

**Lecture 24:**

# **Parallel Deep Neural Networks**

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**Parallel Computer Architecture and Programming  
CMU 15-418/15-618, Fall 2018**

# Training/evaluating deep neural networks

Technique leading to many high-profile AI advances in recent years

Speech recognition/natural language processing

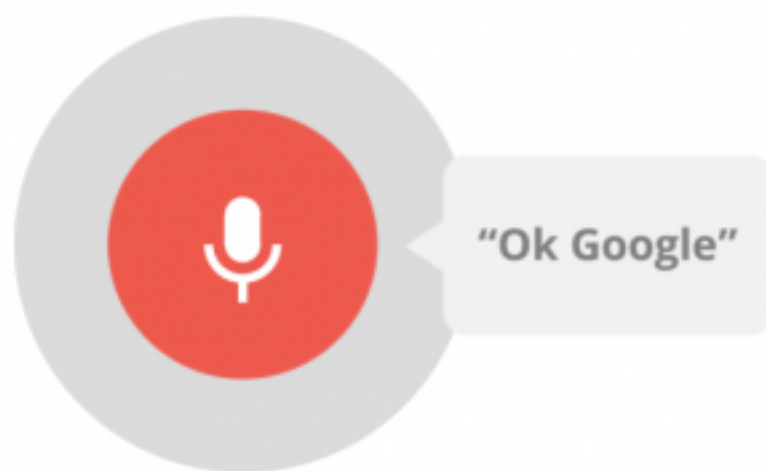
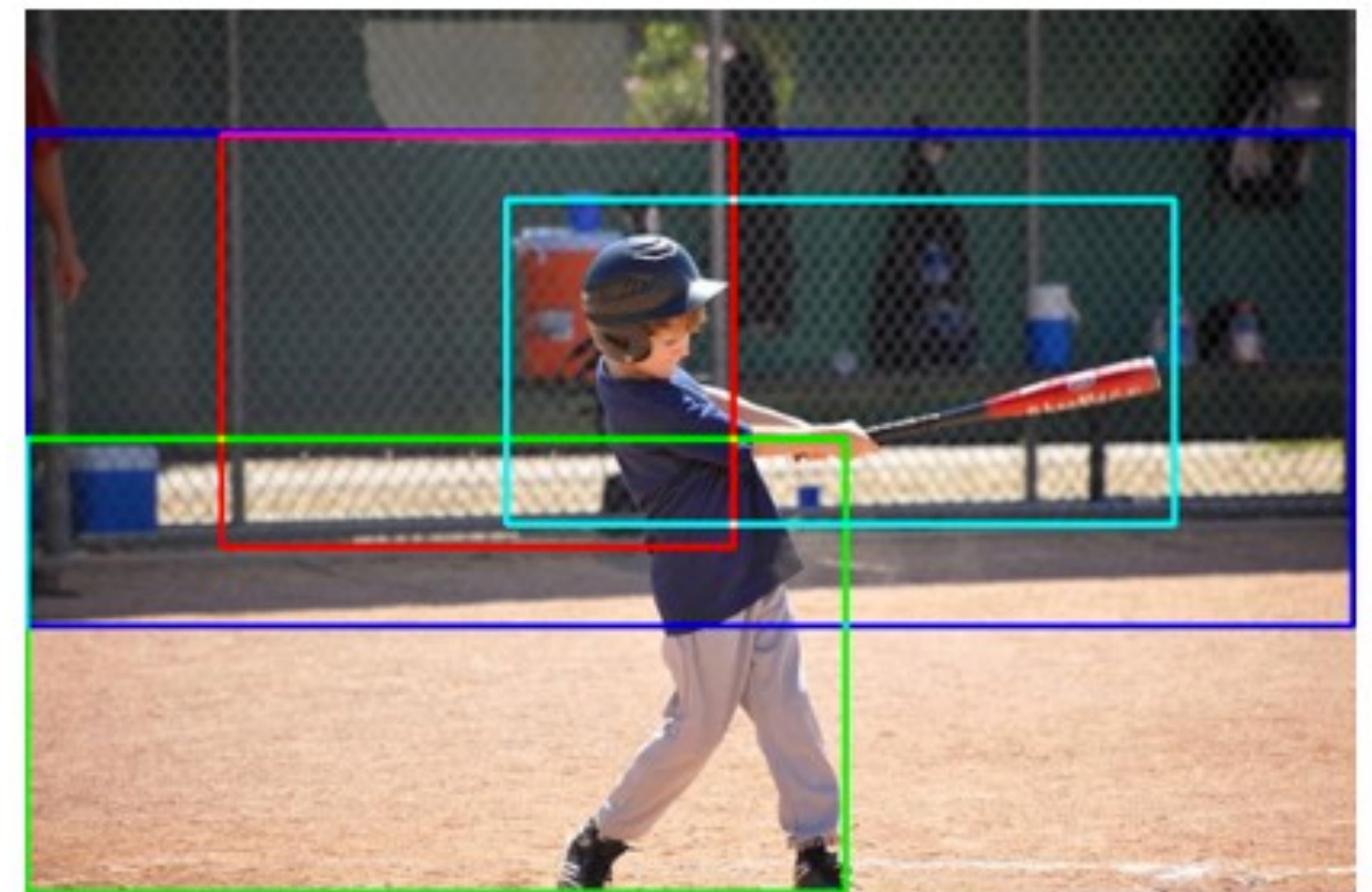
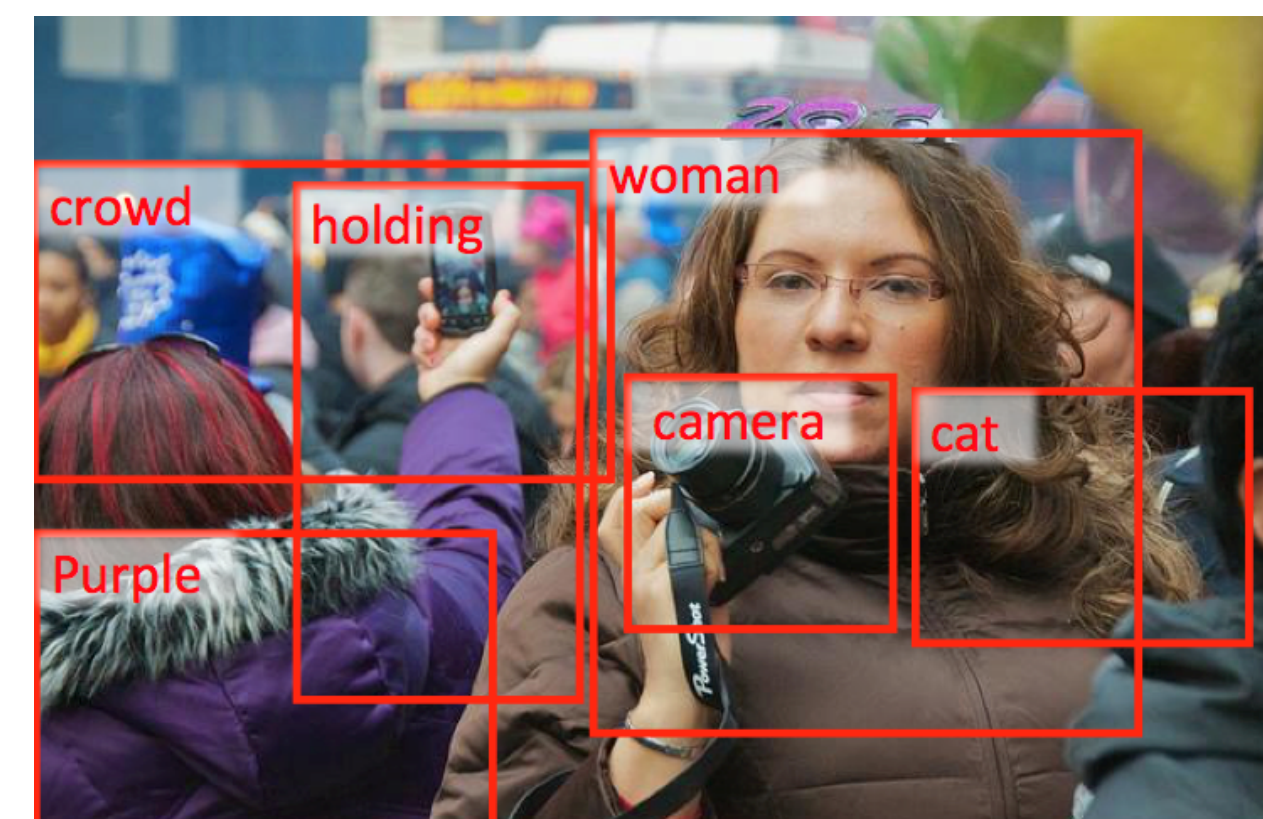


Image interpretation and understanding



[tennis (0.65)] [holding (0.53)] [field (0.59)] [ball (0.79)] [court (0.52)] [boy (0.51)]  
[baseball (0.97)] [player (0.83)] [bat (0.82)] [man (0.80)] [playing (0.65)] [game (0.60)]  
a baseball player swinging a bat at a ball  
a boy is playing with a baseball bat

