Guidelines: Please justify all answers in a succinct (yet complete) manner. In particular, when presenting an algorithm the code if any should be presented at a high level. A full algorithm will contain the input, the output, and any loop invariants.

1. Linear Algebra and Dot Products
   We shall say that a linear transformation is rigid if each elementary vector is mapped to a unit length vector and the image of any pair of distinct elementary vectors is orthogonal.
   
   (a) Give a matrix definition of when a matrix is a rigid transformation.
   (b) Show that dot products are preserved under rigid transformations.

2. Linear Algebra
   Suppose that you are given the vectors $P_1, \ldots, P_k \in \mathbb{R}^d$ that define the subspace
   \[ W = \{\alpha_1 P_1 + \cdots + \alpha_k P_k \mid \alpha_i \in \mathbb{R}\} \]
   show how to write $W$ as the solution to a set of linear constraints, i.e.
   \[ W = \{x \in \mathbb{R}^d \mid Ax = 0\} \]
   Suppose you are given a subspace as a set of constraints $Ax = 0$ show how one could write it as a set of generators.

3. Convex Closure
   We said that a set $C \subset \mathbb{R}^d$ is convex if for any two points $p$ and $q$ in $C$ the line segment $[p, q]$ is contained in $C$. Let $Q = \{P_1, \ldots, P_k\} \subset \mathbb{R}^d$.
   In class we defined to objects:
   
   **Definition 0.1.** (a) The convex closure $CC(Q)$ to be the smallest convex set containing $Q$.
   
   (b) $\text{ConvexComb}(Q) = \{\alpha_1 p_1 + \cdots + \alpha_k p_k \mid \alpha_1 + \cdots + \alpha_k = 1 \text{ and } \alpha_i \geq 0\}$
   
   Show that $CC(Q) = \text{ConvexComb}(Q)$.

4. Size of Priority Queue in Sweep Line Algorithm
   In our analysis of the the sweep line algorithm we said that the size of the priority queue can be at most $|P| + |I|$ which is at most $O(n^2)$. Thus we concluded that the cost per operation was at most $O(\log n)$. Find a better upper bound or find an example match our bound.