1. Barycentric Coordinates

Suppose there is a triangle in 2D space whose vertices are:
\( a = [-2, -1], \ b = [1, -2], \) and \( c = [-1, 3]. \)

(a) Using barycentric coordinates, prove whether each of the following points lie within the triangle.
   i. \([0, 0]\)
   ii. \([0, 1]\)
   iii. \([-\frac{3}{2}, \frac{3}{2}]\)

(b) Consider the point \( p = [-1, 0] \) which lies within the triangle. Suppose the texture coordinate at \( a \) is \([\frac{1}{4}, 0]\), at \( b \) is \([1, \frac{1}{4}]\), and at \( c \) is \([\frac{1}{2}, 1]\).
   What is the texture coordinate at \( p \)?

2. Shading

For this question, we consider the Phong Illumination Model described in class and used by OpenGL fixed-functionality, consisting of an ambient term, a diffuse term, and a specular term.

(a) Given a point light at position \( L \), a viewer at position \( V \), and a surface at point \( P \), what should the normal of the surface be to:
   i. Maximize the amount of diffuse light that reaches the viewer?
   ii. Maximize the amount of specular light that reaches the viewer?
   Justify your answers, perhaps including a picture.

(b) This shading model very poorly approximates most real-life surfaces.
   Explain why Phong shading cannot be used to accurately render:
   i. Human skin
   ii. The moon

3. Surfaces

Given two different 3D points \( v_1 \) and \( v_2 \), what is the implicit equation for the plane whose points are all equidistant to \( v_1 \) and \( v_2 \)? What is the parametric equation? What is the normal to this plane?
4. **Cameras**

Derive an expression for the blur circle diameter, $b$, of a thin lens system with focal length $f$ and aperture diameter $d$. **Hint:** Use the diagram on slide 32 in the lecture notes. You may use $i$, $i'$, $o$, and $o'$ in your answer. Based on this expression argue whether the depth of field increases or decreases with:

(a) Aperture diameter

(b) Distance of the object to the plane of focus

5. **Viewing**

Consider a perspective projection function $P([x, y, z]^T) = [\alpha x, \alpha y]^T$ for some fixed $\alpha$. Prove that two parallel lines $a(t) = [x_1, y_1, z_1]^T + t[u, v, w]^T$ and $b(t) = [x_2, y_2, z_2]^T + t[u, v, w]^T$ have the same vanishing point under $P$. 