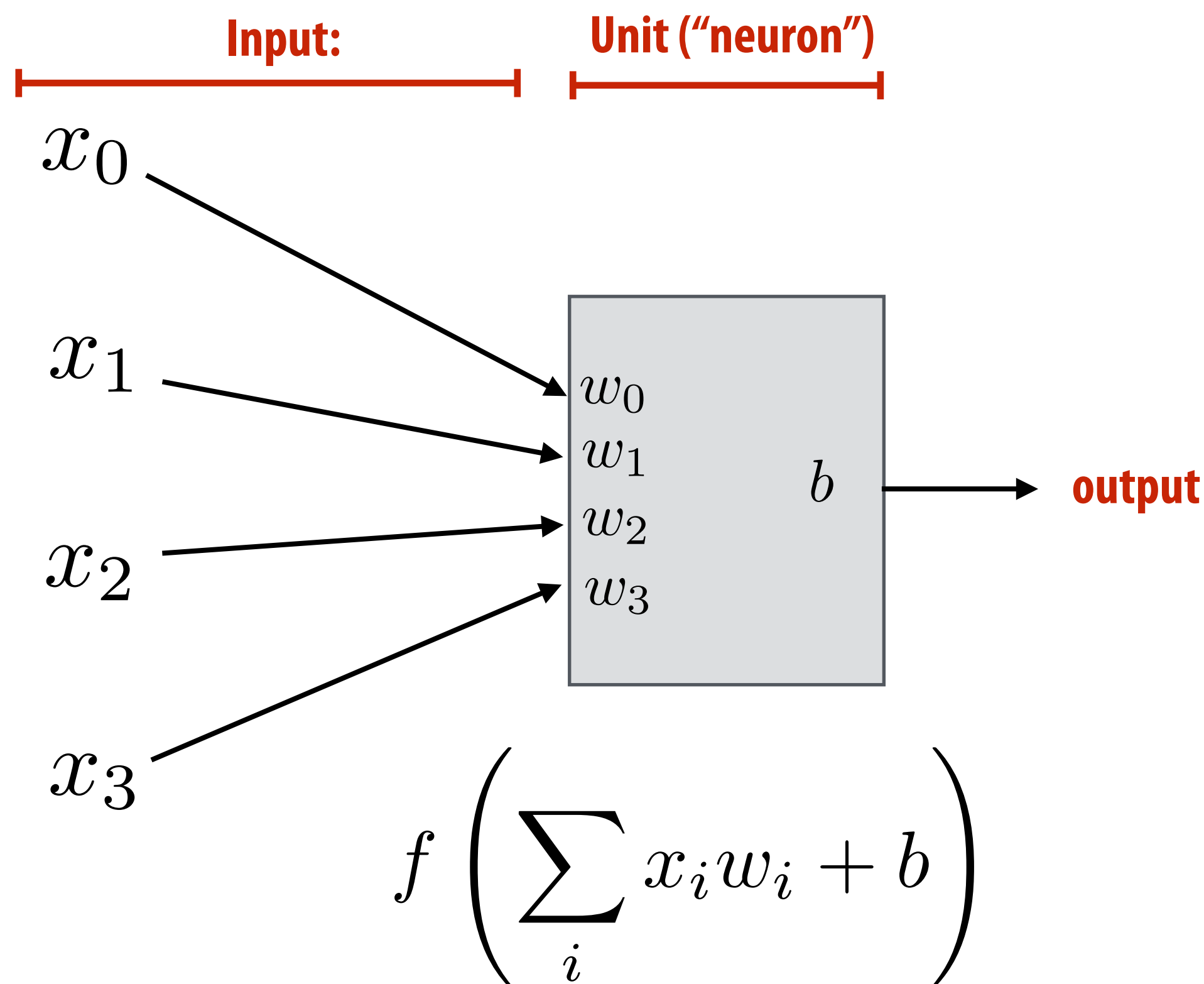




# What is a deep neural network?

## A basic unit:

Unit with  $n$  inputs described by  $n+1$  parameters  
(weights + bias)



**Example: rectified linear unit (ReLU)**

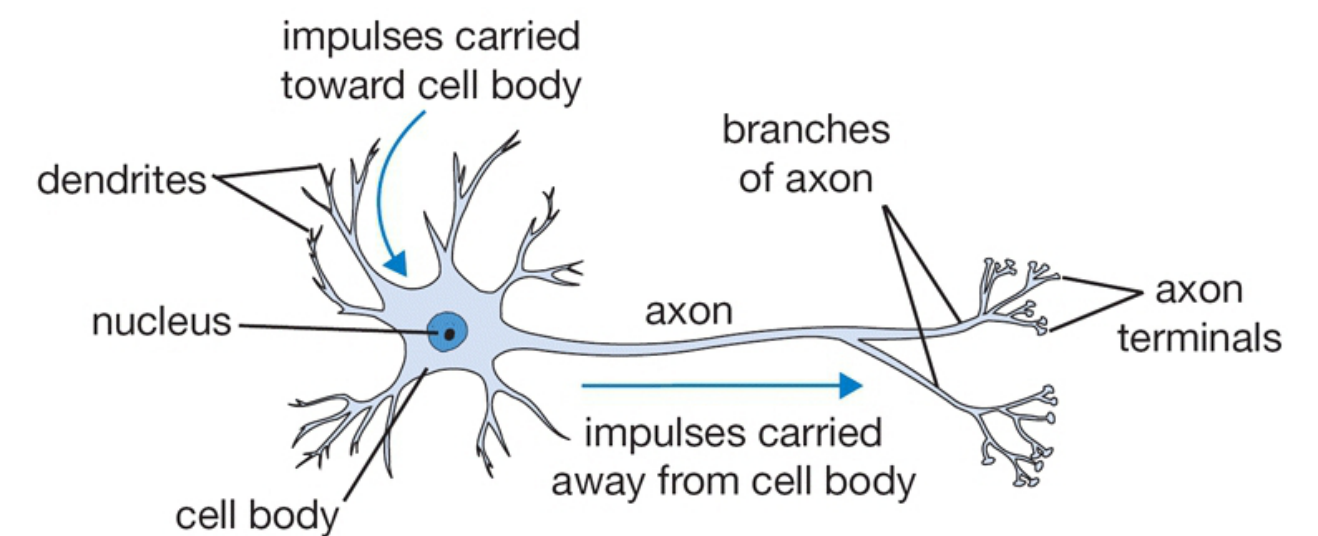
$$f(x) = \max(0, x)$$

**Basic computational interpretation:**

**It's just a circuit!**

**Biological inspiration:**

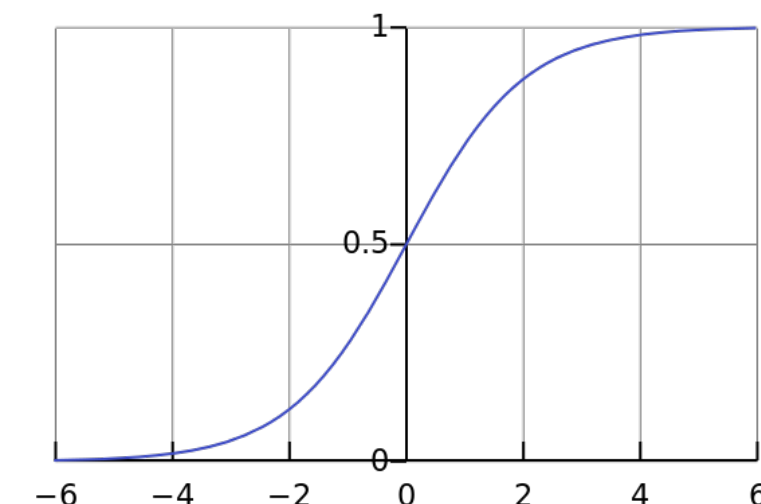
**unit output corresponds loosely to activation of neuron**



**Machine learning interpretation:**

**binary classifier:** interpret output as the probability of one class

$$f(x) = \frac{1}{1 + e^{-x}}$$



# Two Distinct Issues with Deep Networks

## ■ Evaluation

- often takes milliseconds

## ■ Training

- often takes hours, days, weeks



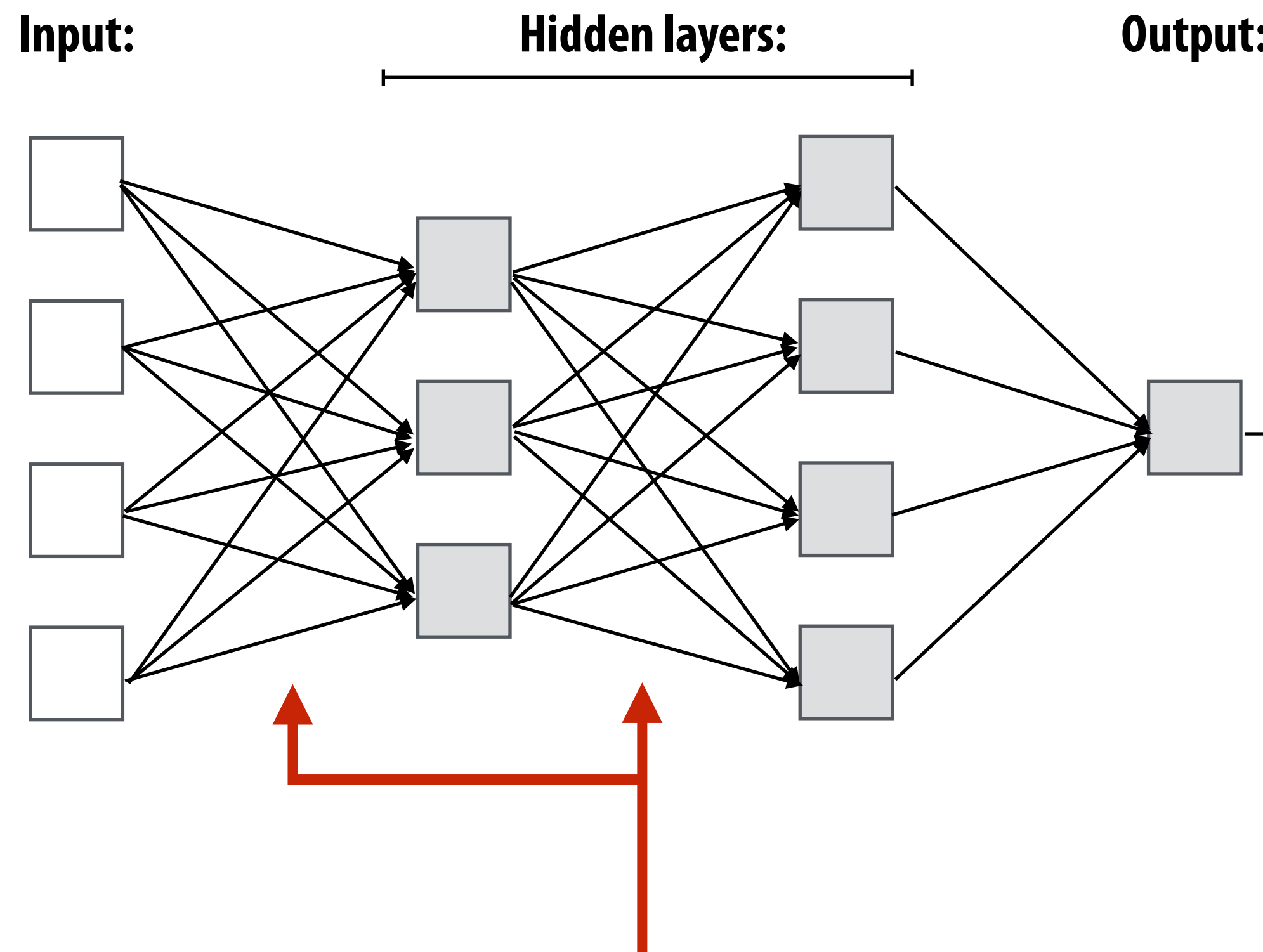
# What is a deep neural network? topology

