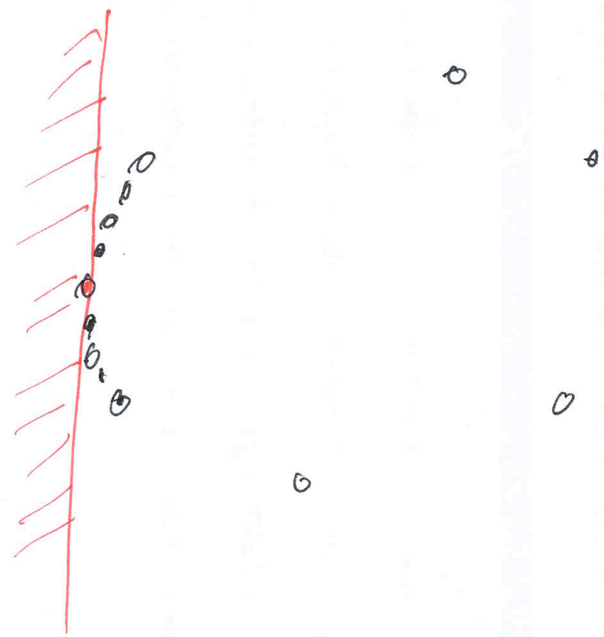
Idea: Choosing $p \in P$ closest to a centerpoint!

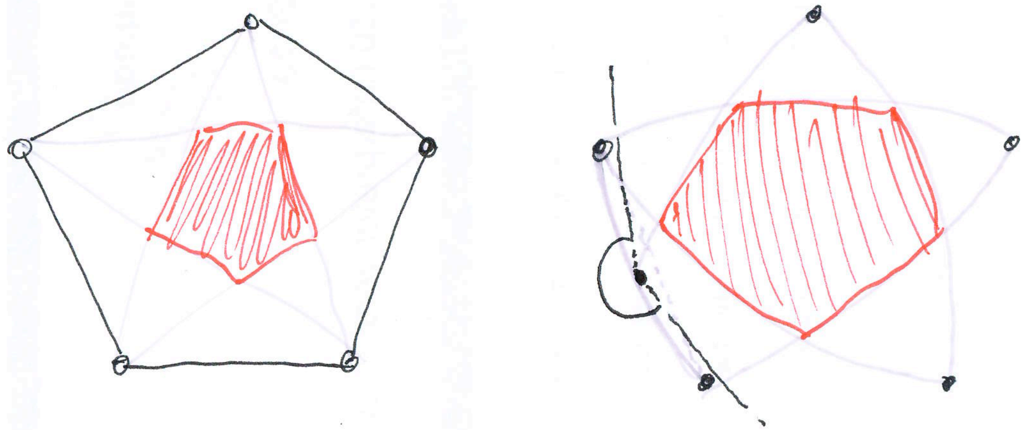


Def k-depth level $\equiv$ set of points of depth $\geq k$ (AKA k-hull)

Def The center-region $\equiv \left\lfloor \frac{n}{d+1} \right\rfloor$-hull

Def α-wedge depth of x w.r.t P

$$D_\alpha(x) = \min_{\substack{\alpha \text{ wedges } W \\ =(x,r)}} |W \cap P|$$



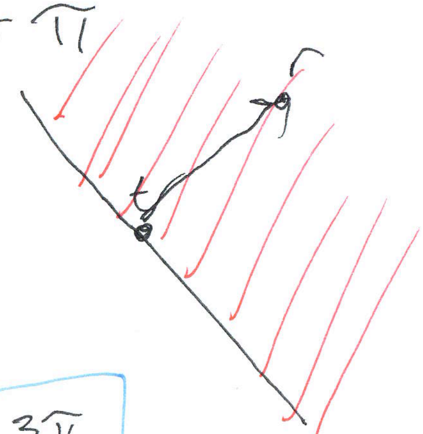
Def a α-wedge is defined

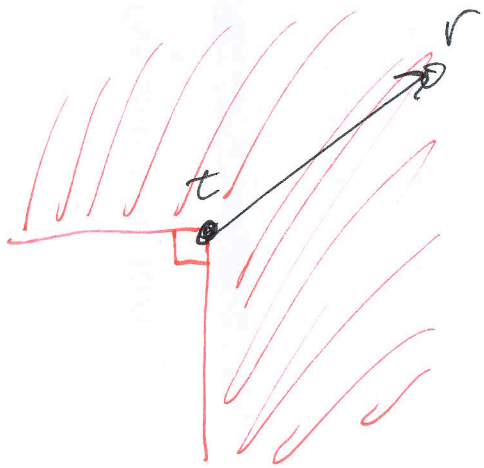
points t, r

$$\alpha\text{-wedge} = \{ x \in \mathbb{R}^d \mid \angle(xtr) \le \frac{\alpha}{2} \}$$

$\alpha = \frac{\pi}{2}$

$\alpha = \pi$





$\boxed{\alpha = \frac{3\pi}{2}}$

magic

Def A <u>centervertex</u> is a point $p \in P$ s.t. $D_{\frac{3\pi}{2}}(p) \geq \frac{n}{d+1}$

Thm Centervertices always exist!

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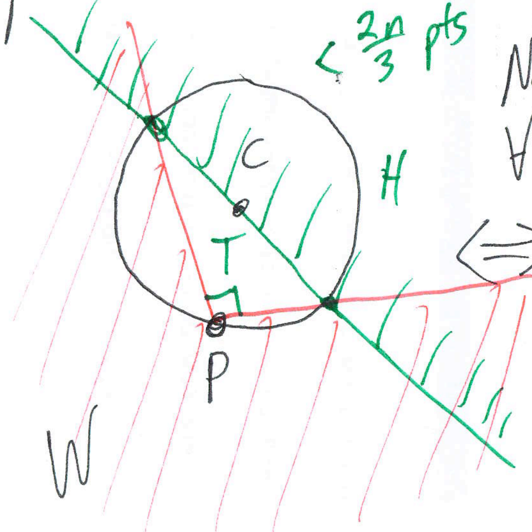
pf (In 2D)

Choose Closest $p \in P$ to a centerpoint.

