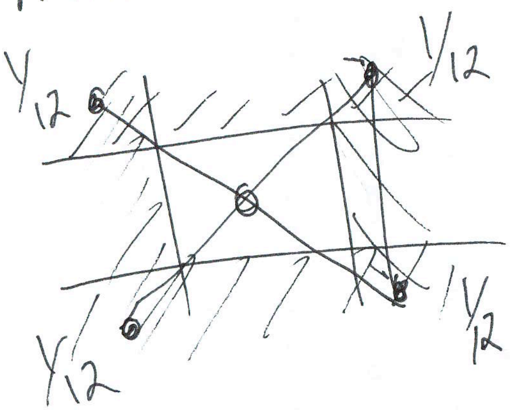
Centerpoints

Linear-time Centerpoints in $\mathbb{R}^2$

Prune + Search



$$4\left(\frac{1}{12}\right) = \frac{1}{3}$$

$$\frac{3}{4}\left(\frac{1}{3}\right) = \frac{1}{4} \text{ pts removed}$$

$$T(n) = O(n) + T\left(\frac{3n}{4}\right) = O(n)$$

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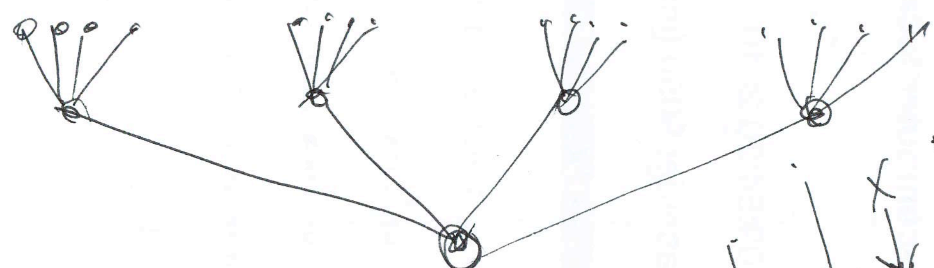
Centerpoints in d-dimensions

