15-451/651 Algorithms, Fall 2017
Recitation #10 Worksheet

Points in Rectangles.

You are given a set P of n points (x_i, y_i) with integer coordinates and a set R of m axis-aligned rectangles specified by their four edges (i.e. you are given a_j, b_j, c_j, d_j, such that rectangle j consists of all points such that $a_j \leq x < b_j$ and $c_j \leq y < d_j$).

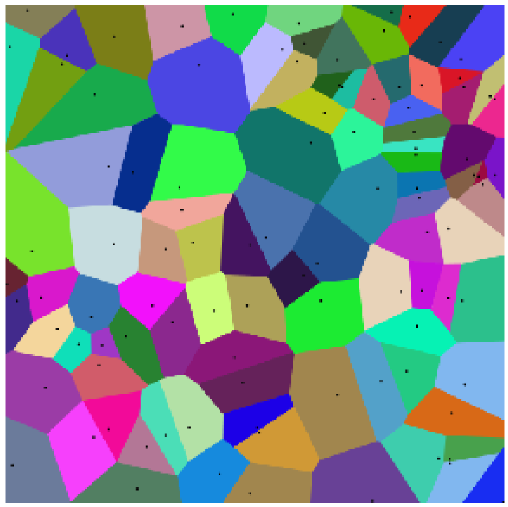
Give an algorithm that can determine the number of points in P that are contained in each rectangle in $O((n+m)\log m)$ time.

Width of a Set of Points.

You are given a set of n points on a plane. Define a *strip* as the area between two parallel lines, and the *width* of that strip to be the perpendicular distance between those two lines. In $O(n \log n)$ time, find a strip of minimum width that contains all of the points.

Painted Voronoi Diagram.

You are given a set P of n distinct points $p_1, p_2, \ldots p_n$ and n distinct colors $c_1, c_2, \ldots c_n$. In a *painted voronoi diagram*, the region of the unit square that is closer to point p_i than any other point in P is painted with color c_i. For example:



Consider the following painting algorithm:

> **Painting Algorithm:** Insert the points one at a time in order. When a point is inserted, paint the region of the unit square closer to this new point than any other point inserted so far.

Note that one unit of paint is needed to paint the entire square.

1. Devise a sequence of n points and prove that for this sequence the painting algorithm uses $\Omega(n)$ paint.

2. Suppose that we randomly permute the points before applying the painting algorithm. Show that the expected amount of paint used by this algorithm is at most $1 + \ln n$.