Spatial Data Structures

Hierarchical Bounding Volumes
Grids
Octrees
BSP Trees
Speeding Up Computations
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• Ray Tracing
  – Spend a lot of time doing ray object intersection tests
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• Hidden Surface Removal – painters algorithm
  – Sorting polygons front to back
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  – Quickly determine if two objects collide
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Spatial data-structures

$n^2$ computations
Spatial Data Structures

- We’ll look at
  - Hierarchical bounding volumes
  - Grids
  - Octrees
  - K-d trees and BSP trees

- Good data structures can give speed up ray tracing by 10x or 100x
Bounding Volumes

- Wrap things that are hard to check for intersection in things that are easy to check
  - Example: wrap a complicated polygonal mesh in a box
  - Ray can't hit the real object unless it hits the box
  - Adds some overhead, but generally pays for itself.
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- But you don’t want expensive intersection tests!

![Bounding Volumes Diagram](image)
Bounding Volumes

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- Use the ratio of the object volume to the enclosed volume as a measure of fit.

\[
\text{Cost} = n \times B + m \times I
\]

- \( n \) - is the number of rays tested against the bounding volume
- \( B \) - is the cost of each test (Do not need to compute exact intersection!)
- \( m \) - is the number of rays which actually hit the bounding volume
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Hierarchical Bounding Volumes

• Still need to check ray against every object --- $O(n)$
• Use tree data structure
  – Larger bounding volumes contain smaller ones
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Check intersect root
If not return no intersections
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  check intersect left sub-tree
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Hierarchical Bounding Volumes

• Many ways to build a tree for the hierarchy
• Works well:
  – Binary
  – Roughly balanced
  – Boxes of sibling trees not overlap too much
Hierarchical Bounding Volumes

- Sort the surfaces along the axis before dividing into two boxes
- Carefully choose axis each time
- Choose axis that minimizes sum of volumes
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Hierarchical Bounding Volumes

• Works well if you use good (appropriate) bounding volumes and hierarchy

• Should give $O(\log n)$ rather than $O(n)$ complexity
  ($n=$ # of objects)

• Can have multiple classes of bounding volumes and pick the best for each enclosed object
Hierarchical bounding volumes

Spatial Subdivision

- Grids
- Octrees
- K-d trees and BSP trees
3D Spatial Subdivision

- Bounding volumes enclose the objects (object-centric)

- Instead could divide up the space—the further an object is from the ray the less time we want to spend checking it
  - Grids
  - Octrees
  - K-d trees and BSP trees
Grids

- Data structure: a 3-D array of cells (voxels) that tile space
  - Each cell points to list of all surfaces intersecting that cell

- Intersection testing:
  - Start tracing at cell where ray begins
  - Step from cell to cell, searching for the first intersection point
  - At each cell, test for intersection with all surfaces pointed to by that cell
  - If there is an intersection, return the closest one
Grids

- Cells are traversed in an incremental fashion
- Hits of sets of parallel lines are very regular
More on Grids

- Be Careful! The fact that a ray passes through a cell and hits an object doesn’t mean the ray hit that object in *that* cell

- Optimization: cache intersection point and ray id in “mailbox” associated with each object

- Step from cell to cell
- Get object intersecting cell
- Find closest intersection
- If found intersection --- done
More on Grids

- Grids are a poor choice when the world is nonhomogeneous (clumpy)
  - many polygons clustered in a small space

- How many cells to use?
  - too few $\Rightarrow$ many objects per cell $\Rightarrow$ slow
  - too many $\Rightarrow$ many empty cells to step through $\Rightarrow$ slow

- Non-uniform spatial subdivision is better!
Octrees

- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”
Octrees

- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get “simple enough”

- Octree is the 3-D generalization of quadtree
  - node (cell) is a cube, recursively split into eight equal sub-cubes
  - for ray tracing:
    - stop subdivision based on number of objects
    - internal nodes store pointers to children, leaves store list of surfaces
  - more expensive to traverse than a grid
  - but an octree adapts to non-homogeneous scenes better

```c
trace(cell, ray) {  // returns object hit or NONE
  if cell is leaf, return closest(objects_in_cell(cell))
  for child cells pierced by ray, in order       // 1 to 4 of these
    obj = trace(child, ray)
    if obj!=NONE return obj
  return NONE
}
```
Which Data Structure is Best for Ray Tracing?

