15-456 Computational Geometry, Spring 2013

Homework 2 (60 pts) Due: 26 April 2013

Guidelines: Please justify all answers in a succienct (yet complete) manner. In particular, when presenting an algorithm the code if any should be presented at a high level. A full algorithm will contain the input, the output, and any loop invariants.

Question	Points	Score
1	5	
2	5	
3	30	
4	30	
5	30	
Total:	100	

(5) 1. **Homology Computation** Compute the Betti numbers of a sphere by computing the simplicial homology of a hollow tetrahedron. In addition, give the generators of the nontrivial homology groups.

(5) 2. **Triangulating a Torus** A torus can be obtained from the unit square by identifying opposite sides, as shown in Figure 1. Give a simplicial complex that represents a torus.

(30) 3. **Finding a bases for the cycle space**

Let $G = (V, E)$ be a connected 1D simplicial complex(a graph).

1. Show that the dimension of the cycle space (The kernel of ∂_1) is the number of non-tree edges and give a bases for the space. That is let T be a spanning tree of G find a bases of the cycle space by considering the edges E $E(T)$.
2. Use Euler's formula for a planar graph to determine Betti one.

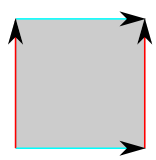

Figure 1: This representation of the torus has one 2-cell, two edges, and one vertex. In this question, you are asked to triangulate this complex.

(30) 4. **Sauers Lemma is tight** Show that Sauers lemma is tight for each VC dimension. Specifically, provide a finite range space that has the number of ranges as claimed by Lemma 5.9 That is: for any $\delta > 0$ there exists a finite range space (X, R) of VC dimension δ with $|X| = n$ such that $|R| = \mathcal{G}_\delta(n)$.

(30) 5. **Another algorithm for k-center clustering** Consider the algorithm that, given a point set P and a parameter k, initially picks an arbitrary set $C \subset P$ of k points. Next, it computes the closest pair of points $c, f \in C$ and the point s realizing $||P_C||_\infty$. If $d(s, C) > d(c, f)$, then the algorithm sets $C \leftarrow C - c + s$ and repeats this process till the condition no longer holds.

1. Prove that this algorithm outputs a k-center clustering of radius $\leq 2opt_\infty(P, k)$.
2. What is the running time of this algorithm?