15-451 — Algorithms — Spring 2006

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Homework #6

Due: Tuesday April 18, 2006.

Ground rules:

- This is an oral presentation assignment. You should work in groups of three. At some point before **April 14**, your group must sign up for a 1-hour time slot on the signup sheet on the course web page.
- Please write up (and turn in during your presentation) Solutions to the problems. You need not be overly formal or verbose. Just summarize the important ideas of your solution.
- During the presentations, you may be asked to explain why your algorithm is correct.

Problems:

1 Traveling Salesman Tours in the Unit Square

There are n points $p_1, p_2, \ldots, p_n$ inside the unit square $\{(x, y) \mid 0 \le x \le 1 \;,\; 0 \le y \le 1\}$. Give an algorithm that constructs a Traveling Salesman Tour among these points whose length is at most $a + b\sqrt{n}$, for some constants a and b. Your algorithm should try to minimize b. The running time of your algorithm should be $O(n \log n)$. (A Traveling Salesman Tour is simply a permutation of the points. The length of the tour is the sum of the Euclidean distance between successive points, plus the distance between the last point and the first one.)

HINTS: There's a very simple algorithm to do this. To get started, find a TST for the $\sqrt{n}$ by $\sqrt{n}$ array of points in the unit square.

2 Arithmetic Progression of Bits

Given an array of n bits $A[0], A[1], \ldots A[n-1]$, give an $O(n \log n)$ algorithm that finds an an "arithmetic progression" of 1 bits in the array (if there is one). That is, if finds constants a and b (with $b > 0$ and $0 \le a$ and $a + 2b \le n - 1$) such that $A[a] = 1$ and $A[a+b] = 1$ and $A[a+2b] = 1$ – or proves that no such constants exist. HINT: Think FFT.

3 Move-Half-Way-To-Front

In class we analyzed the Move-To-Front algorithm for the list update problem. We showed that it's 4-competitive. In this problem we'll consider the MHF algorithm that moves the accessed element half way to the front. More specifically, if an element x in position i is accessed (the positions are numbered $1, 2, \ldots, n$ from front to back), then x is swapped $\lceil \frac{i-1}{2} \rceil$ times with its neighbor to the front. In other words, x moves past $\lceil \frac{i-1}{2} \rceil$ elements, but still has $\lfloor \frac{i-1}{2} \rfloor$ elements in front of it.

Chose a potential function, and choose a constant c (as small as possible), and use them to prove that MHF is c-competitive.