiosity

- Consider lambertian surfaces and sources.
- Radiance independent of viewing direction.
- Consider total power leaving per unit area of a surface.
- Can simulate soft shadows and color bleeding from diffuse surfaces.
- Used abundantly in heat transfer literature
Irradiance, Radiosity

- Irradiance $E$ is the power **received** per unit surface area
  - Units: $W/m^2$

- Radiosity
  - Power per unit area **leaving** the surface (like irradiance)

*Figure 2.8: Projection of differential area.*
Planar piecewise constancy assumption

- Subdivide scene into small “uniform” polygons

Table in room sequence from Cohen and Wallace
Power Equation

- Power from each polygon:

\[ \forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^{N} \Phi_j F(i \rightarrow j) \]

- Linear System of Equations:

- \( \Phi_i \): power of patch i (unknown)
- \( \Phi_{ei} \): emission of patch i (known)
- \( \rho_i \): reflectivity of patch i (known)
- \( F(j \rightarrow i) \): form-factor (coefficients of matrix)
Form Factor

- $F_{j \rightarrow i} =$ the fraction of power emitted by $j$, which is received by $i$
- **Area**
  - if $i$ is smaller, it receives less power
- **Orientation**
  - if $i$ faces $j$, it receives more power
- **Distance**
  - if $i$ is further away, it receives less power
Form Factor

\[ F(j \rightarrow i) = \frac{1}{A_j} \int \int \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} \, V(x, y) \, dA_y \, dA_x \]

- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder

© Kavita Bala, Computer Science, Cornell University
Form Factors Invariant

\[ F(j \rightarrow i) = \frac{1}{A_j} \int \int \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_y \, dA_x \]

\[ F(i \rightarrow j) = \frac{1}{A_i} \int \int \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_x \, dA_y \]

\[ F(i \rightarrow j) A_i = F(j \rightarrow i) A_j \]
Form Factor Computation

\[ F(j \rightarrow i) = \frac{1}{A_j} \int \int \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_y \, dA_y \]

- Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.
- In general, computing double integral is hard.
- Use Monte Carlo Integration.
Being Smart about Form Factors

Form factors depend only on scene geometry. If geometry is constant, they only need to be calculated once.

Solution of the radiosity system is independent of viewing conditions, so if only the viewer position changes, it only needs to be solved once—can walk around the scene in real-time after it’s initially generated.
Being Smart about Form Factors

Form factors are complicated. Full numeric approximation of these is expensive—many special cases may be solved analytically.

Because we assume that radiosity is constant across a patch, two patches are typically assumed to be fully inter-visible or not at all inter-visible. That means that patches have to be small enough to resolve shadows and other complexities.
How to perform visibility testing?

Two basic methods, both of which have aliasing problems:

- Raycasting (typically slow)
- Hemicube method (z-buffer exploit)

Anti-aliasing may be performed in both cases
Hemicube Visibility Testing

Render the entire scene from the perspective of the center of the current patch.
Rather than color, store patch identifiers, using the z-buffer to determine visibility.
Takes advantage of graphics hardware.

R. Ramamoorthi
Hemicube in Action

http://www.siggraph.org/education/materials/HyperGraph/radiosity/overview_2.htm
Hemicube in Action

Power $\rightarrow$ Radiosity

$$\Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^{N} \Phi_j F(j \rightarrow i)$$

Divide by $A_i$

$$\frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(j \rightarrow i)}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \rightarrow j) A_i}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \rightarrow j)}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \rightarrow j)$$
Linear System of Radiosity Equations

\[ \forall \text{patches } i: \quad B_i = B_{ei} + \rho_i \sum_{j} F_{i \rightarrow j} B_j \]

\[
\begin{bmatrix}
1 - \rho_1 F_{1 \rightarrow 1} & -\rho_1 F_{1 \rightarrow 2} & \cdots & -\rho_1 F_{1 \rightarrow n} \\
-\rho_2 F_{2 \rightarrow 1} & 1 - \rho_2 F_{2 \rightarrow 2} & \cdots & -\rho_2 F_{2 \rightarrow n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n F_{n \rightarrow 1} & -\rho_n F_{n \rightarrow 2} & \cdots & 1 - \rho_n F_{n \rightarrow n}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= 
\begin{bmatrix}
B_{e1} \\
B_{e2} \\
\vdots \\
B_{en}
\end{bmatrix}
\]

