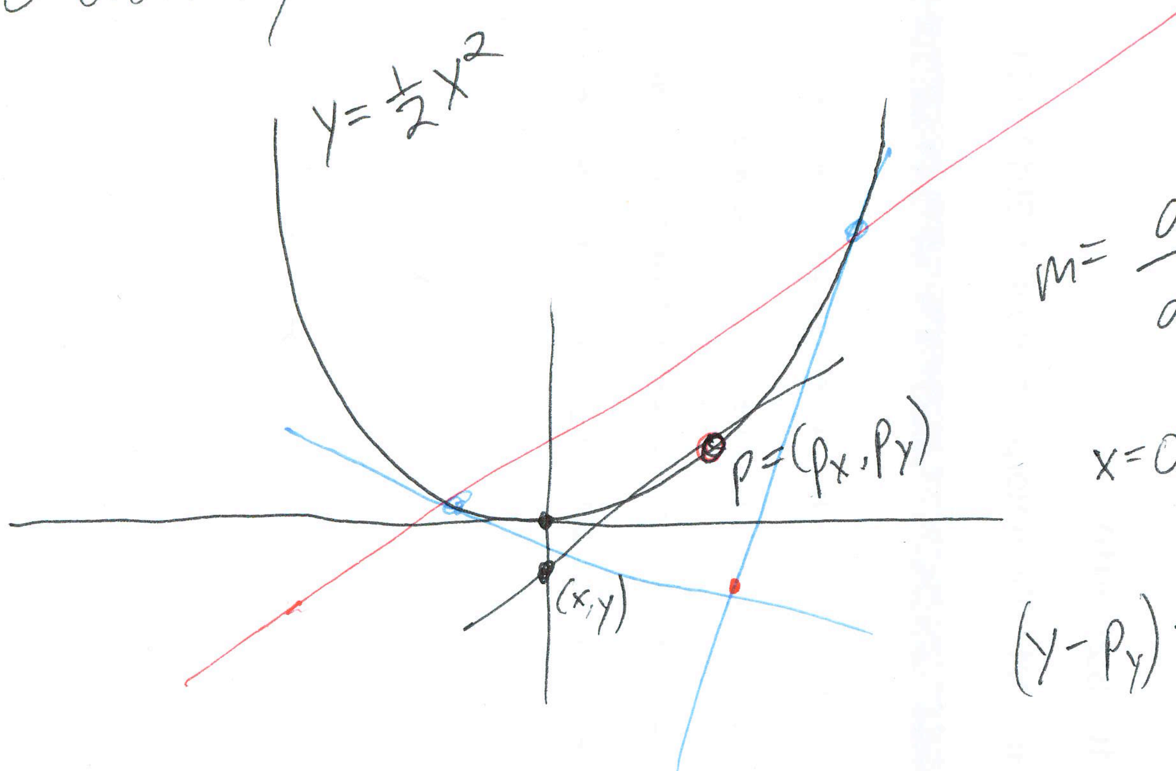
Duality without Coordinates



$$m = \frac{dy}{dx} = p_x$$

$x = 0$

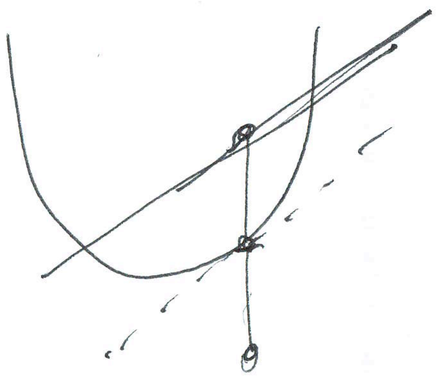
$$(y - p_y) = m(x - p_x)$$

$$= \cancel{x^2 - xp_x}$$

$$y - p_y = \cancel{p_x x} - p_x^2$$

$$y = p_y - p_x^2 = p_y - 2p_y$$