This network has: 4 inputs, 1 output, 7 hidden units

“Deep” = at least one hidden layer

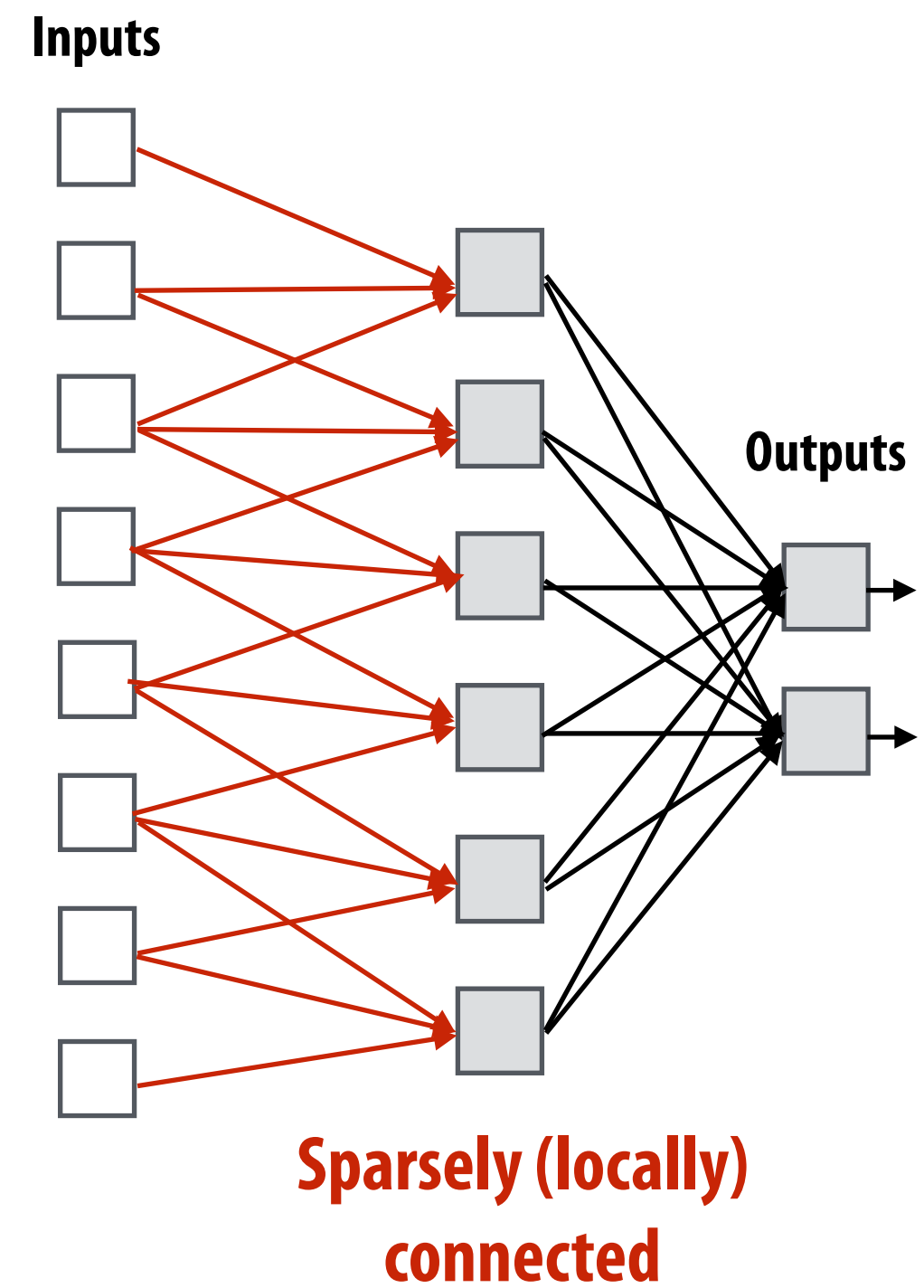
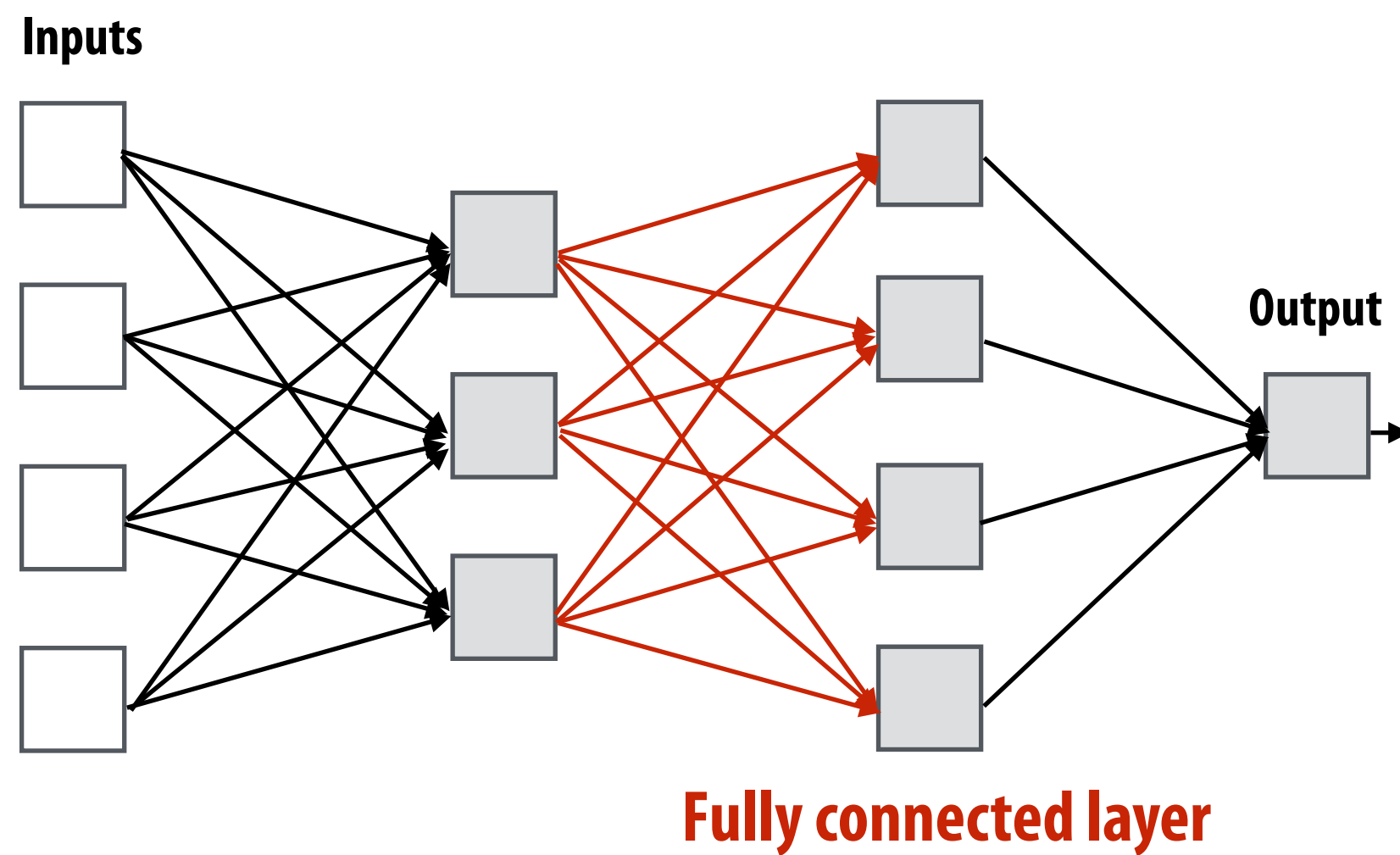
Hidden layer 1: 3 units x (4 weights + 1 bias) = 15 parameters

