Def: A graph is polytopal if the vertices and edges correspond to the 1-skeleton of a 3-polytope.

Thm [Steinitz] A graph is polytopal iff it is planar, simple, and 3-connected.
Fact: \( K_n \) the complete graph is \( n \)-polytopal

neighborly polytopes
Map for Today

Tutte's Alg
+ M-C correspondence
+ Fix Body issue

Steinitz's Thm
If $G$ is polytopal then it is simple, planar, and 3-connected.

Edges are line segments uniquely defined by endpoints.
Monotone Paths
Equil Stress
\[ \Rightarrow \]
Recip Diagrams
\[ \Rightarrow \]
Liftings

If outer face is flat after lifting from Tutte + MC
Then we're done.

\[ \Rightarrow \infty \text{- face} \]
If the outer face is a $\Delta \Rightarrow$ Done
Claim: Either G has a \( \Delta \) face or its dual has a \( \Delta \).

\[
\frac{E}{F} \geq \frac{4}{2} = 2 \quad V - E + F = 2
\]

\[
\frac{E}{V} \geq \frac{4}{2} = 2 \quad V - E + F \leq \frac{E}{2} - E + \frac{E}{2} = 0
\]

\[
2 \leq 0
\]

*Polar Polytope*
Construction

If $\exists \Delta$ face
3 Fix that $\Delta$ as outer face
  Run Tutte's Alg.
  Graph search to assign heights in lifting
3 else
  # dualize the graph
  # do this
  # polarize
3