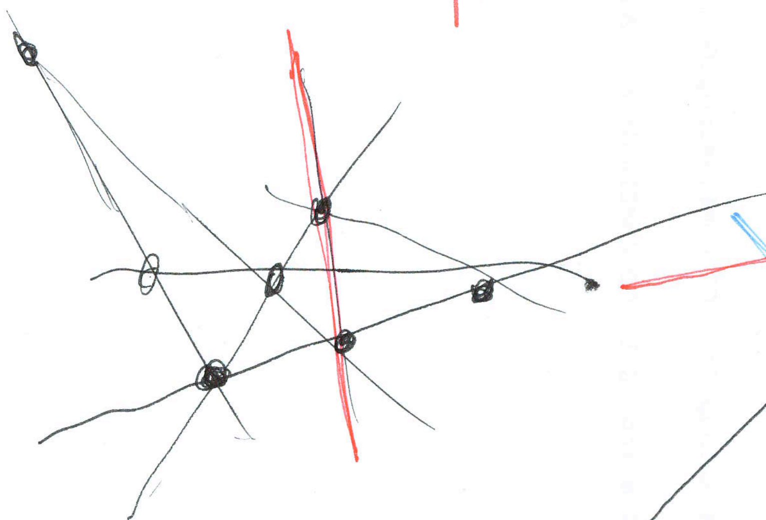
$$b = y = -p_y$$



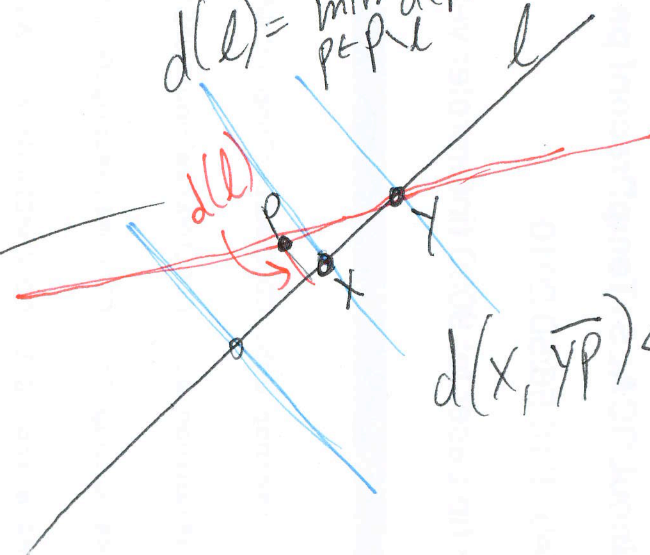
Sylvester-Gallai Thm

Given $n \geq 3$ ~~points~~ lines P not all on a ~~line~~ point

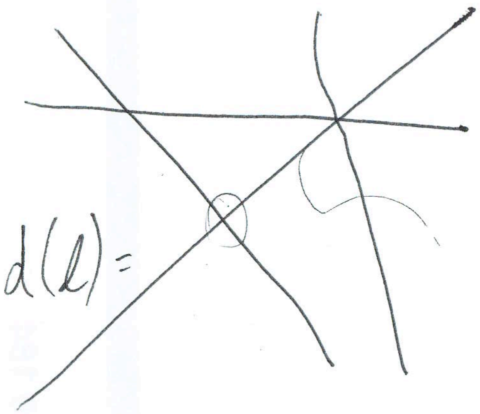
$\exists$ ~~line~~ point ℓ passing thru exactly 2 ~~points.~~ lines