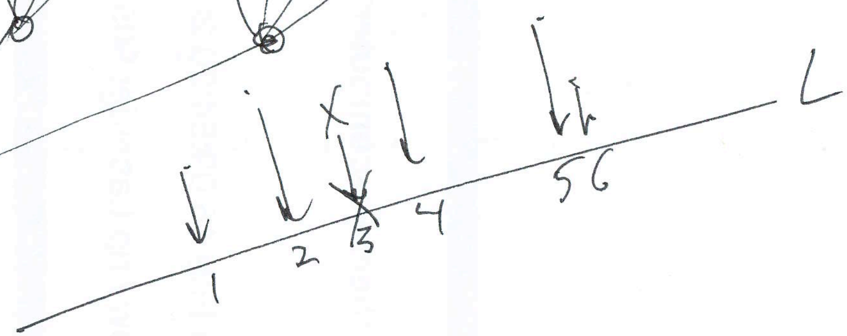
Known $O(n^d)$

$(\log n)^{\log d}$

All Arguments should be indep of L

Partition P into $\frac{n}{d+2}$ sets

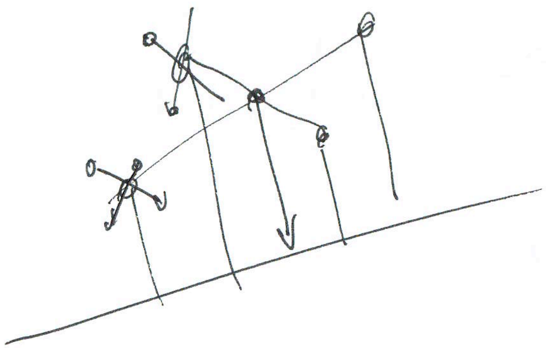




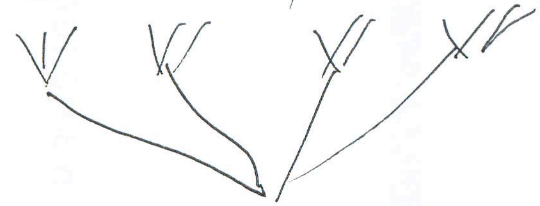
Line L

Project $P \to L$

Center point $\to L$ will have rank $\geq \frac{n}{d+1}$



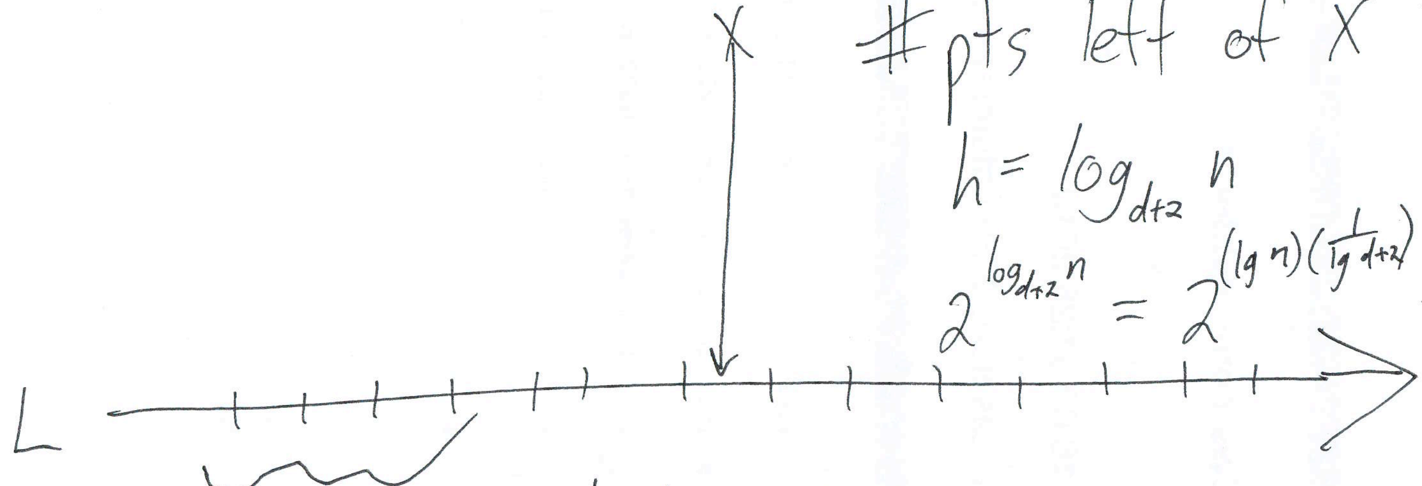
(d+2)-ary tree



$\#$pts left of X is $\geq 2^h$

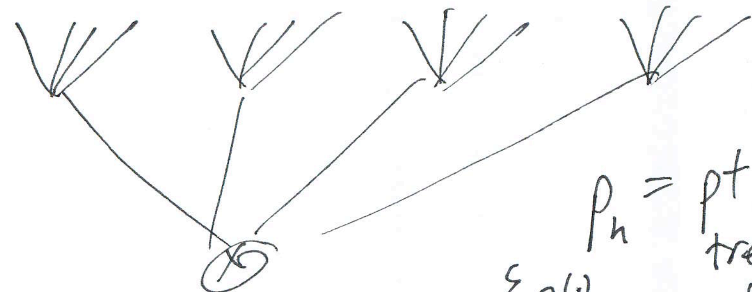
$h = \log_{d+2} n$

$2^{\log_{d+2} n} = 2^{(\lg n)\left(\frac{1}{\lg d+2}\right)} = n^{\left(\frac{1}{\lg d+2}\right)}$



$\underbrace{\quad\quad}_{}$ k pts $\Rightarrow$ depth(x) $\geq$ k

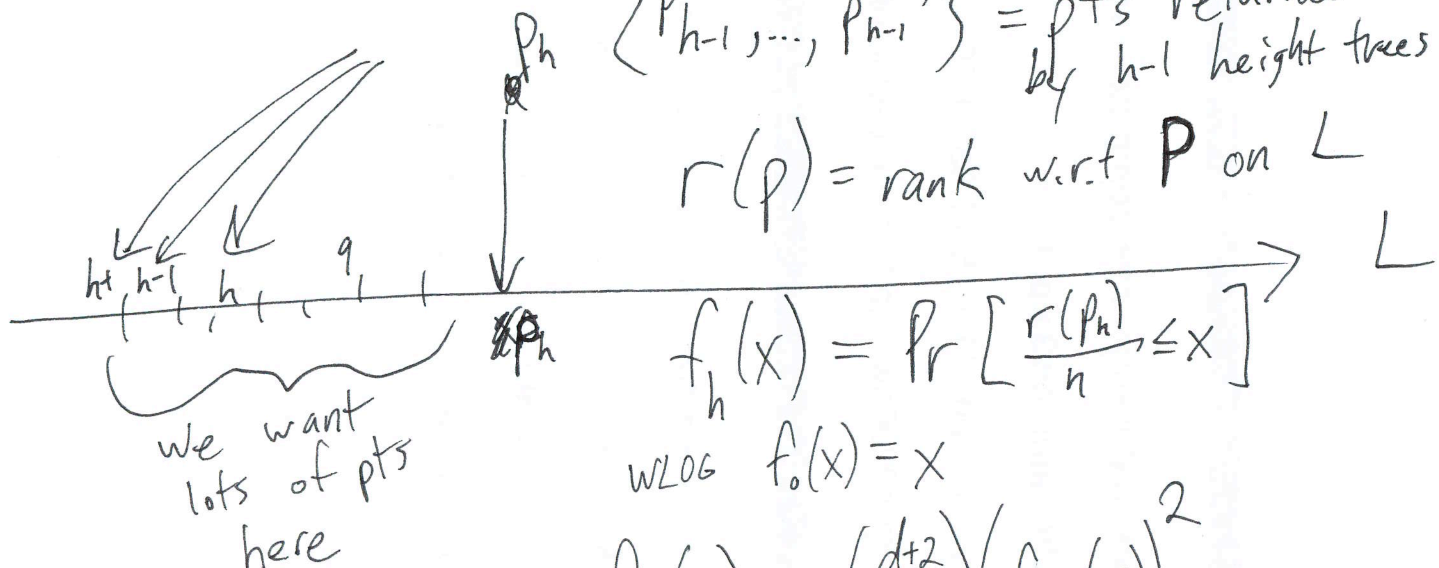
Randomness is your friend.



p_h = pt returned by a tree of height h

$\{p_{h-1}^{(1)}, \ldots, p_{h-1}^{(d+2)}\}$ = pts returned by $h-1$ height trees

$r(p)$ = rank w.r.t $\mathbf{P}$ on L



$$f_h(x) = \Pr\left[\frac{r(p_h)}{n} \le x\right]$$

WLOG $f_0(x) = x$

$$f_h(x) \le \binom{d+2}{2}\left(f_{h-1}(x)\right)^2$$

$$\le \binom{d+2}{2}^{-1}\left(\binom{d+2}{2}x\right)^h$$

want $X = \frac{1}{2\binom{d+2}{2}} = \frac{1}{(d+1)(d+2)} = 1/2^h$

$\Rightarrow \text{depth}(p_h) \geq \frac{n}{(d+1)(d+2)}$ w.p. $\geq \left(1 - \frac{1}{2^h}\right)$