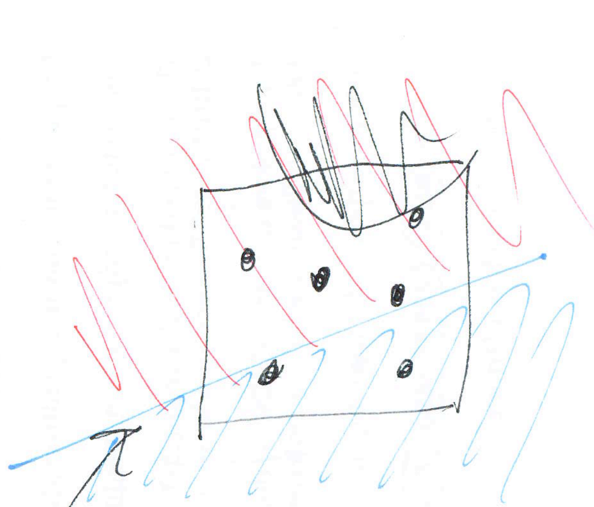
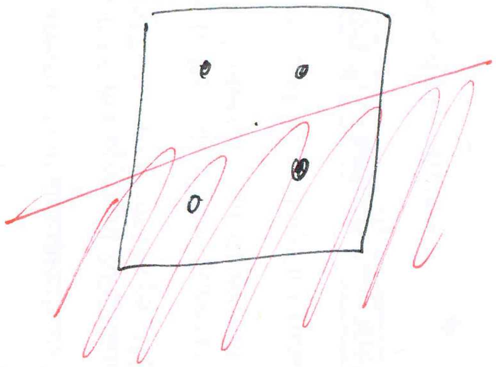
Application: Ray tracing



What makes a good super sample?

Assumption

pixels intersected by a single halfspace?



pixel $U = [0,1] \times [0,1]$

h : half-plane

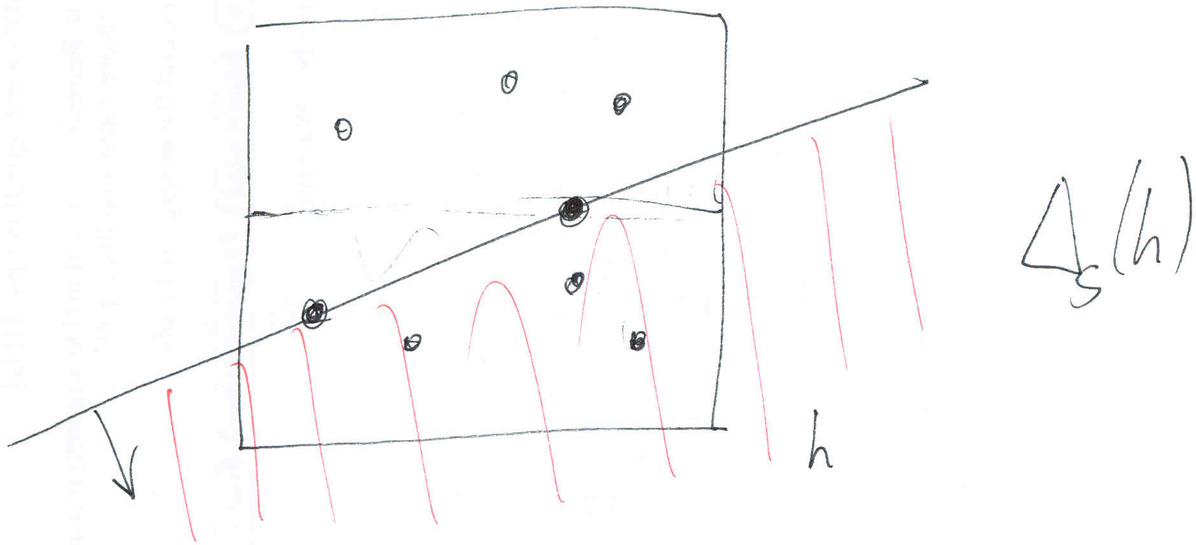
$S \subset U$: the sample

$$\mu(h) = \text{area}(U \cap h)$$

$$\mu_S(h) = \frac{|h \cap S|}{|S|}$$

Discrepancy $\Delta_S(h) = |\mu(h) - \mu_S(h)|$

$$\Delta_H(S) = \sup_{h \in \mathcal{H}} \Delta_S(h)$$



<u>Obs</u>: Some point in S is on bdy(h) if h maximizes $\Delta_S(h)$

When only one pt of S is on bdy(h) we can test $\Delta_S(h)$ in $O(n^2)$ time.