$$d(\ell) = \min_{p \in P \setminus \ell} d(p, \ell)$$



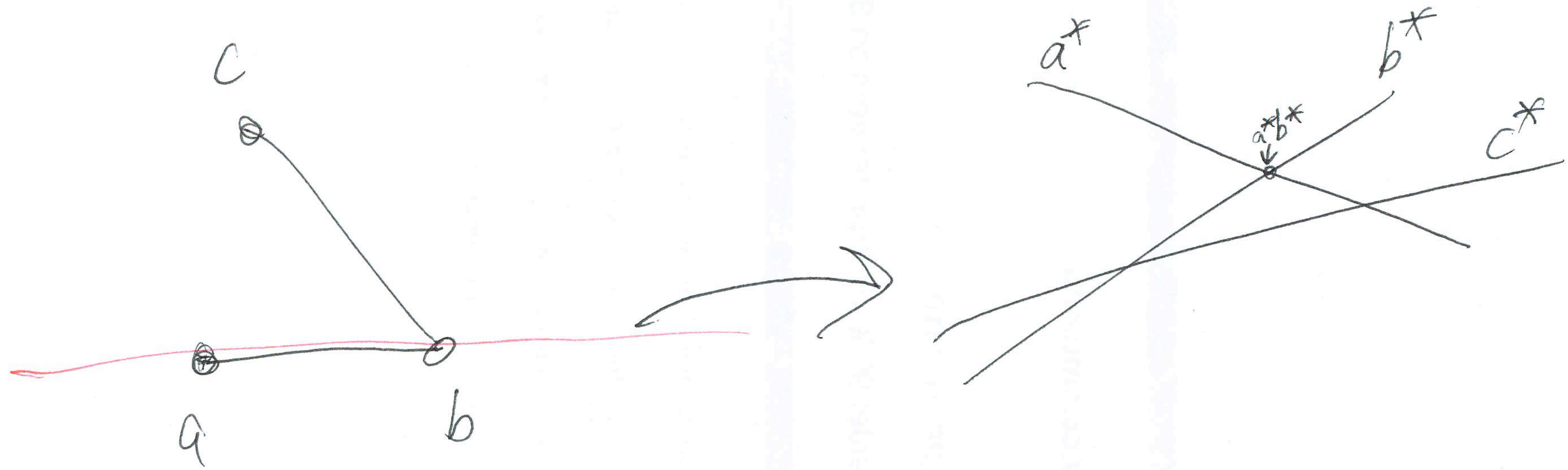
$$d(x, \overline{YP}) < d(\ell) =$$



Dual Version

Given $n \geq 3$ lines not all passing thru a common point, + not all parallel.

$\exists$ point p intersecting exactly 2 lines



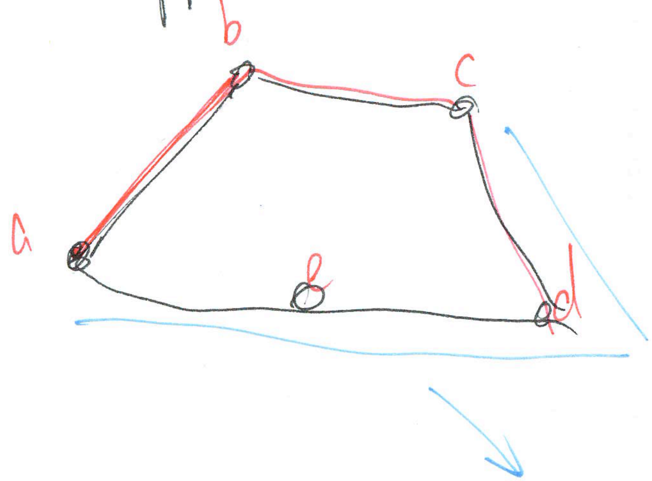
$ccw(a,b,c)$?

