

15-456/852 Computational Geometry, Fall 2017

Homework 3 (90 pts)

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Due: Nov 6 by class time

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Question	Points	Score
1	30	
2	30	
3	30	
Total:	90	

(30) 1. Updates to Cell-Chains

In this problem we will show how to update our cell-chain representation as we change the topological structure.

1. Show the poset for the case of a simple triangle in the plane as shown in figure 1.

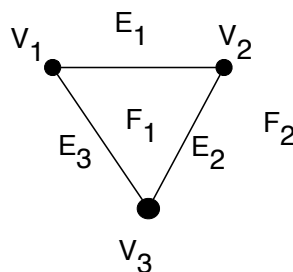


Figure 1: A Triangle in the Plane.

List the switch operators $\alpha_0, \alpha_1, \alpha_2$ for this poset.

2. Show how the poset and switch operators are modified when we add a new edge E in the face F_1 attached to the vertex V_1 with the other vertex of being a new vertex V .
3. Suppose now we would like to delete some cell, say, a vertex V . In the process of deleting V we must first delete all the edges common the V . This may in turn require us to delete faces common to these edges if the dimension is more than two, etc. We will write code for two cases separately 1) when there is more than one edge common to V and 2) when there is exactly one edge.

Write pseudocode for these two case when the ambient dimension is two.

(30) 2. **Staircases**

Let P be a set of n points in the plane. A point $p \in P$ is Pareto-optimal if no other point in P is both above and to the right of p . The sorted sequence of Pareto-optimal points describes a staircase with all the points in P below and to the left. The staircase layers of P are defined recursively as follows. The first staircase layer is just the staircase; for all $k > 1$, the k th staircase layer is the staircase of P after the points in the first $k - 1$ staircase layers have been deleted.

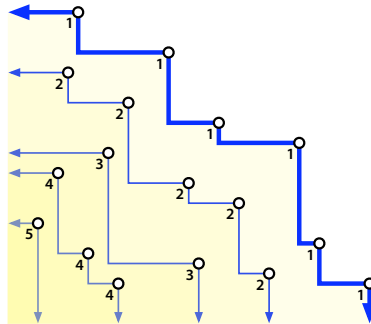


Figure 2: set of points with five staircase layers.

1. Describe and analyze an algorithm to compute the staircase layers of P in $O(n \log n)$ time. Your algorithm should label each point with an integer indicating which staircase layer contains it, as shown in the Figure 2 above.
2. Describe and analyze an algorithm to compute the staircase of P in $O(n \log p)$ time, where p is the number of Pareto-optimal points. [Hint: There are at least two different ways to do this.]

(30) 3. **Bichromatic Closest Pair**

Suppose we have two sets of points in the unit interval, n red points R and m green points G . The goal of this problem is to find the closest pair of points with one point from R and the other one from G . The algorithm should run in expected linear time, $O(n + m)$. We shall develop a hashing based algorithm similar to the 2D closed pair algorithm presented in class.

1. Suppose we are given the closest bichromatic distance α . Give a linear expected time algorithm to find a pair of points realizing this distance.
Hint: Try looking for a pair such that one point is in one bucket and the other is in a neighboring one.
2. Similar to the closest pair algorithm from class define a function $\text{TEST}(\alpha, R, G)$ and give an efficient algorithm for it.
3. Give your complete algorithm with proof of correctness and run time analysis.

Extension: Find a 2D version of this algorithm.