So, WMA there ≥ 2 pts of S on $bdy(h^*)$

naive: $O(n^3)$

sweepline: $O(n^2 \log n)$

today: $O(n^2)$

Duality

points $p = (p_x, p_y) \in \mathbb{R}^2$

$\Updownarrow$

lines $l = \{y = mx + b\} \Rightarrow (m, b) \in \mathbb{R}^2$

Dual of p $p^* = \{y = p_x X - p_y\}$

$l^* = (m, -b)$

$p^{**} = (p^*_x, -p^*_y) = (p_x, p_y)$

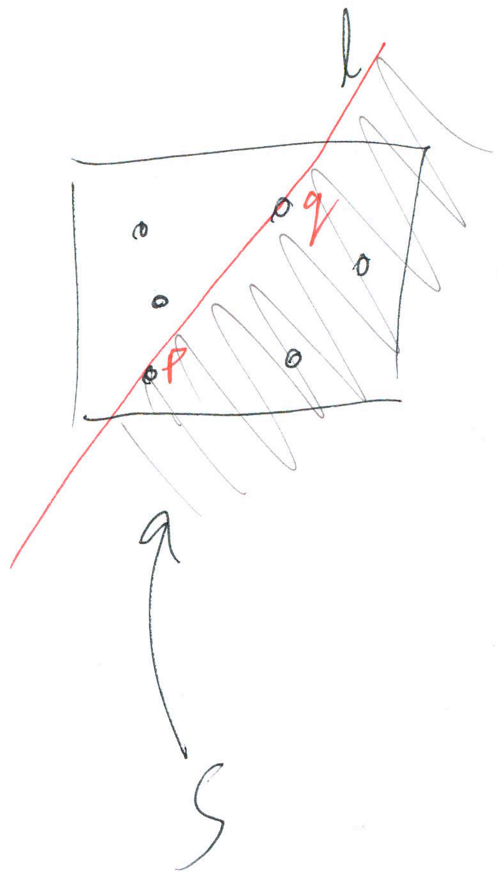
__Lem__ $p \in \ell \iff \ell^* \in p^*$

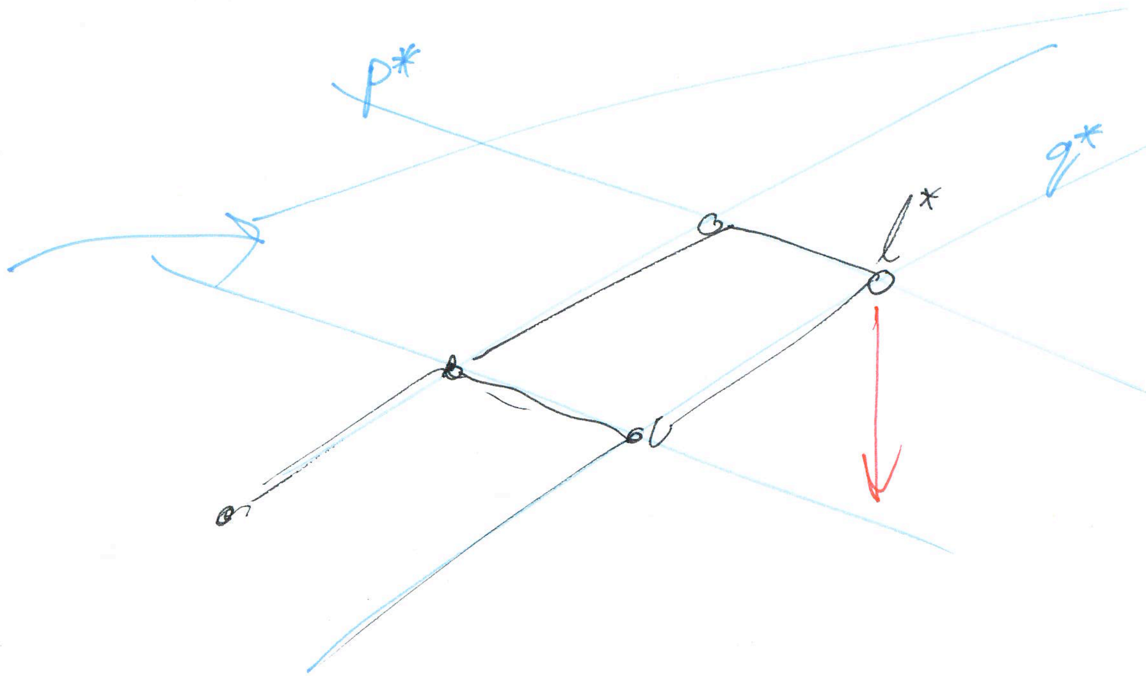
__pf__

$$
\begin{aligned}
p \in \ell &\iff p_y = \ell_m p_x + \ell_b \\
&\iff -\ell_b = p_x \ell_m - p_y \\
&\iff \ell^*_y = p^*_m \ell^*_x + p^*_b \\
&\iff \ell^* \in p^*
\end{aligned}
$$