- Known
- Unknown

- Matrix Inversion to Solve for Radiosities.
Iterative approaches

- Jacobi iteration
- Start with initial guess for energy distribution (light sources)
- Update radiosity/power of all patches based on the previous guess

\[ B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \rightarrow j) \]

new value  \hspace{1cm} \text{old values}

- Repeat until converged
Radiosity “Pipeline”

Scene Geometry

Form factor calculation

Solution of Radiosity Eq

Radiosity Image

Visualization

Viewing Conditions

Reflectance Properties
• Classical Approach
• No Interpolation
• Classical Approach
• Low Res
• Classical Approach
• High Res
• More accurate
• Classical Approach
• High Res
• Interpolated
Progressive Solution

A Progressive Refinement Approach to Fast Radiosity Image Generation, Cohen et al 88

For all j:
\[ B_j = B_j + B_1 (\rho_j A_{j1}/A_j) \]

where: \( E_{j1} = E_{j1} A_{j1}/A_j \)

Figure 1: Gathering vs. Shooting
PROGRESSIVE SOLUTION

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.
Sample Scenes
Sample Scenes

From Cohen, Chen, Wallace and Greenberg 1988
Sample Scenes
Sample Scenes
Radiosity Summary

Classic radiosity = finite element method

Assumptions
- Diffuse reflectance
- Usually polygonal surfaces

Advantages
- Soft shadows and indirect lighting
- View independent solution
- Precompute for a set of light sources
- Useful for walkthroughs
Review: Local vs. Global Illumination

• Global illumination: Ray tracing
  – Realistic specular reflection/transmission
  – Simplified diffuse reflection*

• Global illumination: Radiosity
  – Realistic diffuse reflection
  – Diffuse-only: No specular interaction*

[Diagram showing indirect, direct, and both lighting effects]
Radiosity Examples

Raytracing Examples

http://www.povray.org/
Raytracing Examples

http://www.povray.org/
Radiosity Examples

Image vs. Object Space

- **Image space:** Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)

- **Object space:** Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL
A Better Idea: The Best of Both Worlds

Combine radiosity and raytracing

Goal: Represent four forms of light transport:

- Diffuse $\rightarrow$ Diffuse
- Diffuse $\rightarrow$ Specular
- Specular $\rightarrow$ Diffuse
- Specular $\rightarrow$ Specular

Two-pass approach, one for each method
First Pass: Enhanced Radiosity

Diffuse -> Diffuse
- Normal diffuse reflection model
- Diffuse transmission (translucent objects) – requires modified form factor

Specular -> Diffuse
- Specular transmission (transparent objects, e.g. windows) – involves extended form factor
- Specular reflection (reflective objects, e.g. mirrors) – create actual “mirror image” environment with copies of all patches. Expensive!
Enhanced Radiosity - Evaluation

- Only accounts for a single specular reflection (try creating “mirror image” environments for two mirrors facing each other)
- Accurate diffuse model
- Equations solved as in the classical method
- Still viewer-independent
Second Pass: Enhanced Raytracing

- Specular -> Specular
  - Reflection and transmission as in classical method
- Diffuse -> Specular
  - Use the radiosity calculated in the first pass
  - Integrate incoming light over a hemisphere (or hemicube), or approximate with a tiny frustum in the direction of reflection
  - Recurse if visible surface is specular
First Pass Result

http://www.cg.tuwien.ac.at/research/rendering/rays-radio/
Second Pass Result
(radiosity info. not yet used, just raytracing)
Combined (Final) Result
Two-Pass Global Illumination: Evaluation

Very expensive. Takes the cost of radiosity added to the cost of raytracing and then throws even more calculations into the mix. Many approximations remain, particularly in specular -> diffuse and diffuse -> specular transport.