Hidden layer 2: 4 units x (3 weights + 1 bias) = 16 parameters



**Note fully-connected topology in this example**

# What is a deep neural network? topology

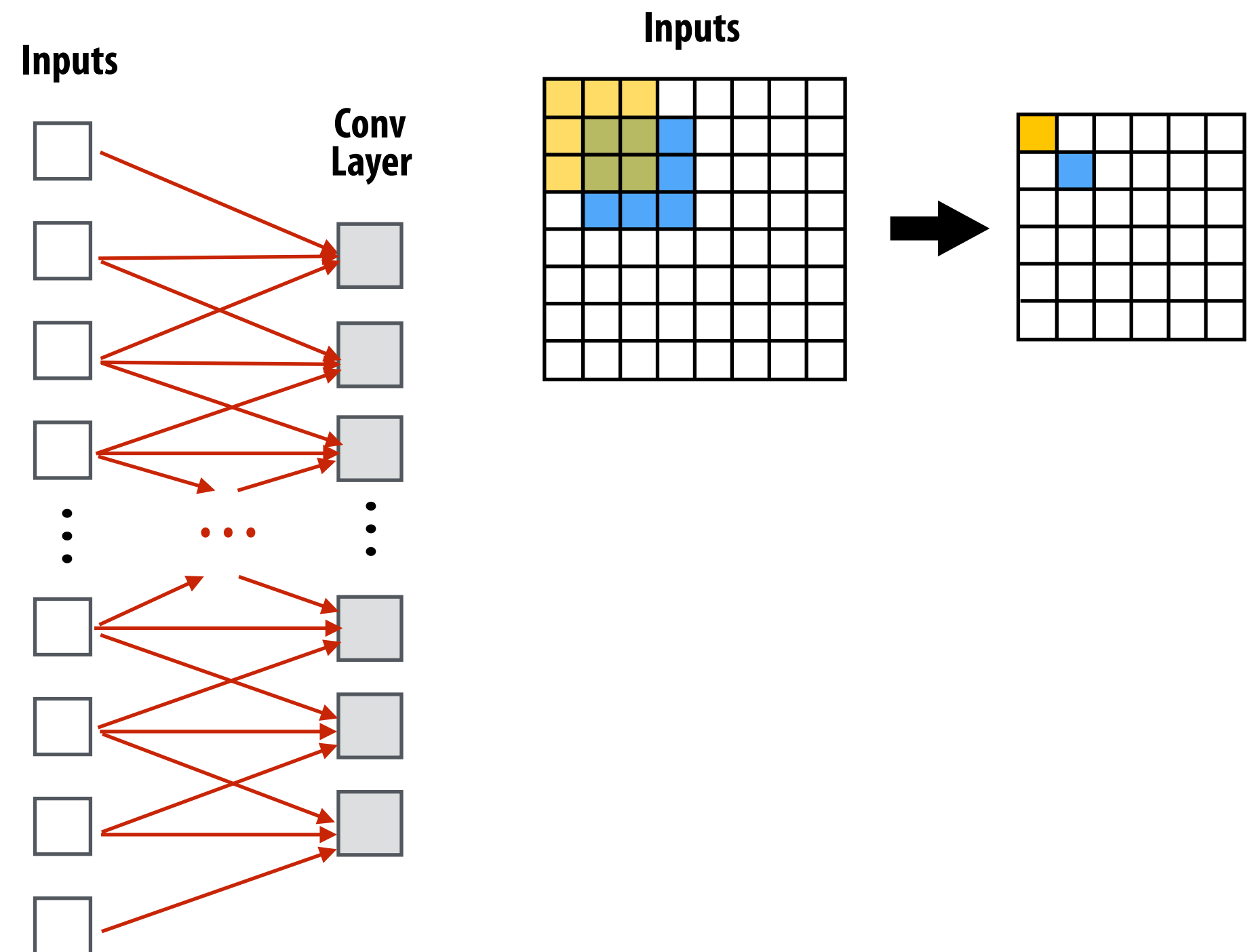


# Recall image convolution (3x3 conv)

```
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1.0/9, 1.0/9, 1.0/9,
                  1.0/9, 1.0/9, 1.0/9,
                  1.0/9, 1.0/9, 1.0/9};

for (int j=0; j<HEIGHT; j++) {
  for (int i=0; i<WIDTH; i++) {
    float tmp = 0.f;
    for (int jj=0; jj<3; jj++)
      for (int ii=0; ii<3; ii++)
        tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
    output[j*WIDTH + i] = tmp;
  }
}
```



**Convolutional layer: locally connected AND all units in layer share the same parameters (same weights + same bias):**  
(note: network diagram only shows links due to one iteration of  $i$  loop)

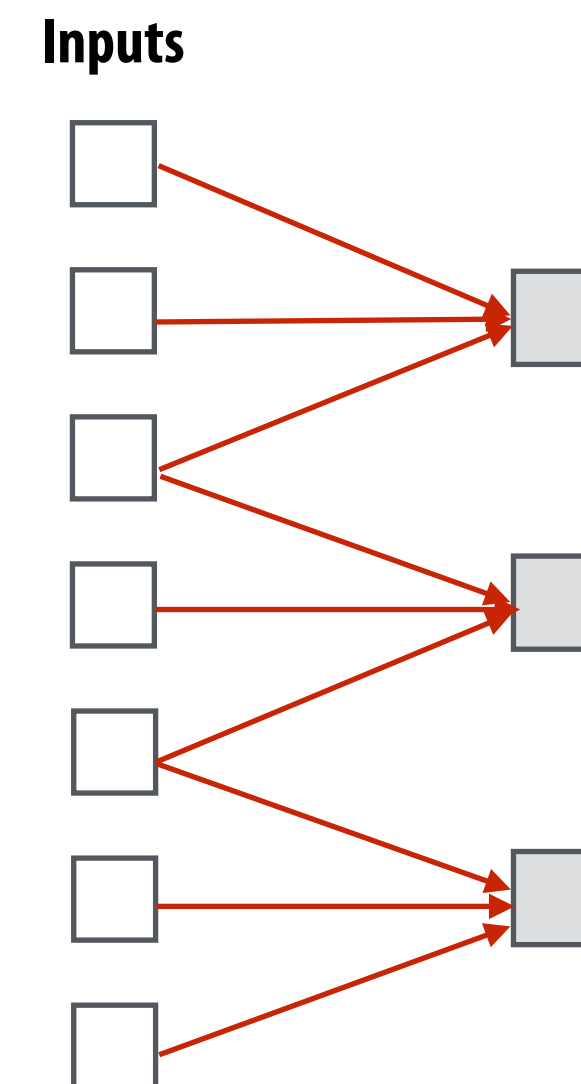
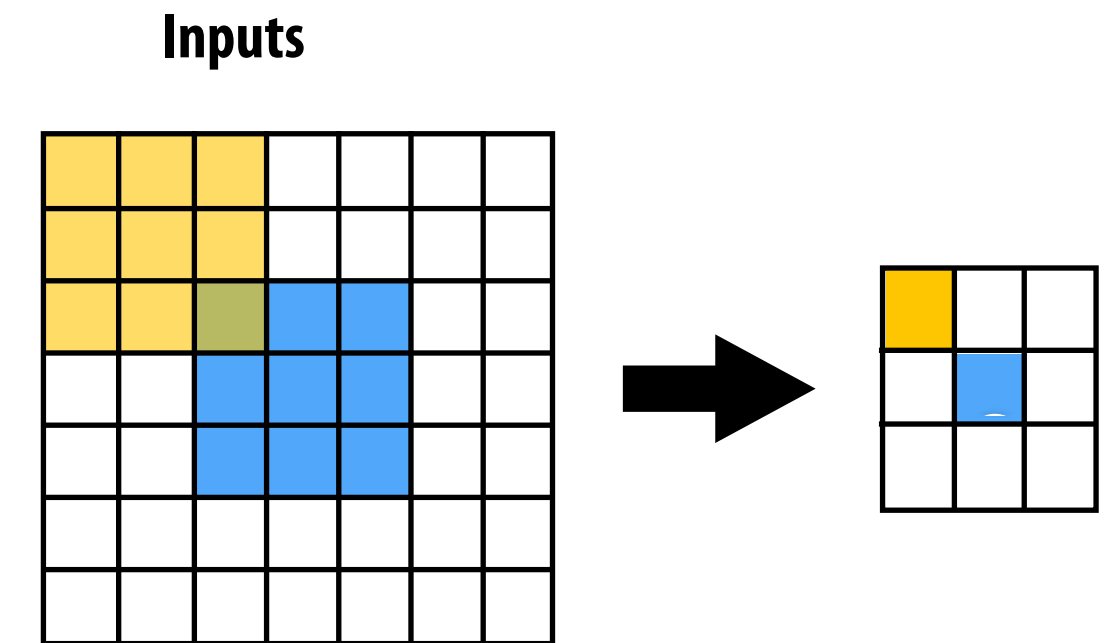


# Strided 3x3 convolution

```
int WIDTH = 1024;
int HEIGHT = 1024;
int STRIDE = 2;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[(WIDTH/STRIDE) * (HEIGHT/STRIDE)];

float weights[] = {1.0/9, 1.0/9, 1.0/9,
                  1.0/9, 1.0/9, 1.0/9,
                  1.0/9, 1.0/9, 1.0/9};

for (int j=0; j<HEIGHT; j+=STRIDE) {
  for (int i=0; i<WIDTH; i+=STRIDE) {
    float tmp = 0.f;
    for (int jj=0; jj<3; jj++)
      for (int ii=0; ii<3; ii++) {
        tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
        output[(j/STRIDE)*WIDTH + (i/STRIDE)] = tmp;
      }
  }
}
```