__Lem__ p above $\ell \iff \ell^*$ above p.

__pf__ $p_y \geq \ell_m p_x + \ell_b$

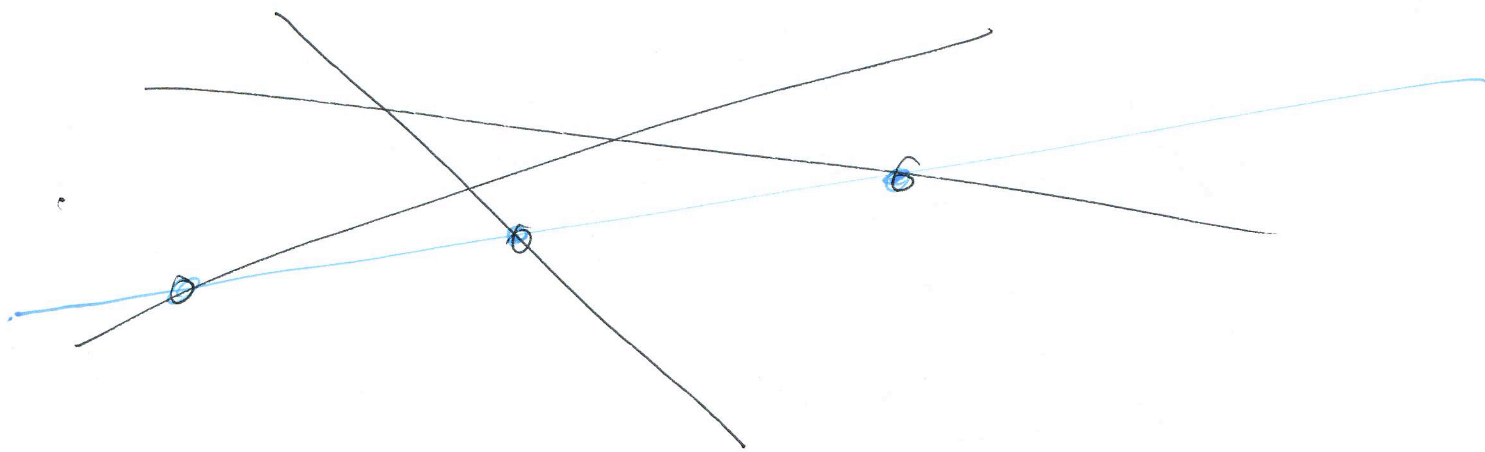




$A(S^*)$ = Arrangement of S^*

Computing the Arrangement

Incremental Construction



