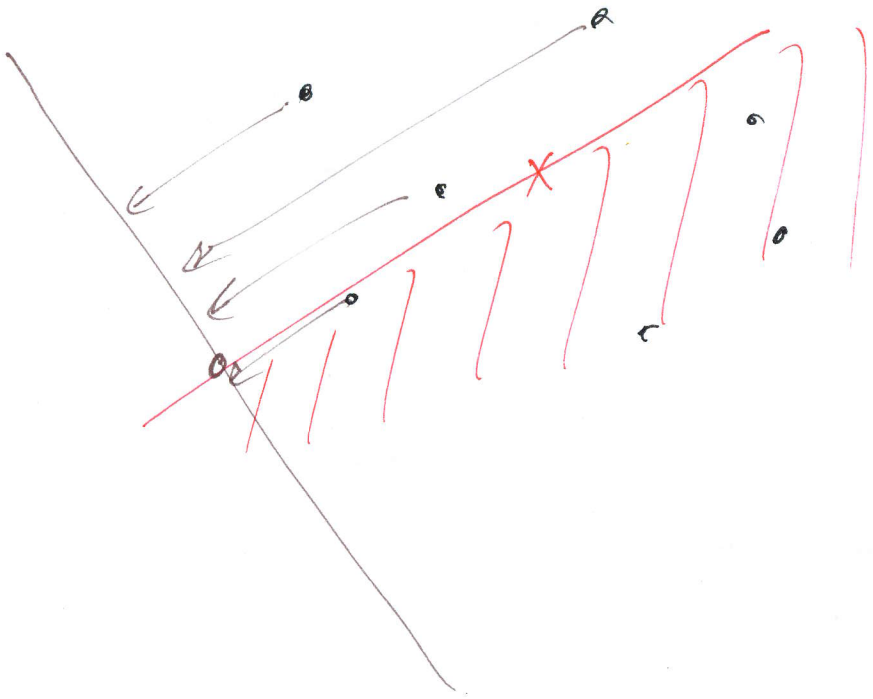
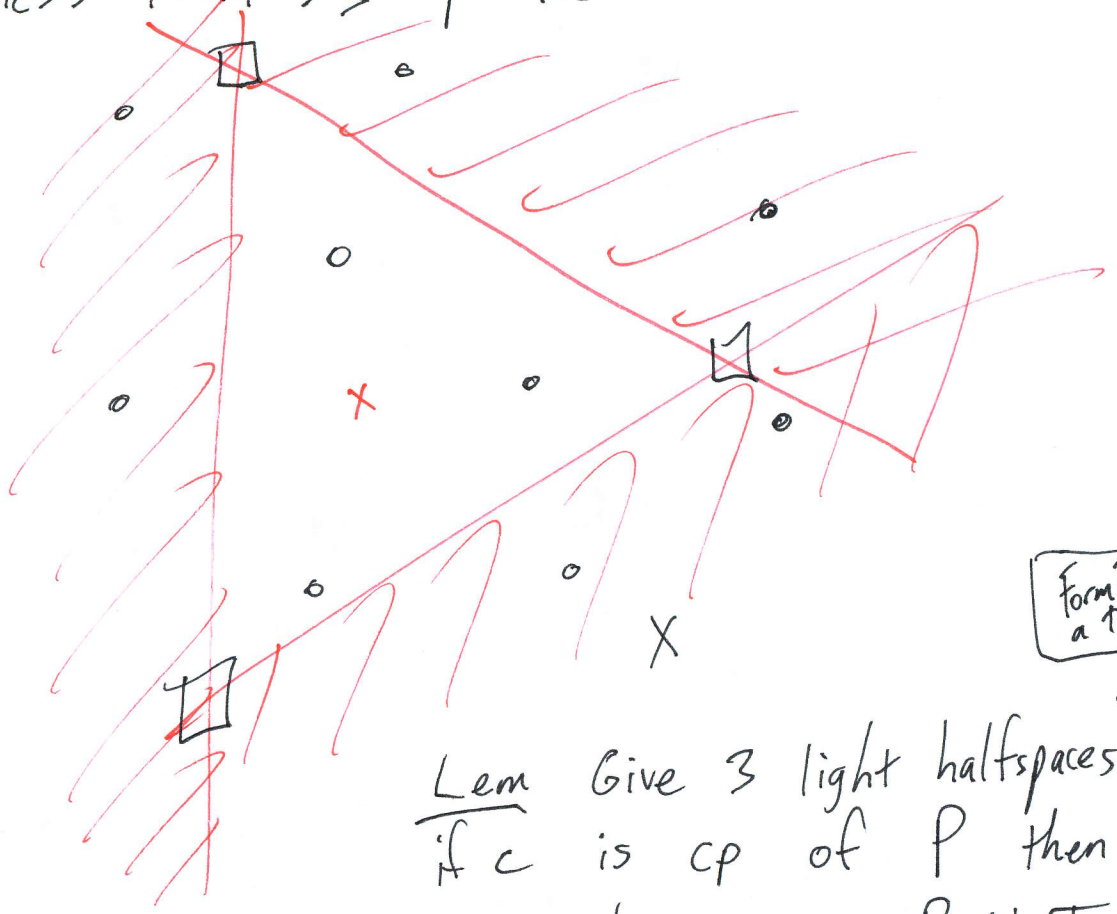
Centerpoints in the plane

