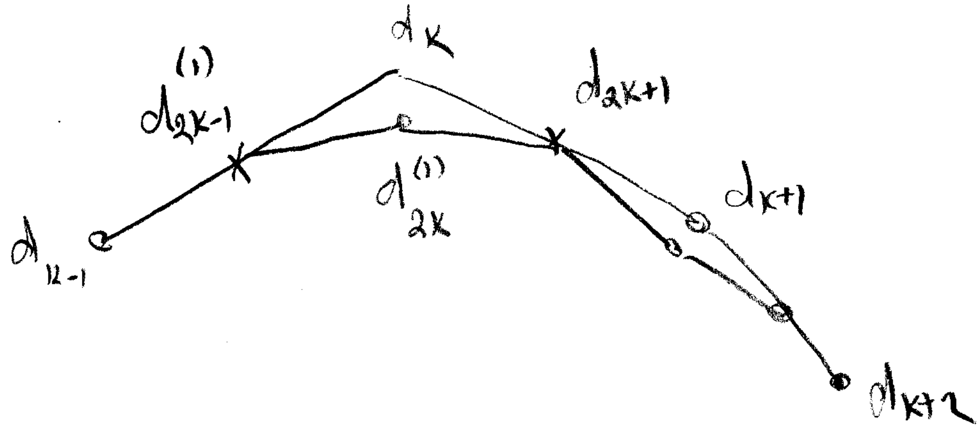
13-456
4/24/13

Recursive Subdivision

1D first Polygon $d_1 \cdots d_n$

Define $d_1^{(1)} \cdots d_{2n}^{(1)}$



$$d_{2k+1}^{(1)} = \frac{1}{2} d_i + \frac{1}{2} d_{i+1}$$

$$d_{2i}^{(1)} = \frac{1}{8} d_{i-1} + \frac{3}{4} d_i + \frac{1}{8} d_{i+1}$$

$$\underbrace{\begin{pmatrix} d_{2k-1}^{(1)} \\ d_{2k}^{(1)} \\ d_{2k+1}^{(1)} \end{pmatrix}}_{D^{(1)}} = \frac{1}{8} \underbrace{\begin{bmatrix} 4 & 4 & 0 \\ 1 & 6 & 1 \\ 0 & 4 & 4 \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} d_{k-1} \\ d_k \\ d_{k+1} \end{pmatrix}}_{D}$$

$$D^{(1)} = AD \qquad D^{(t)} = A^t D_k$$

<u>note</u>: A is stochastic ie row sum are 1, irreducible.

$$A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow A^t \text{ is stochastic, irred}$$

eigenvalues of A are $1, \frac{1}{2}, \frac{1}{4}$

$$E = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 1 & & \\ & \frac{1}{2} & \\ & & \frac{1}{4} \end{pmatrix}$$

$$A = E \Lambda E^{-1}$$

$$E^{-1} = \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$$

Write $A^r = E\Lambda^r E^{-1}$

$$A^\infty = \lim_{r\to\infty} A^r = \lim_{r\to\infty} E\Lambda^r E^{-1} = E \lim_r \Lambda E^{-1}$$

$$= E\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} E^{-1}$$

$$= \frac{1}{6}\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} E^{-1} = \frac{1}{6}\begin{pmatrix} 1 & 4 & 1 \\ 1 & 4 & 1 \\ 1 & 4 & 1 \end{pmatrix}$$

Thus d_k "goes to" the point $\frac{1}{6}d_{k-1} + \frac{4}{6}d_k + \frac{1}{6}d_{k+1}$

So does it neighbors.

2D Surfaces

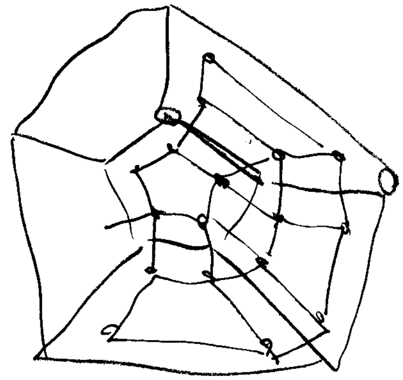
discuss Doo-Sabin

Catmull-Clark

Midpoint Sub

Loop

Doo-Sabin



Rule: 1) for each face F with $v_1 \cdots v_k$ vertices

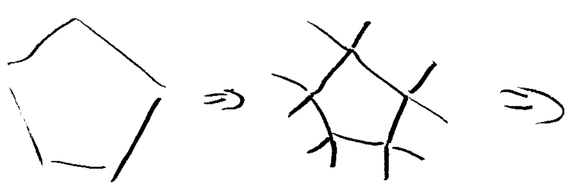
$$v_i^{(1)} = \sum \alpha_{ij} v_j$$

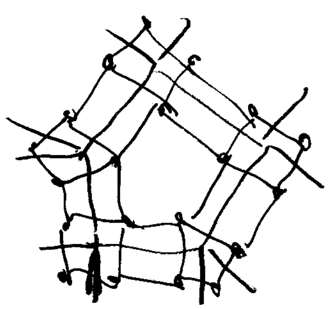
$$\alpha_{ii} = \frac{n+5}{4n} \qquad \alpha_{ij} = \frac{3 + 2\cos\left(\frac{2\pi(i-j)}{n}\right)}{4n}$$

2) stitch new face together.