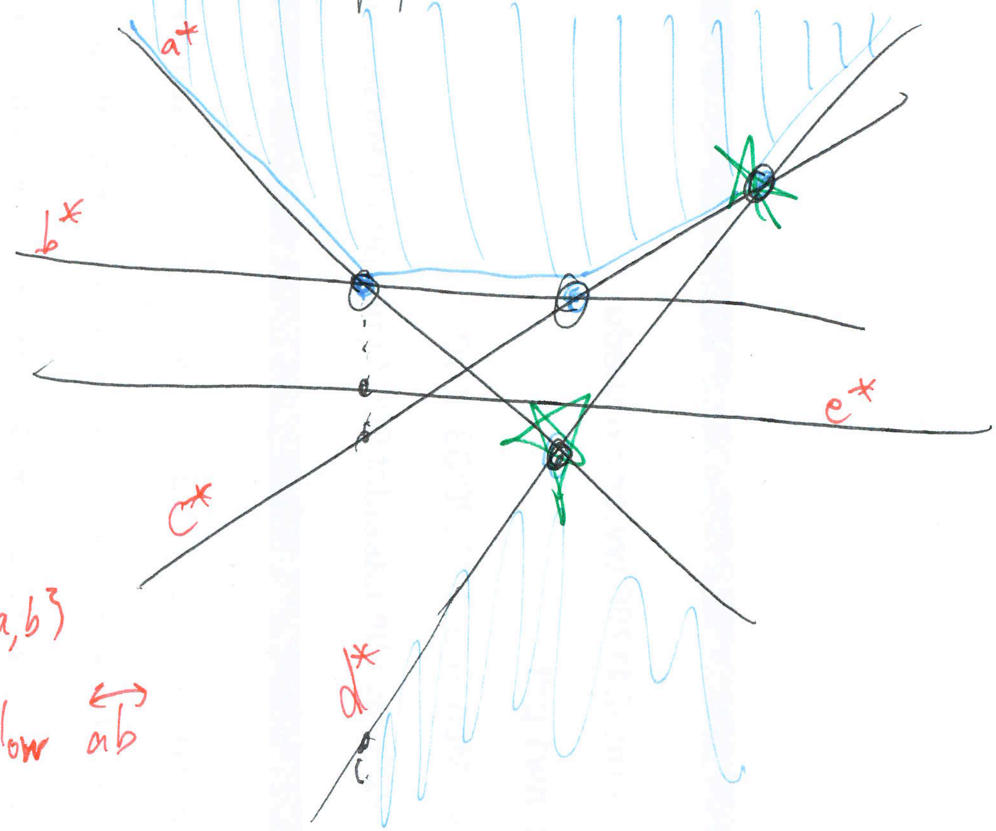
is c above $\overrightarrow{ab}$

is a^*b^* above c^*

Convex Hull (Dualized)