n total lines

ith insertion takes $O(i)$

$$\sum_{i=1}^{n} O(i) = O(n^2)$$

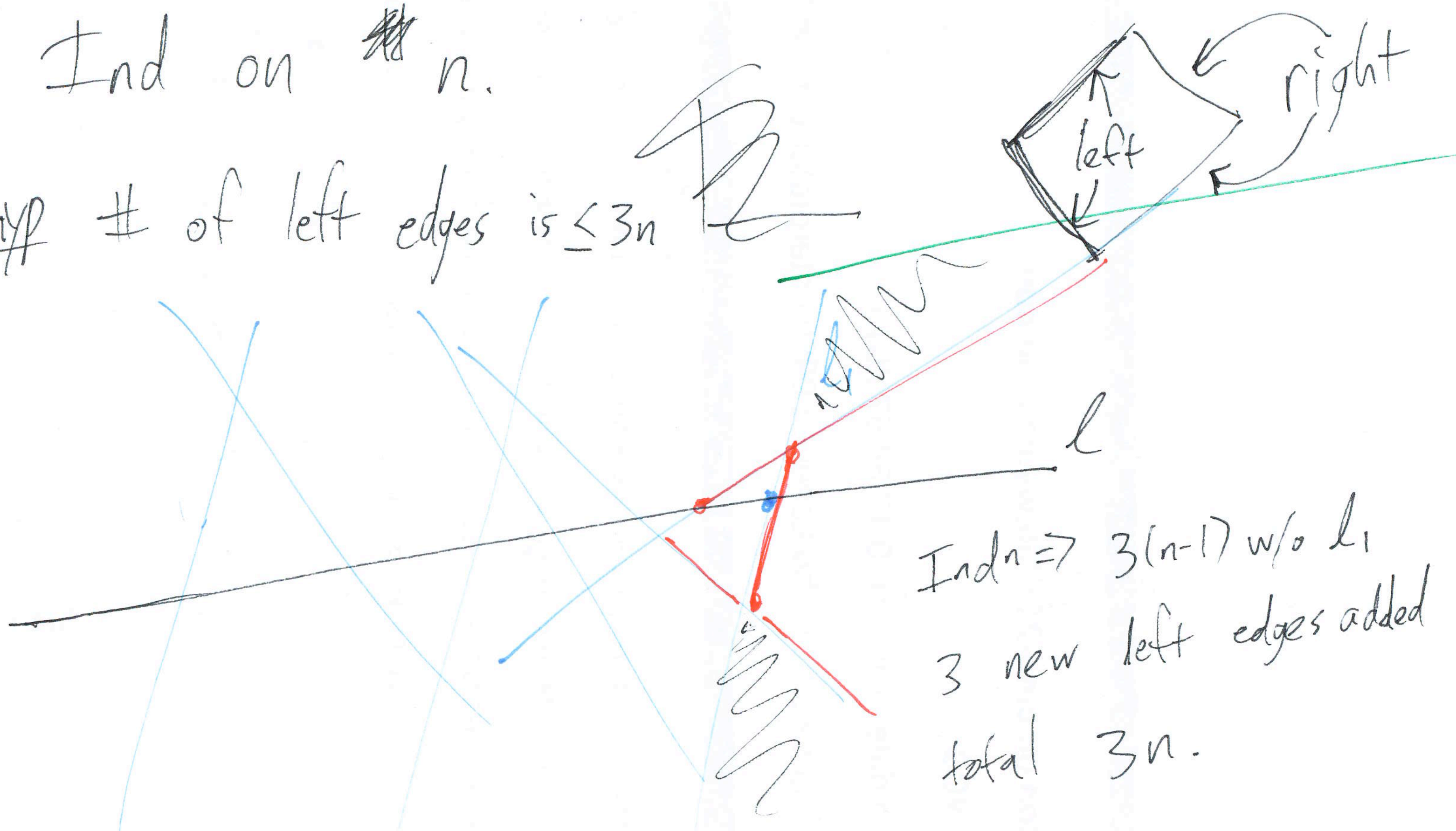
Thm [Zone Thm]

The # edges in the zone(ℓ) is $O(n)$

pf

Ind on # n.