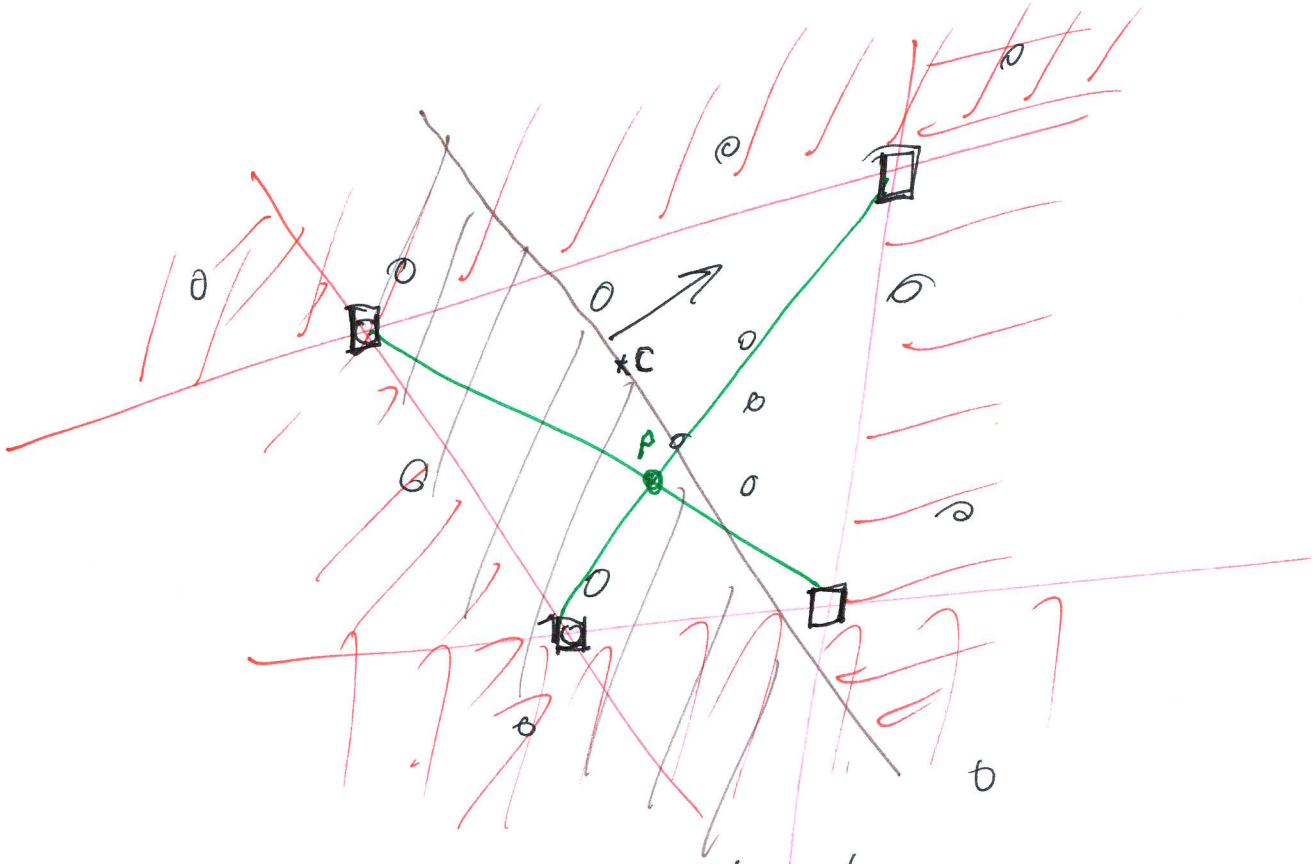


$\geq \frac{n}{3}$ pts

Define a halfspace to <u>light</u> if it contains less than $n/3$ points



<u>Lem</u> Give 3 light halfspaces if c is cp of P then c is also a cp $P \cup T$



Lem Given 4 light HSs
if c is a center point of $P \cup \{p\}$
then it's a cp of $P \cup Q$

Pf Let h be a HS containing c
if $p \in h$ $n' = n+3$ $n' = |P \cup Q|$ $n = |P \cup \{p\}|$

