Recap of this week’s lectures:

- LP

Review Seidel’s Algorithm

Algorithm 1 2D random incremental LP algorithm

**Input:** $m_1, m_2, h_1, h_2, \ldots, h_n, c$

**Output:** 2D-LP($h_1, h_2, \ldots, h_n, c$)

1: Step 1: $v_0 \leftarrow$ 2D-LP($m_1, m_2, c$) (i.e. $v_0 = CH(m_1) \cap CH(m_2)$)
2: Step 2: Randomly order $h_1, h_2, \ldots, h_n$
3: Step 3:
4: for $i = 1$ to $n$ do
5: if $v_{i-1} \in h_i$ then
6: $v_i \leftarrow v_{i-1}$
7: else (Make and solve 1D-LP problem)
8: $L \leftarrow CH(h_i)$ (Boundary of $h_i$)
9: for $j = 1$ to $i - 1$ do
10: $h'_j \leftarrow L \cap h_j$
11: end for
12: $c' \leftarrow \text{projection}(c, L)$ \(^1\) (Note: $c' \neq 0$)
13: $v_i \leftarrow 1D$-LP($h'_1, h'_2, \ldots, h'_{i-1}, c'$)
14: end if
15: if $v_i$ is “undefined” then
16: Report “No Solution” and halt
17: end if
18: end for
19: return $v_n$

How can we generalize this to higher dimensions?

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\(^1\)Here projection is a function mapping a point in 2D space into a line.
Finding infeasible constraints

For an infeasible $d$ dimensional LP, we can find $d + 1$ constraints that make it infeasible. In other words there exists $d + 1$ rows of $A$ such that if $A$ only consisted of those rows, the LP is infeasible. We will prove this by construction.

Describe an algorithm that takes an LP and either finds that it is feasible, or finds the $d + 1$ constraints as above.
Bounding box

1. Show that if there exists $v$ such that $c^Tv > 0$ and $Av \leq 0$, the LP is unbounded.

2. Assume that an LP is unbounded if and only if such a $v$ exists (proof omitted). Design an algorithm to find a bounding box of the LP.

3. Go through an instance of this algorithm for an unbounded 2D LP and then go through an instance of this algorithm for a bounded 2D LP.