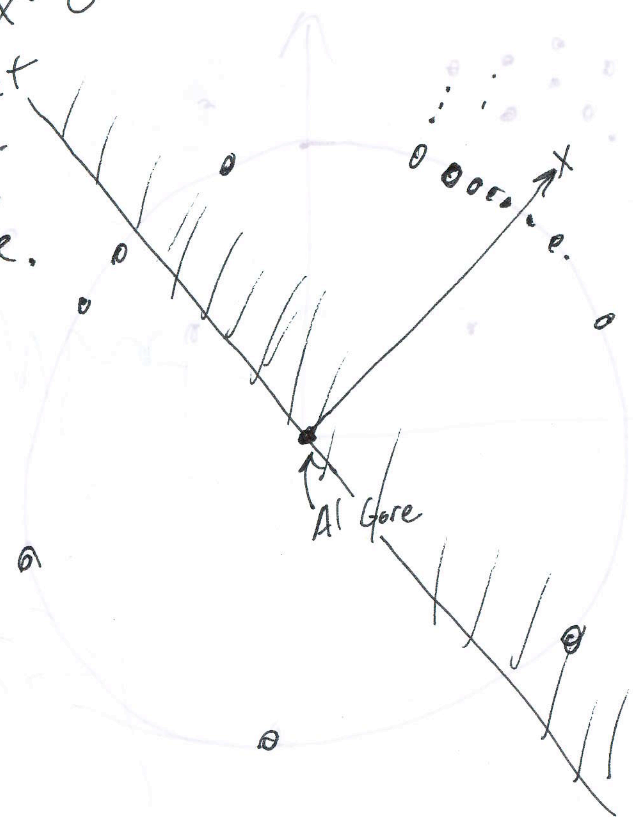
Maximum Feasible Subsystem

robot suffrage

Given system $Ax > 0$

Is there a subset of $\geq b$ constraints that are feasible.



P: voters

Choose $n \in \mathbb{R}^d$ to maximize

$$|\{p \in P : n \cdot p > 0\}|$$

$$h = \{x \in \mathbb{R}^d : n \cdot p \leq 0\}$$

python vs ruby

$$h = \underset{HS\ h'}{\operatorname{argmin}} |\{h' \cap P\}|$$

TukeyDepth

Given $P \subseteq \mathbb{R}^d$ and $b \in \mathbb{Z}$, $\underset{=0}{x} \in \mathbb{R}^d$

Is ? $D_\Pi(x) \geq b$

$I \in \text{TukeyDepth}(\cancel{x, P, b})$

$I = (P, b)$

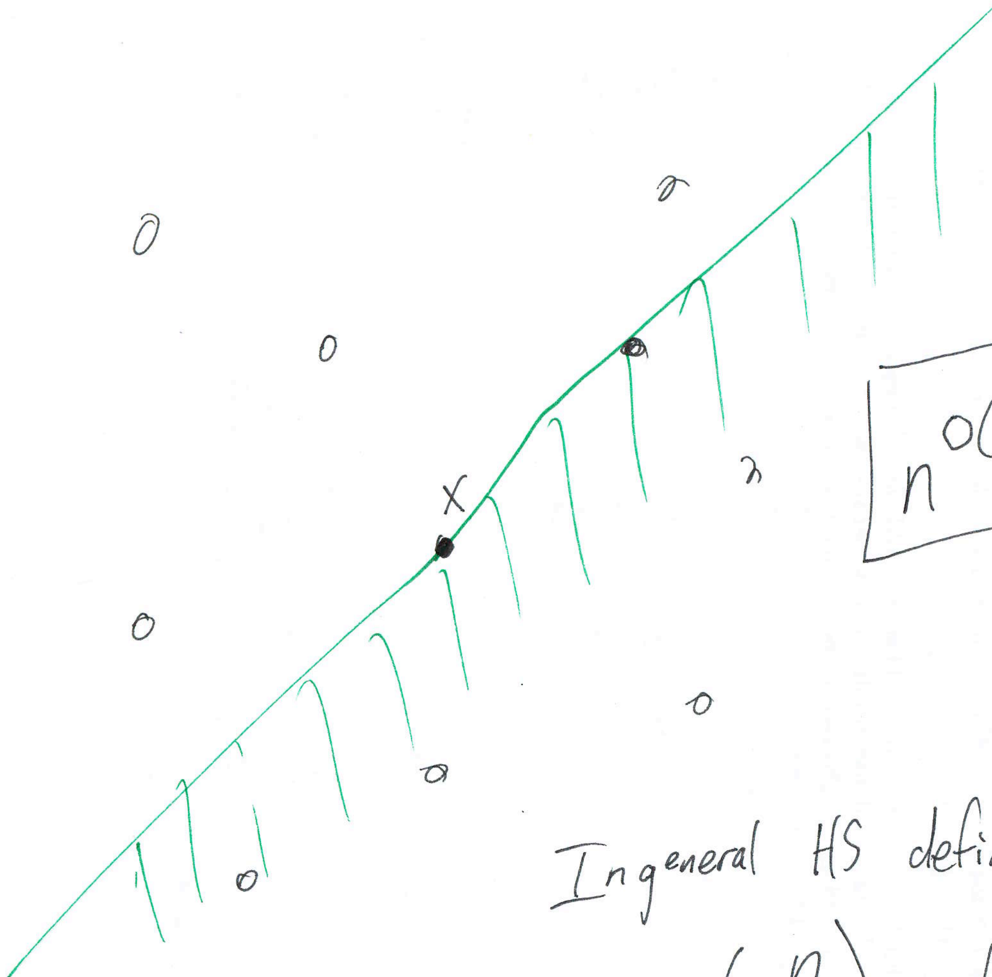
$I' = (P, n-b)$

Claim: $I' = (P, n-b) \in \overline{MaxFS}$

pf If $D_\Pi(x) < b$ then $\exists$ halfspace h s.t.

$|h \cap P| < b \iff \exists \bar{h}$ s.t. $|\bar{h} \cap P| \geq n-b$

$\Longrightarrow$ TukeyDepth is coNP-Complete



$$\boxed{n^{O(d)}}$$

In general HS defined by d pts

$$\binom{n}{d+1} \text{ choices } = n^{O(d)}$$

$$\begin{bmatrix} | & - \; p'_1 \; - & | \\ p_1 & \cdots & p_n \\ | & - \; p'_d \; - & | \end{bmatrix}$$

Selection

1) Geometric Medians + Depth
2) Centerpoints [They Exis!]
   - Prove via Helly + Radon
   - Prove Tverberg's Thm
3) Equivalence Tverberg + Centerpoints in $\mathbb{R}^d$
4) Computing cps in the plane. $O(n)$ time
5) Approx CPs in high dimensions $\approx \frac{n}{d^2}$
6) Centervertex thm for Wedge Depth $\alpha = \frac{3\pi}{2}$
7) Hardness of depth testing