$n = |P \cup \{p\}|$ $n' = |P \cup Q|$

pf Let h be a halfspace containing c

if $p \in h$ then

$\{p\}$ ↙ Radon sets from Q ↙

$$|h \cap (P \cup Q)| \geq |h \cap (P \cup \{p\})| - 1 + 2$$

$$\geq \frac{n'}{3}$$

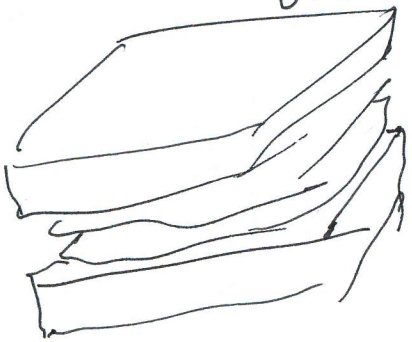
if $p \notin h$ then

From Q ↙

$$|h \cap (P \cup Q)| \geq |h \cap (P \cup \{p\})| + 1$$

$$\geq \frac{n'}{3}$$

Ham
Cheese
Bread

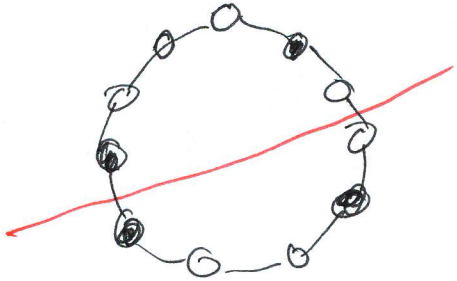


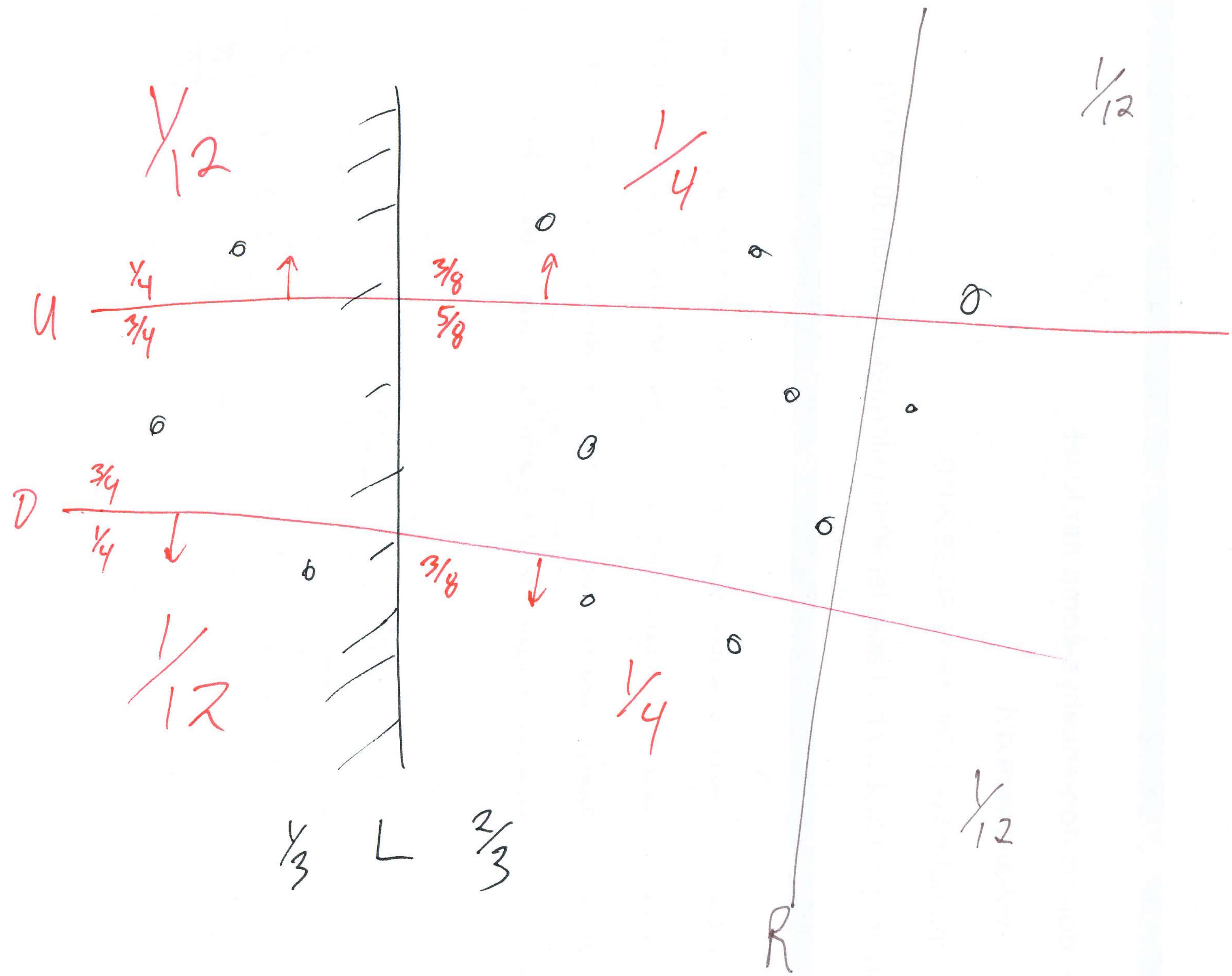
Ham Sandwich Thm

Given d sets in $\mathbb{R}^d$

$\exists$ hyperplane that bisects each set

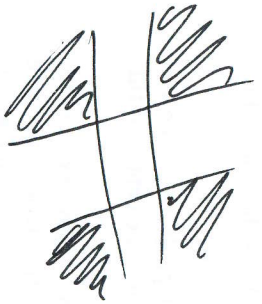
In the plane we can find HS cuts in $O(n)$ time.




1/12
1/4
1/12
U
1/4
3/4
3/8
5/8
D
3/4
1/4
3/8
1/12
1/4
1/12
1/3
L
2/3
R

Algorithm

Find L, U, D, R [$O(n)$ time]



Pick one point from each of

$(L \cap U), (L \cap D), (R \cap U), (R \cap D)$

Replace them with their Radon pt.

Repeat until one set is empty

Repeat until only a few pts left

$$T(n) = T\left(\frac{3n}{4}\right) + O(n) = O(n)$$