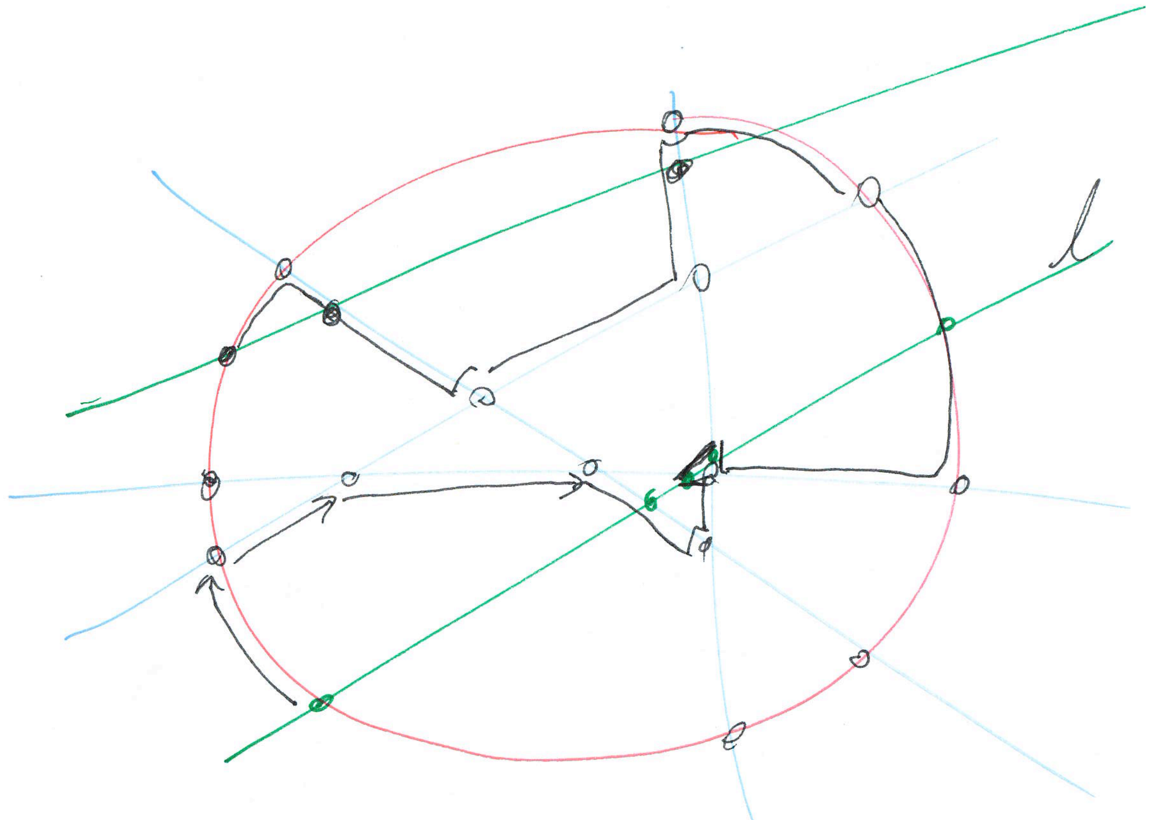
hyp # of left edges is $\leq 3n$



Indn $\Rightarrow$ $3(n-1)$ w/o ℓ_1

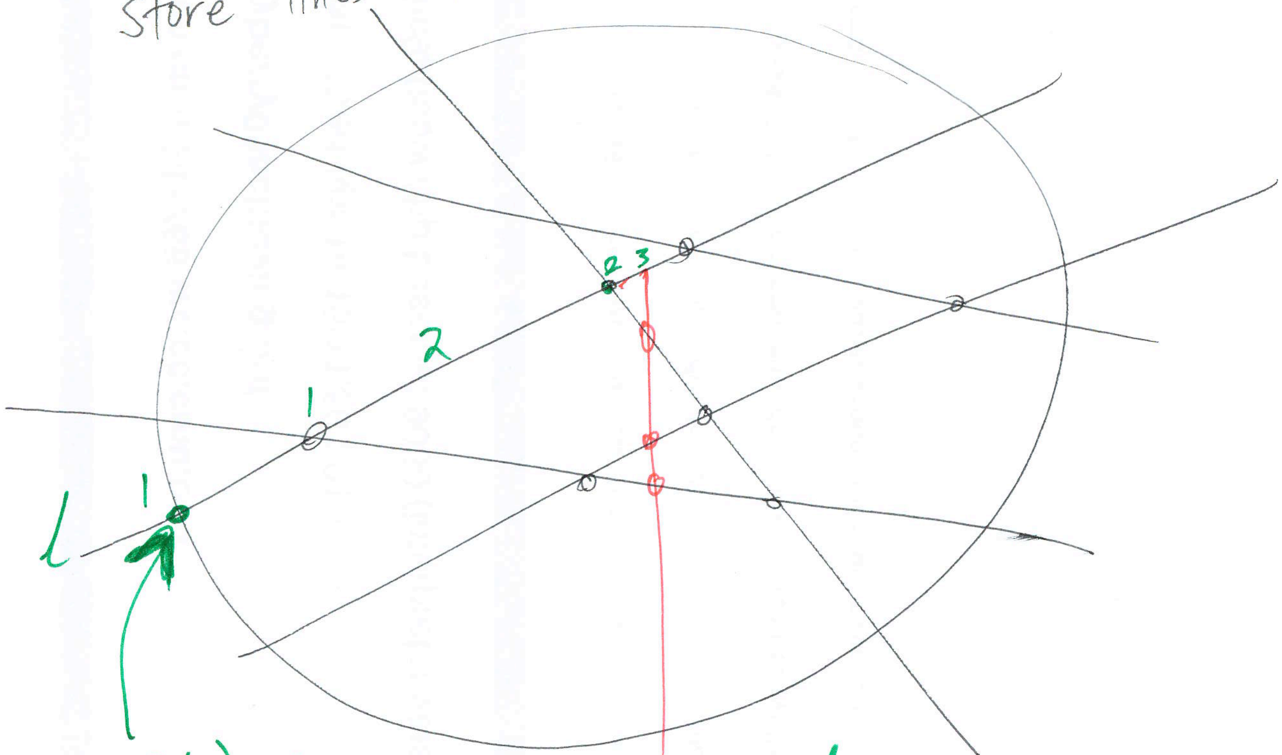
3 new left edges added

total $3n$.



The faces that ℓ cuts through is zone(ℓ)

Preprocess the arrangement:
store lines below each vertex.



O(n) for first point on ℓ

- easy to compute for subsequent vertices