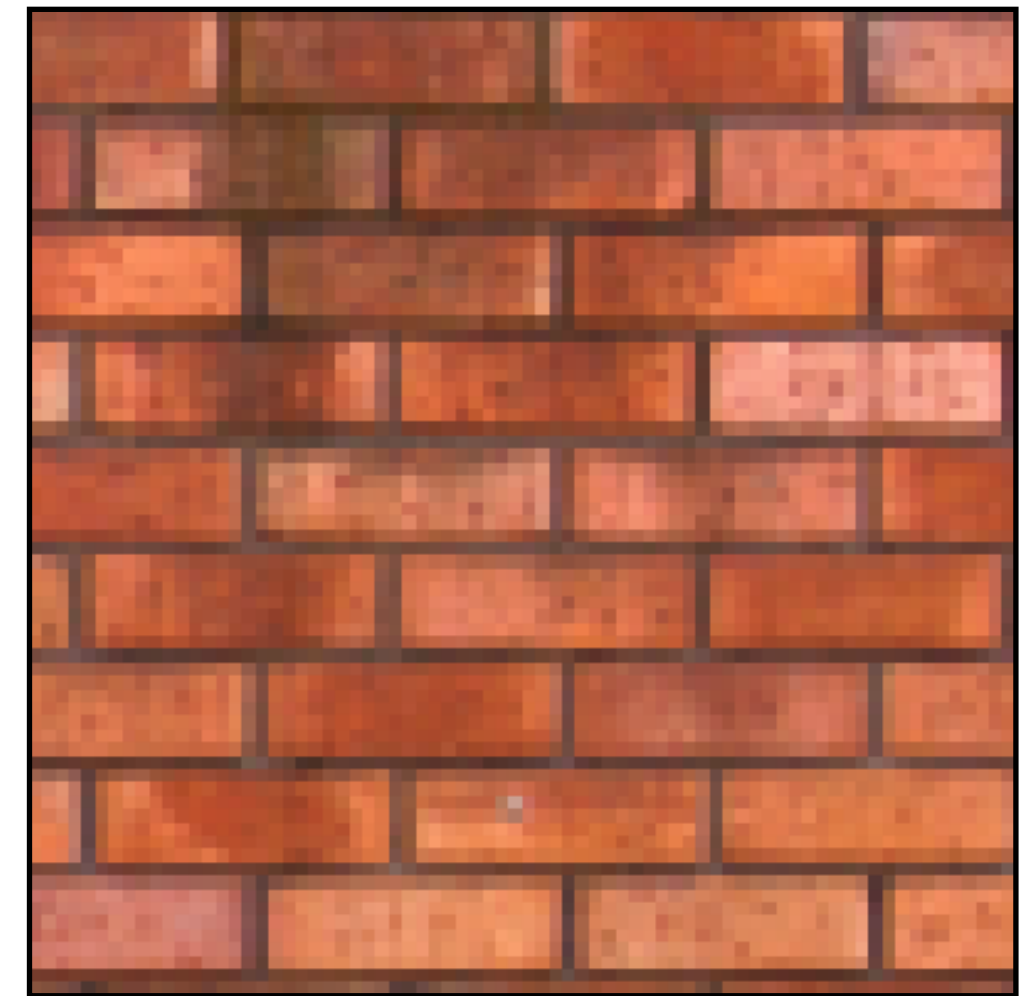
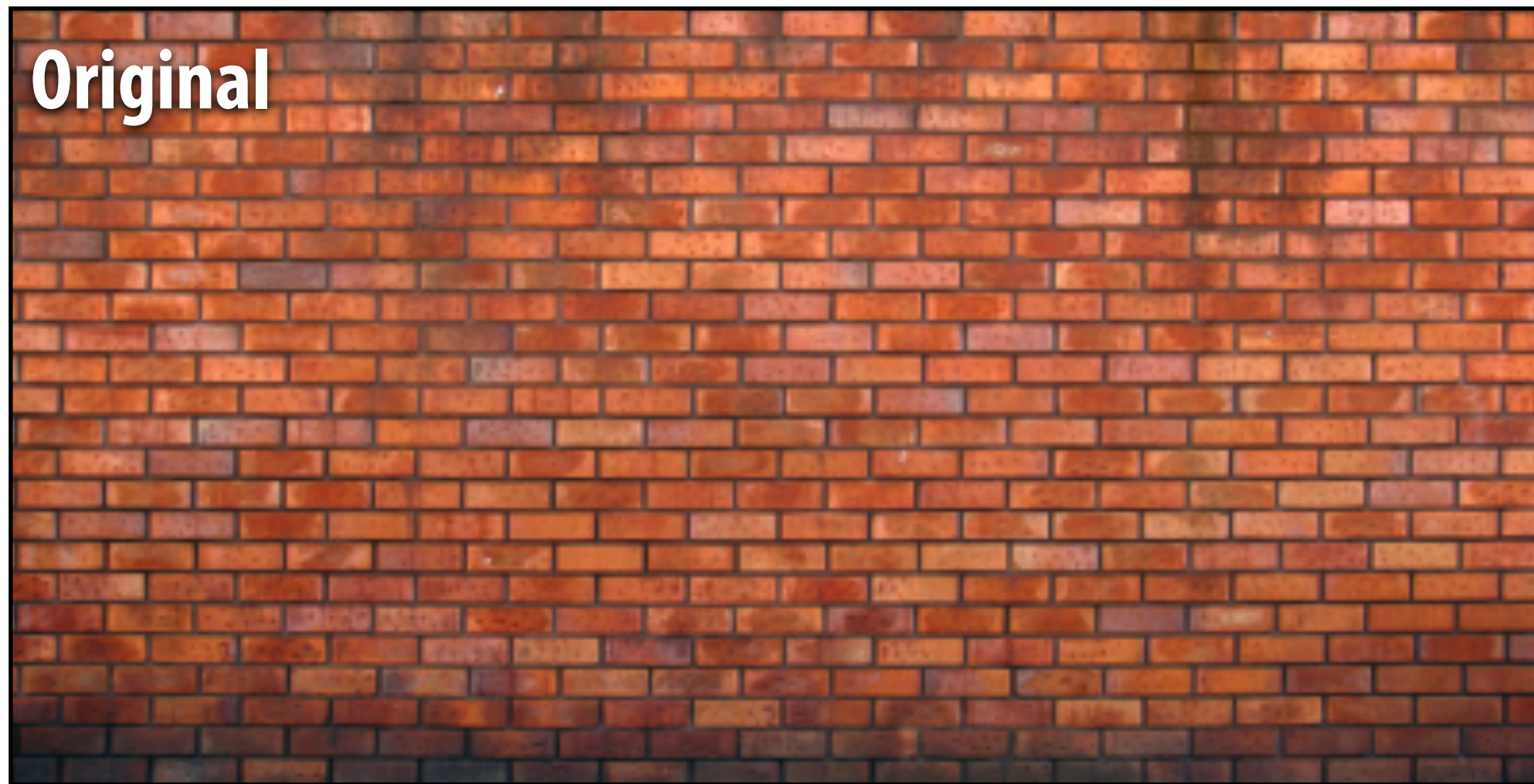


**Convolutional layer with stride 2**

# What does convolution using these filter weights do?

$$\begin{bmatrix} .075 & .124 & .075 \\ .124 & .204 & .124 \\ .075 & .124 & .075 \end{bmatrix}$$

“Gaussian Blur”



