Duality without Coordinates

\[ y = \frac{1}{2} x^2 \]

\[ m = \frac{dy}{dx} = px \]

\[ x = 0 \]

\[ (y - p_y) = m (x - p_x) \]

\[ y = \frac{1}{2} (2x) = x^2 \]

\[ y = \frac{1}{2} x^2 = p_y^2 - p_x^2 \]

\[ y = p_y - p_x^2 = p_y - 2p_x \]

\[ b = y = -p_y \]
Sylvester-Gallai Thm

Given \( n \geq 3 \) points \( P \) not all on a line

\( \exists \) line \( l \) passing thru exactly 2 points.

\( d(l) = \min_{p \in P} d(p, l) \)

\( d(x, \overline{YP}) < d(l) = \)

Dual Version

Given \( n \geq 3 \) lines not all passing thru a common point, + not all parallel.

\( \exists \) point \( P \) intersecting exactly 2 lines
ccw(a, b, c) ?
is c above $\overrightarrow{ab}$
is $\overrightarrow{ab}$ above $c^*$
Convex Hull (Dualized)

Upper Hull ← Upper Envelope

\[ \overline{ab} \text{ on CH} \iff \forall p \in P \setminus \{a, b\} \]

\[ p \text{ is below } \overline{ab} \]
Half-plane Range Search
(Reporting)
Counting

\[ A(P^+) \rightarrow \text{Build in } O(n^2) \]

\[ \text{Leverage Planar Point Location } O(\log n) \]
The space of lines that separate $\mathbf{x}, \mathbf{y}_i$ from $\mathbf{A}, \mathbf{B}\mathbf{y}_i$.

This is convex in the dual