Picture Each old 4face becomes a square mesh.

5-face?



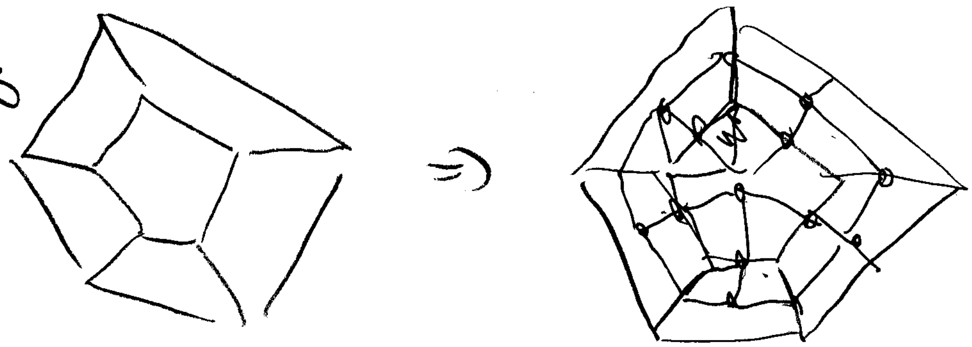


squares around a 5-gon

Catmull-Clark

Make new vertex, edge, face vertices.

eg



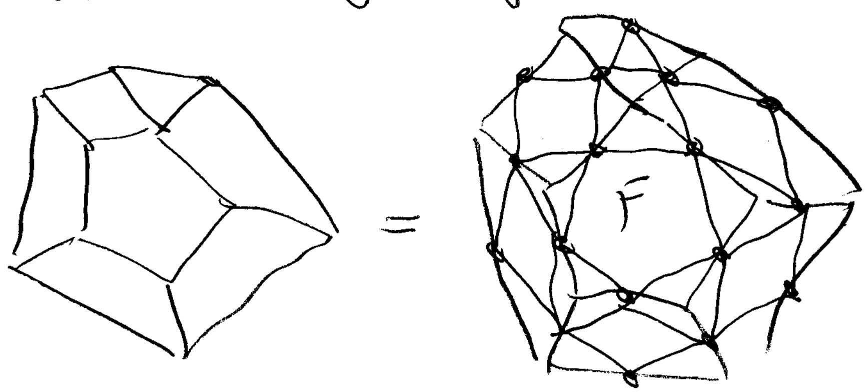
~~face point~~: $f_j^{(1)}$ = centroid of face vertices

new edge: $e_j^{(1)}$ = centroid of edge vertices + face point

mess!

Midpoint

newpoints are edge midpoints.



$P_1 \cdots P_n$ cyclic order of points on F

$$\begin{pmatrix} P_1^{(1)} \\ \vdots \\ P_n^{(n)} \end{pmatrix} = \underbrace{\begin{pmatrix} 1/2 & 1/2 & & \\ & 1/2 & 1/2 & \\ & & \ddots & \\ 1/2 & & & 1/2 \end{pmatrix}}_{M} \begin{pmatrix} P_1 \\ \vdots \\ P_n \end{pmatrix}$$

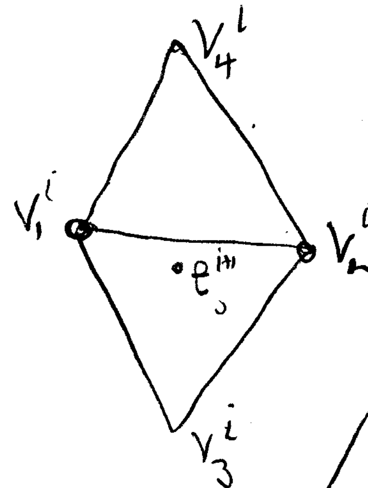
M = circulant

Loop Subdivision

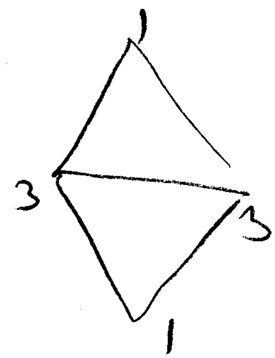
Input Triangulated surfaces

new points

edge points e_i^i = v_1^i to v_2^i



$$e_j^{i+1} = \frac{3}{8}v_1^i + \frac{3}{8}v_2^i + \frac{1}{8}v_3^i + \frac{1}{8}v_4^i$$

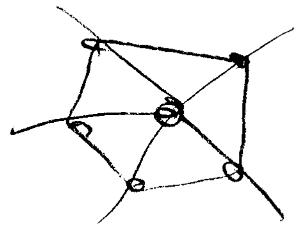


Vertex point v^i with neig $v_1^i - v_n^i$

$$v^{i+1} = (1-n\alpha)v^i + \alpha\sum_{j=1}^{n} v_j^i$$

Where $\alpha = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$ $n > 3$

$\alpha = 3/16$ $n = 3$



~~Cl~~

Claim by eigenvalues

limit of v & neigh of v is

$$v^{\infty} = \frac{3 + 8\alpha(n-1)}{3+8n\alpha} v + \frac{8\alpha}{3+8n\alpha} \sum_{i=1}^{n} v_i$$