Grids
- Easy to implement
- Require a lot of memory
- Poor results for inhomogeneous scenes

Octrees
- Better on most scenes (more adaptive)

Spatial subdivision expensive for animations
- Hierarchical bounding volumes
- Better for dynamic scenes
- Natural for hierarchical objects
k-d Trees and BSP Trees

- Relax the rules for quadtrees and octrees:
  - k-dimensional (k-d) tree
    - don’t always split at midpoint
    - split only one dimension at a time (i.e. x or y or z)
  - binary space partitioning (BSP) tree
    - permit splits with any line
    - in 2-D space split with lines (most of our examples)
    - 3-D space split with planes
    - K-D space split with k-1 dimensional hyperplanes

- useful for Painter’s algorithm (hidden surface removal)
Painters Algorithm

Hidden Surface Elimination
Painters Algorithm

- Need to sort objects back to front
- Order depends on the view point
- Partition objects using BSP tree
- View independent
Building a BSP Tree

- Let’s look at simple example with 3 line segments
- Arrowheads are to show left and right sides of lines.
- Using line 1 or 2 as root is easy.
- (examples from http://www.geocities.com/SiliconValley/2151/bsp.html)
Drawing Objects

- Traverse the tree from the root
- If view point is on the left of the line --- traverse right sub-tree first
- Draw the root
- Traverse left sub-tree

* a BSP tree using 2 as root
Building the Tree 2

Using line 3 for the root requires a split
Triangles

Use plane containing triangle $T_1$ to split the space.
If view point is on one side of the plane draw polygons on the other side first.
$T_2$ does not intersect plane of $T_1$. 
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Triangles

Split Triangle
Building a Good Tree - the tricky part

• A naïve partitioning of $n$ polygons will yield $O(n^3)$ polygons because of splitting!

• Algorithms exist to find partitionings that produce $O(n^2)$.
  – For example, try all remaining polygons and add the one which causes the fewest splits
  – Fewer splits -> larger polygons -> better polygon fill efficiency

• Also, we want a balanced tree.
Painter’s Algorithm with BSP trees

• Build the tree
  – Involves splitting some polygons
  – Slow, but done only once for static scene

• Correct traversal lets you draw in back-to-front or front-to-back order for any viewpoint
  – Order is view-dependent
  – Pre-compute tree once
  – Do the “sort” on the fly

• Will not work for changing scenes
Drawing a BSP Tree

• Each polygon has a set of coefficients:
  \[ Ax + By + Cz + D \]
  
• Plug the coordinates of the viewpoint in and see:
  >0 : front side
  <0 : back facing
  =0 : on plane of polygon

• Back-to-front draw: inorder traversal, do farther child first
• Front-to-back draw: inorder traversal, do near child first

```java
front_to_back(tree, viewpt) {
  if (tree == null) return;
  if (positive_side_of(root(tree), viewpt)) {
    front_to_back(positive_branch(tree, viewpt);
    display_polygon(root(tree));
    front_to_back(negative_branch(tree, viewpt);
  } else { ...draw negative branch first...}
}
```
Drawing Back to Front

- Use Painter’s Algorithm for hidden surface removal

Steps:
- *Draw objects on far side of line 3*
  - *Draw objects on far side of line 2a*
    - Draw line 1
  - *Draw line 2a*
  - *Draw line 3*
  - *Draw objects on near side of line 3*
    - Draw line 2b
Demos

BSP Tree construction
http://symbolcraft.com/graphics/bsp/index.html

• KD Tree construction
Real-time and Interactive Ray Tracing

The OpenRT Real-Time Ray-Tracing Project
http://www.openrt.de/index.php

• Interactive ray tracing via space subdivision
  http://www.cs.utah.edu/~reinhard/egwr/

• Interactive ray tracing with good hardware
  http://www.cs.utah.edu/vissim/projects/raytracing/