$\overline{W} \subseteq T \cup H$

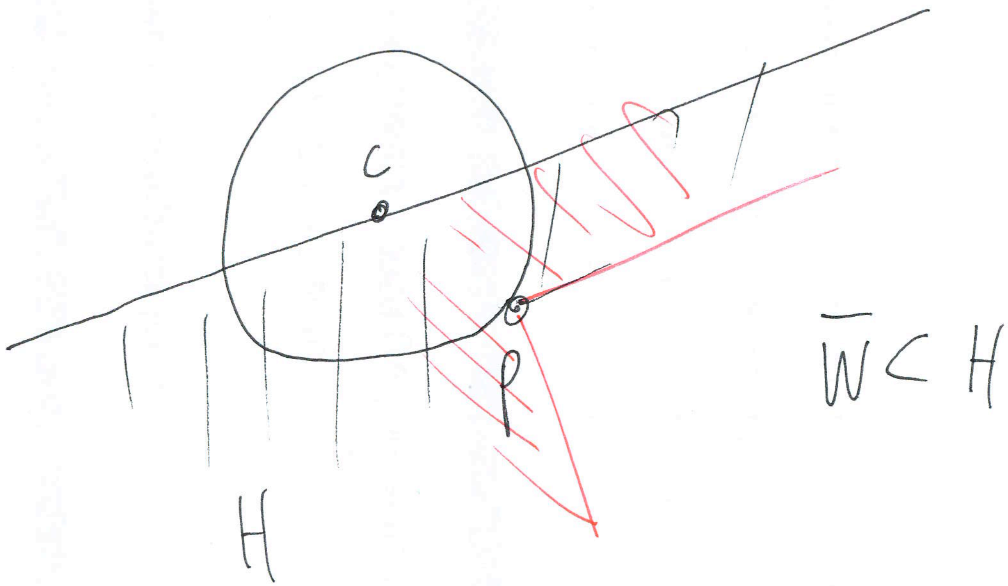
$\Rightarrow |\overline{W} \cap P| < \frac{2n}{3}$



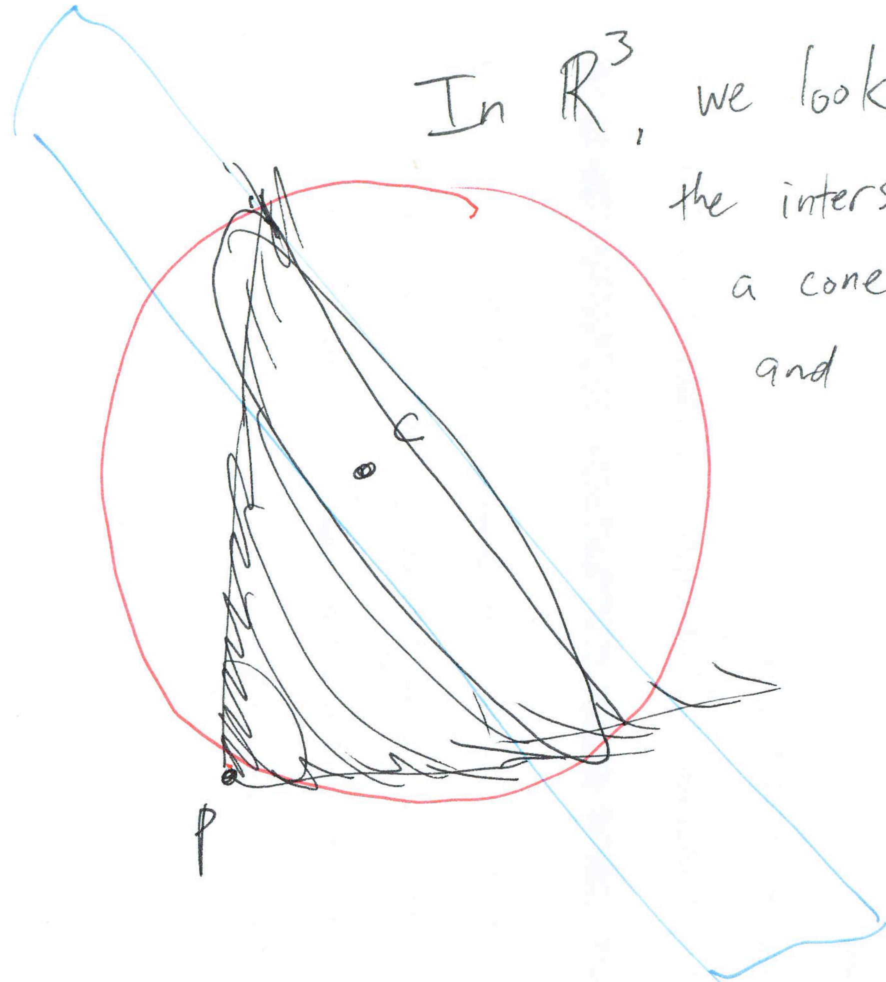
Need to show

$\forall W \quad |W \cap P| \geq \frac{n}{d+1} \quad \frac{n}{3}$

$\Longleftrightarrow |\overline{W} \cap P| < \frac{dn}{d+1} \quad \frac{2n}{3}$



c
p
H
$\bar{W} \subset H$



In $\mathbb{R}^3$, we look at the intersection of a cone, a halfspace, and a sphere.