Upper Hull $\longleftrightarrow$ Upper Envelope



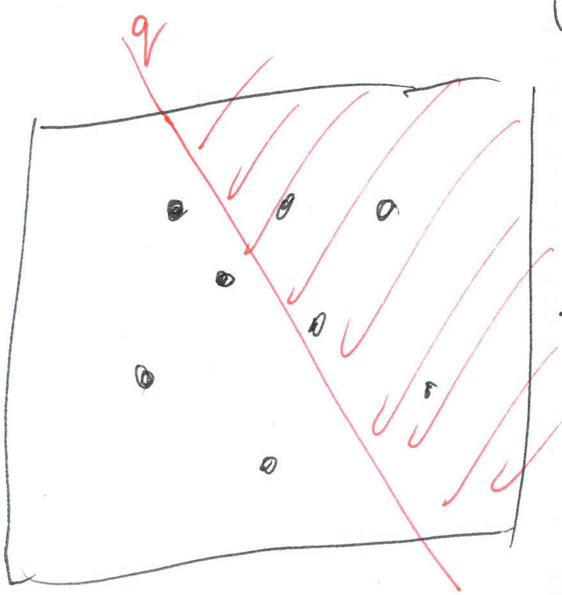


$\overline{ab}$ on CH $\Leftrightarrow$ $\forall p \in P \setminus \{a,b\}$

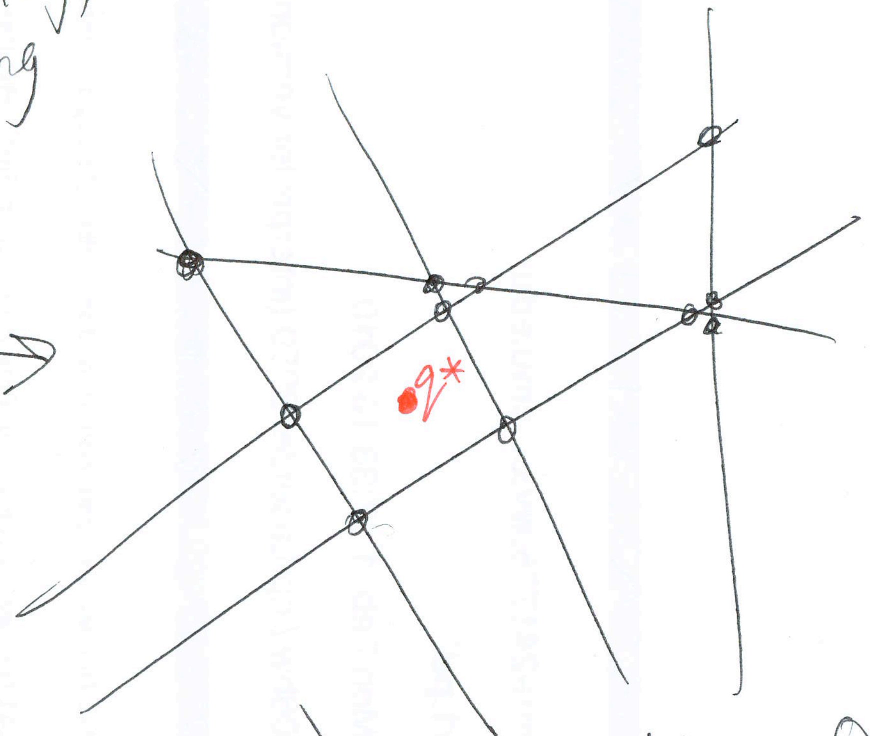
p is below $\overleftrightarrow{ab}$

~~Orthogon~~

Halfplane Range Search (Reporting)
Counting



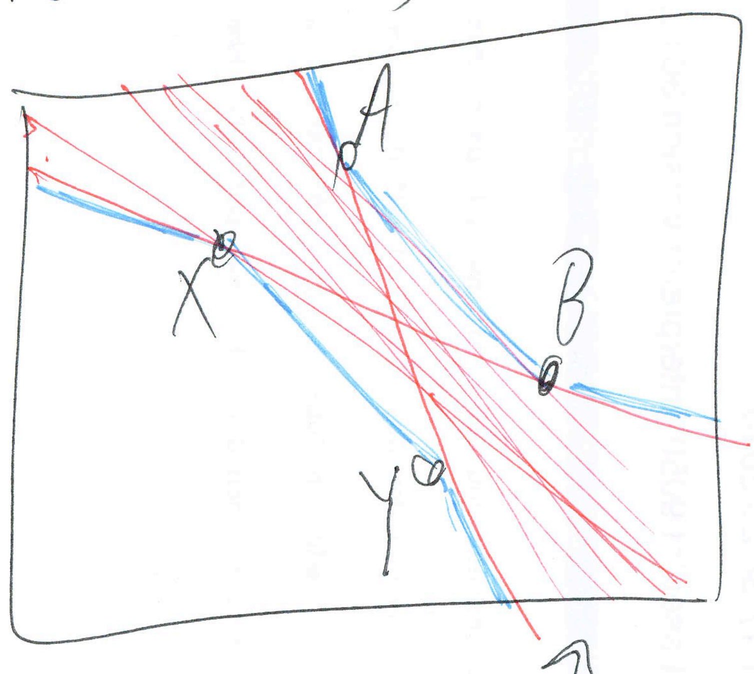
P



$A(P^*) \rightarrow$ Build in $O(n^2)$

Leverage Planar Point Location
$O(\log n)$

The space of lines that separate {X, Y} from {A, B}.



This is convex in the dual