# What does convolution with these filters do?

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

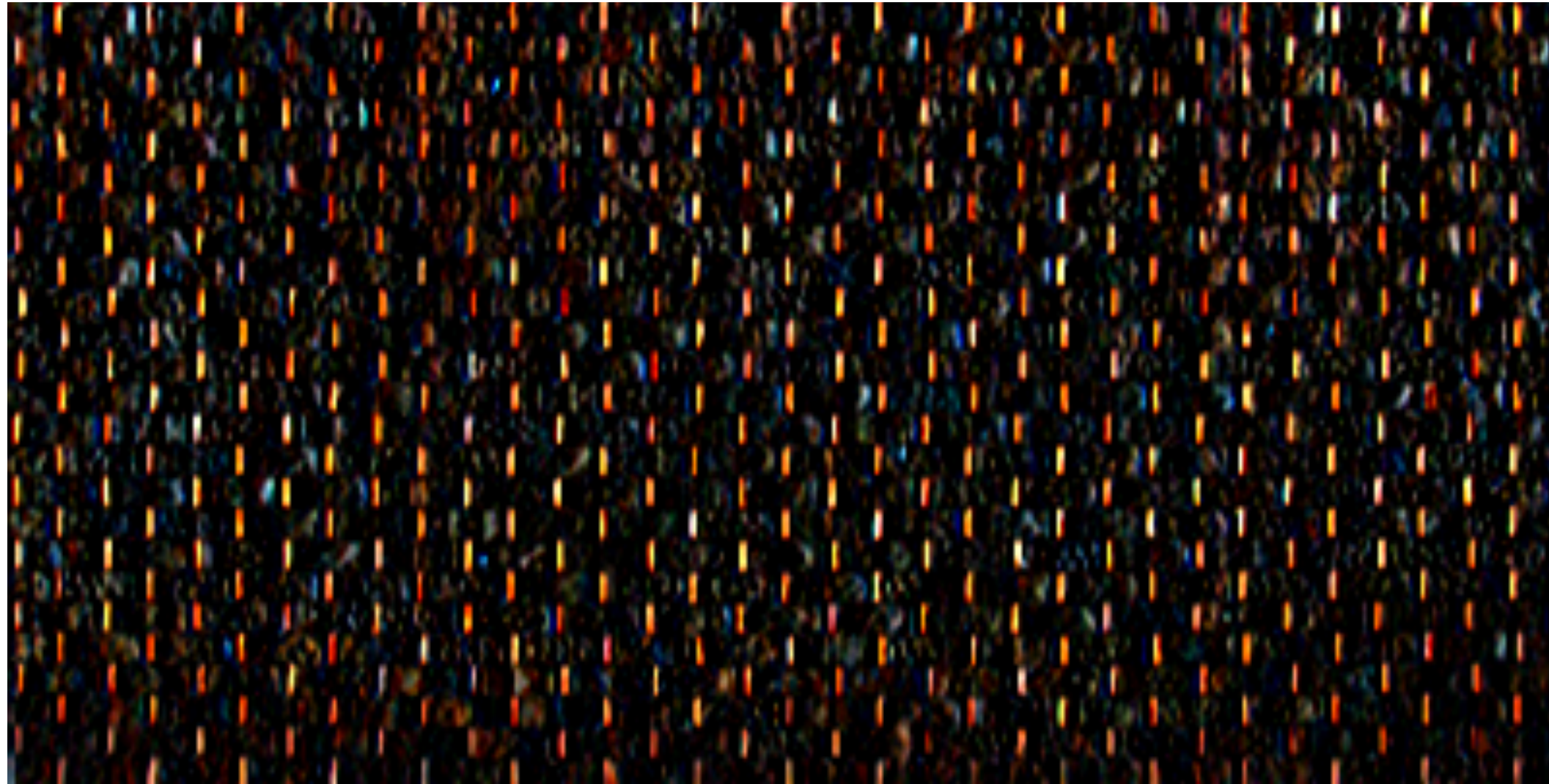
**Extracts horizontal  
gradients**

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

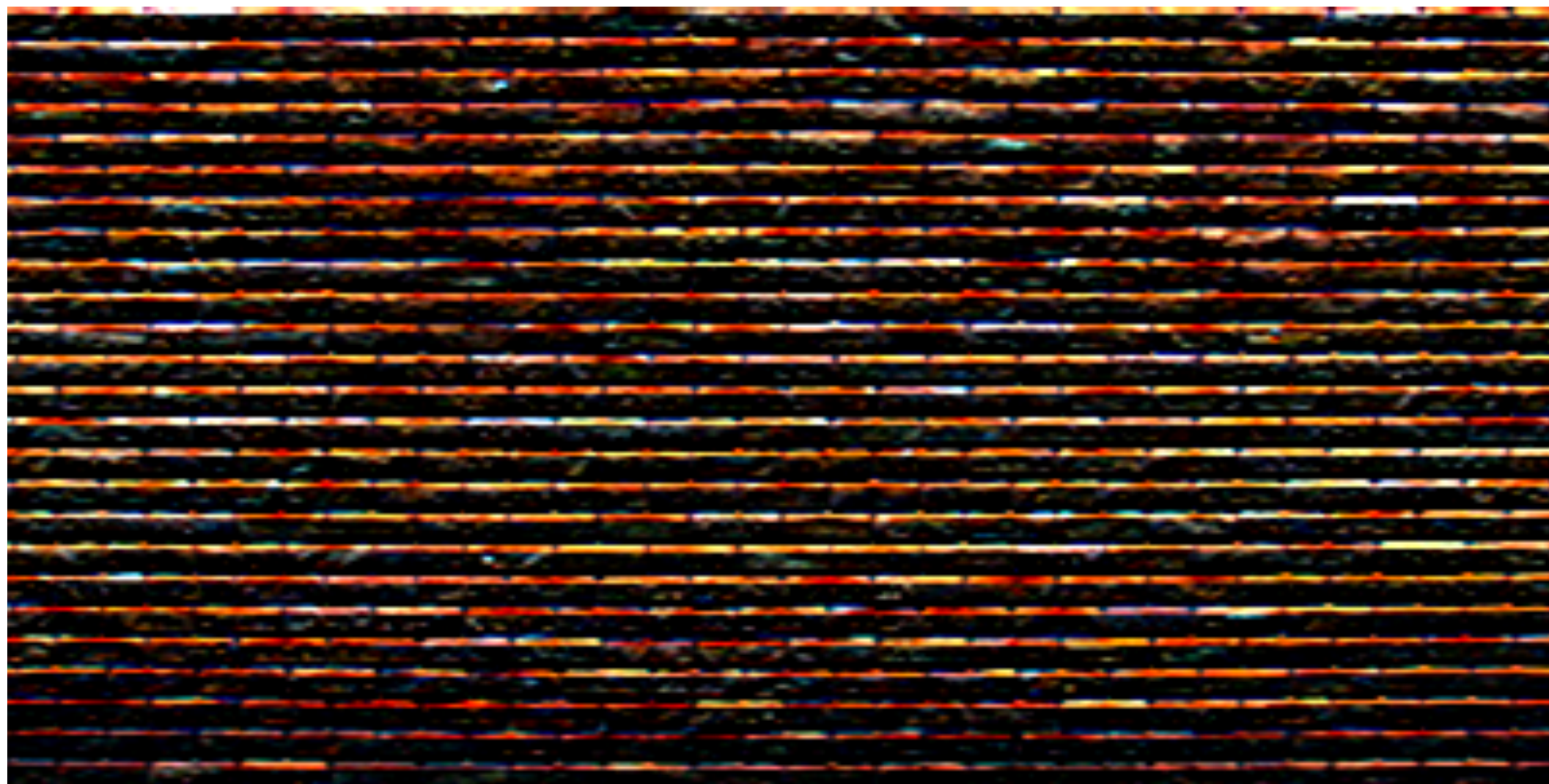
**Extracts vertical  
gradients**



# Gradient detection filters



**Horizontal gradients**

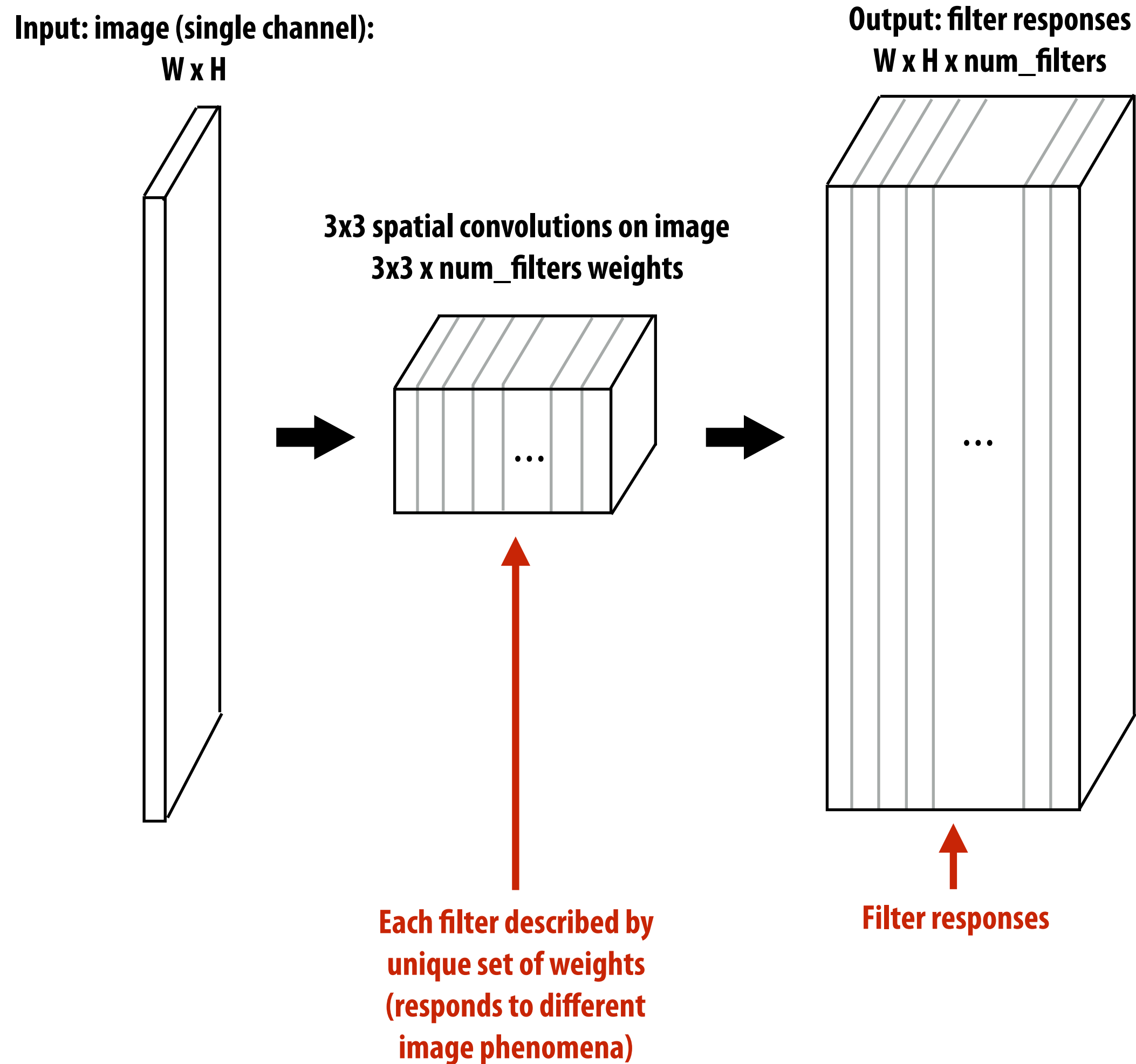


**Vertical gradients**

**Note: you can think of a filter as a “detector” of a pattern, and the magnitude of a pixel in the output image as the “response” of the filter to the region surrounding each pixel in the input image**



