15-462 Homework 1

Due Date: Tuesday, October 13, 2009, at the beginning of lecture

1. **Barycentric Coordinates**

   Suppose there is a triangle in 2D space whose vertices are:
   $a = [-2, -1]$, $b = [1, -2]$, and $c = [-1, 3]$.

   (a) Using barycentric coordinates, prove whether each of the following points lie within the triangle.

      i. $[0, 0]$
      ii. $[0, 1]$
      iii. $[-\frac{3}{2}, \frac{3}{2}]$

   (b) Consider the point $p = [-1, 0]$ which lies within the triangle. Suppose the texture coordinate at a is $[\frac{1}{4}, 0]$, at b is $[1, \frac{1}{4}]$, and at c is $[\frac{1}{2}, 1]$. What is the texture coordinate at p?

2. **Shading**

   For this question, we consider the Phong Illumination Model described in class and used by OpenGL fixed-functionality, consisting of an ambient term, a diffuse term, and a specular term.

   (a) Given a point light at position L, a viewer at position V, and a surface at point P, what should the normal of the surface be to:

      i. Maximize the amount of diffuse light that reaches the viewer?
      ii. Maximize the amount of specular light that reaches the viewer?

      Justify your answers, perhaps including a picture.

   (b) This shading model very poorly approximates most real-life surfaces. Explain why Phong shading cannot be used to accurately render:

      i. Human skin
      ii. The moon

3. **Surfaces**

   Given two different 3D points v_1 and v_2, what is the implicit equation for the plane whose points are all equidistant to v_1 and v_2? What is the parametric equation? What is the normal to this plane?

4. **Cameras**

   Derive an expression for the blur circle diameter, b, of a thin lens system with focal length f and aperture diameter d. **Hint:** Use the diagram on slide 32 in the lecture notes. You may use i, i', o, and o' in your answer. Based on this expression argue whether the depth of field increases or decreases with:

   (a) Aperture diameter

   (b) Distance of the object to the plane of focus

5. **Viewing**

   Consider a perspective projection function $P([x, y, z]^T) = [\alpha\frac{x}{z}, \alpha\frac{y}{z}]^T$ for some fixed α. Prove that two parallel lines $a(t) = [x_1, y_1, z_1]^T + t[u, v, w]^T$ and $b(t) = [x_2, y_2, z_2]^T + t[u, v, w]^T$ have the same vanishing point under P.