# Applying many filters to an image at once

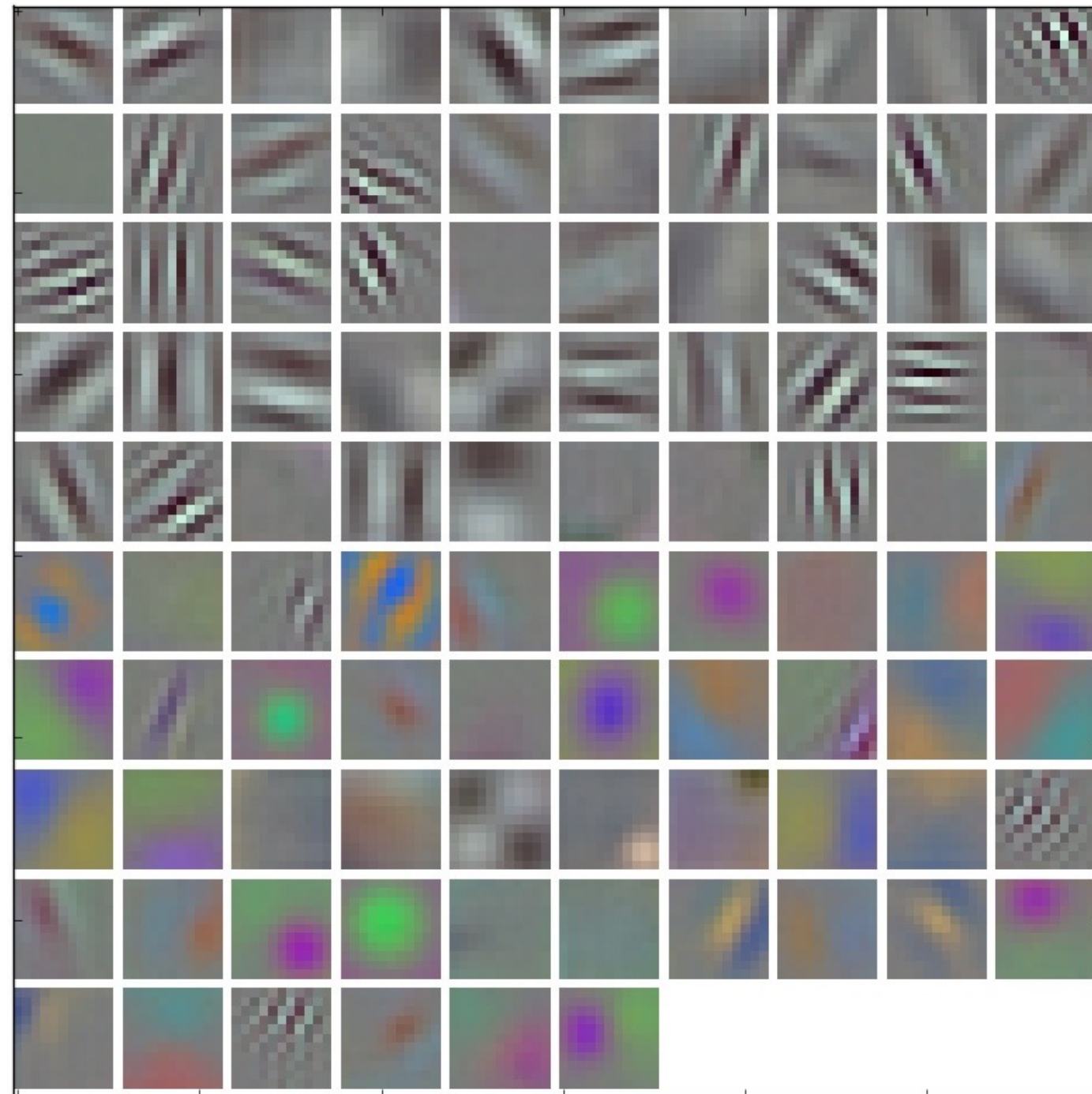


# Applying many filters to an image at once

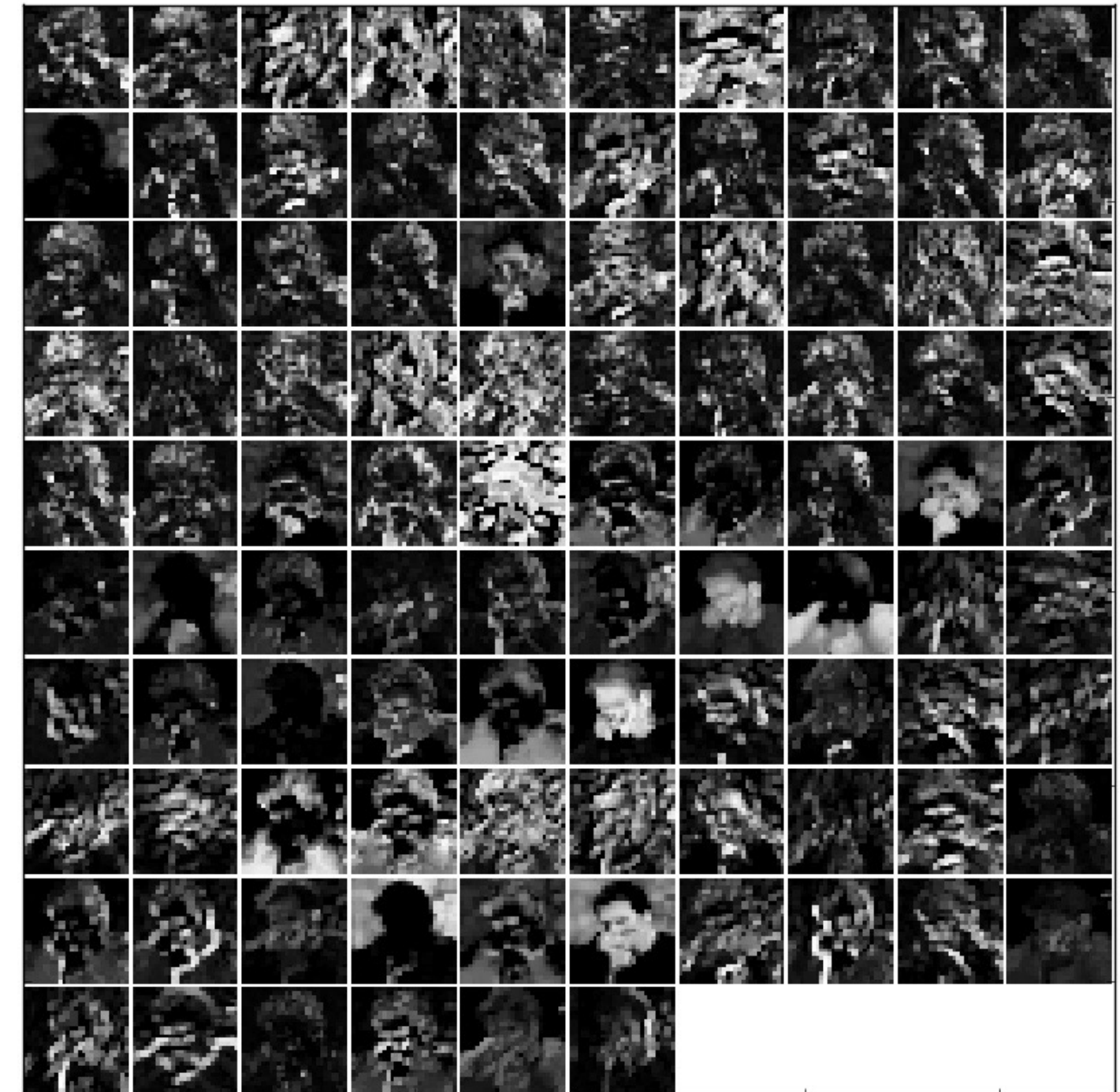
Input RGB image (W x H x 3)



96 11x11x3 filters  
(operate on RGB)

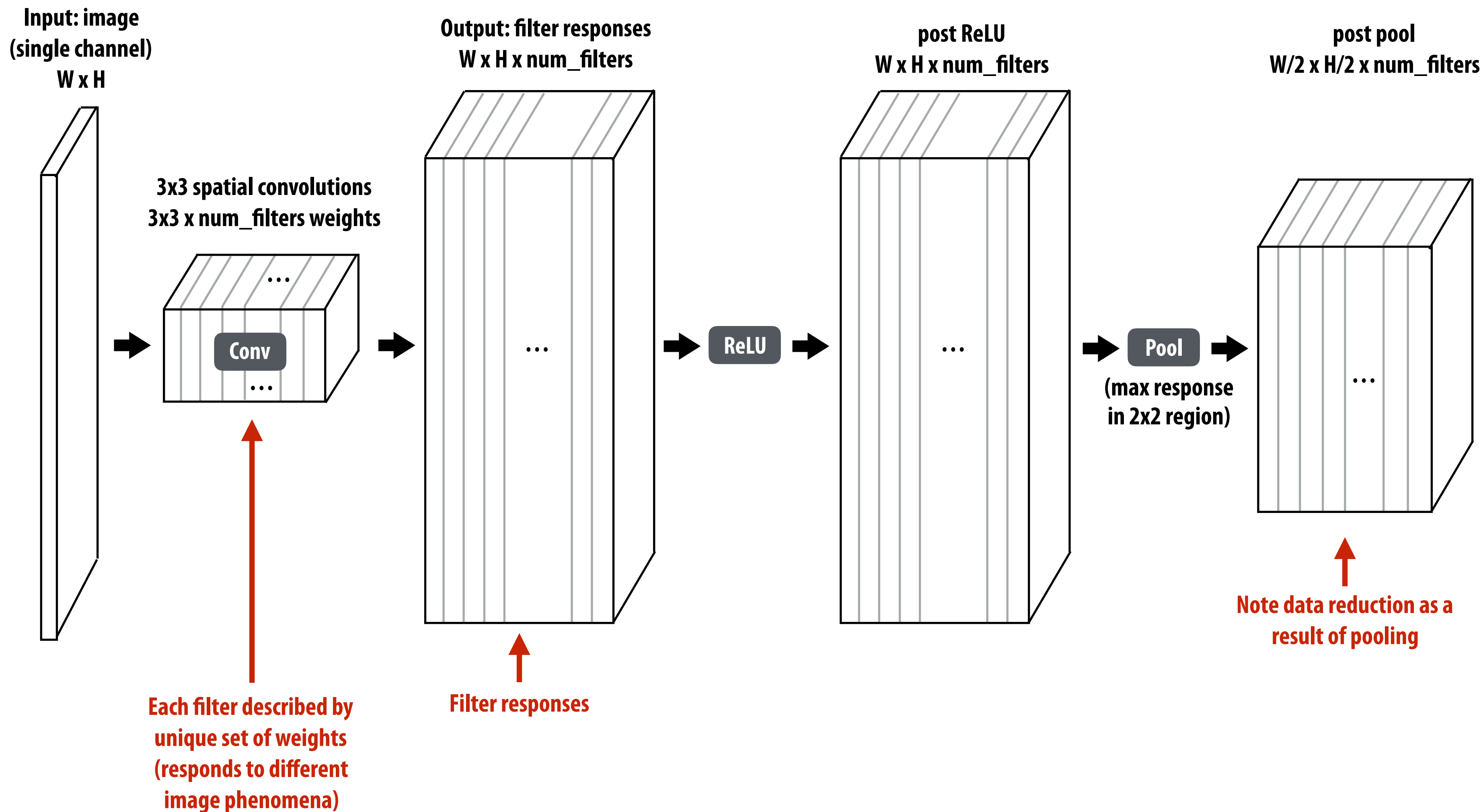


96 responses (normalized)





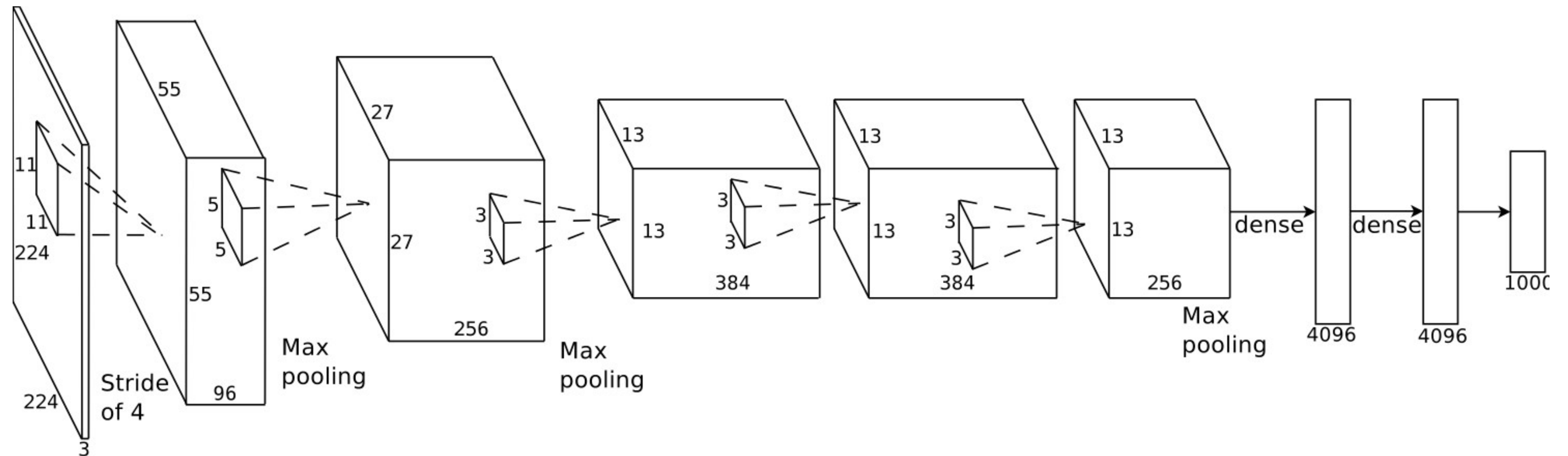
# Adding additional layers



# Modern object detection networks

Sequences of conv + reLU + (optional) pool layers

AlexNet [Krizhevsky12]: 5 convolutional layers + 3 fully connected



VGG-16 [Simonyan15]: 13 convolutional layers

input: 224 x 224 RGB

conv/reLU: 3x3x3x64

conv/reLU: 3x3x64x64

maxpool

conv/reLU: 3x3x64x128

conv/reLU: 3x3x128x128

maxpool

conv/reLU: 3x3x128x256

conv/reLU: 3x3x256x256

conv/reLU: 3x3x256x256

maxpool

conv/reLU: 3x3x256x512

conv/reLU: 3x3x512x512

conv/reLU: 3x3x512x512

maxpool

conv/reLU: 3x3x512x512

conv/reLU: 3x3x512x512

conv/reLU: 3x3x512x512

maxpool

fully-connected 4096

fully-connected 4096

fully-connected 1000

soft-max

# Efficiently implementing convolution layers

# Direct implementation of conv layer

```
float input[INPUT_HEIGHT][INPUT_WIDTH][INPUT_DEPTH];
float output[INPUT_HEIGHT][INPUT_WIDTH][LAYER_NUM_FILTERS];
float layer_weights[LAYER_CONVY, LAYER_CONVX, INPUT_DEPTH];

// assumes convolution stride is 1
for (int img=0; img<IMAGE_BATCH_SIZE; img++)
    for (int j=0; j<INPUT_HEIGHT; j++)
        for (int i=0; i<INPUT_WIDTH; i++)
            for (int f=0; f<LAYER_NUM_FILTERS; f++) {
                output[j][i][f] = 0.f;
                for (int kk=0; kk<INPUT_DEPTH; kk++) // sum over filter responses of input channels
                    for (int jj=0; jj<LAYER_CONVY; jj++) // spatial convolution
                        for (int ii=0; ii<LAYER_CONVX; ii+) // spatial convolution
                            output[j][i][f] += layer_weights[f][jj][ii][kk] * input[j+jj][i+ii][kk];
            }
}
```

**Seven loops with significant input data reuse: reuse of filter weights (during convolution), and reuse of input values (across different filters)**

**But must roll your own highly optimized implementation of a complicated loop nest.**



# Dense matrix multiplication

```
float A[M][K];  
float B[K][N];  
float C[M][N];
```

```
// compute C += A * B
```

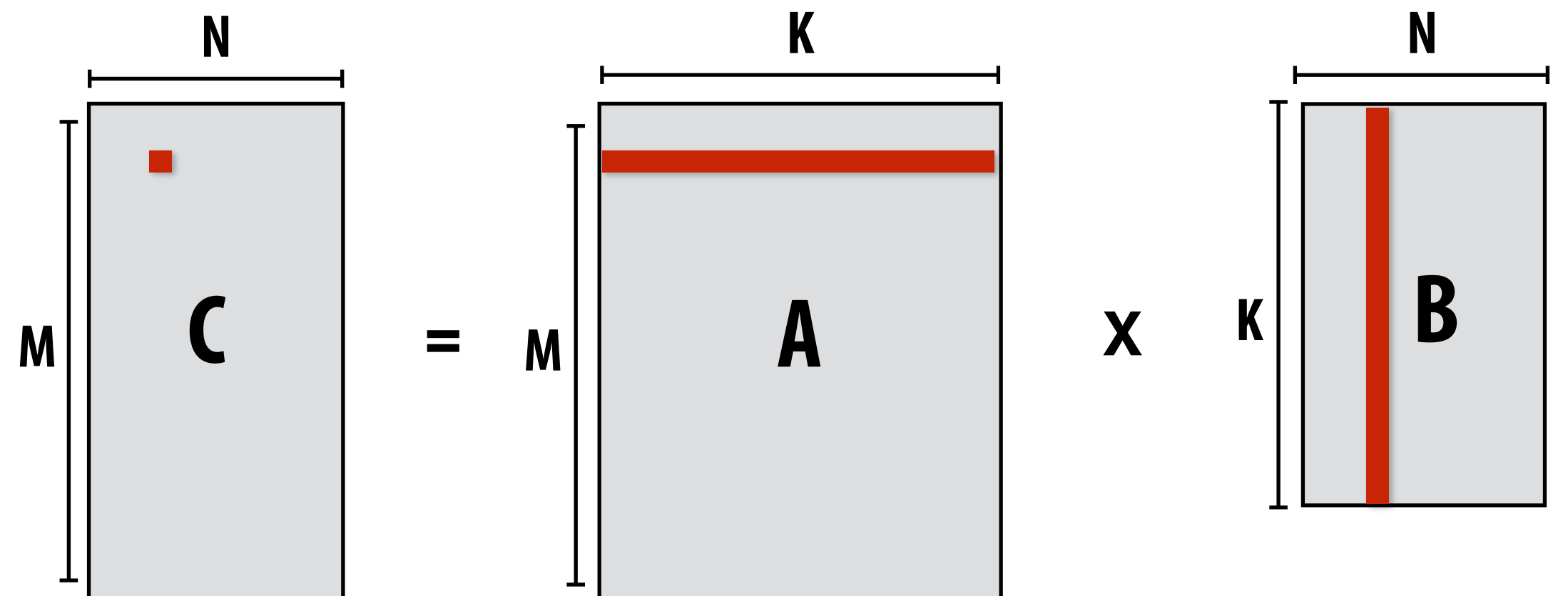
```
#pragma omp parallel for
```

```
for (int j=0; j<M; j++)
```

```
    for (int i=0; i<N; i++)
```

```
        for (int k=0; k<K; k++)
```

```
            C[j][i] += A[j][k] * B[k][i];
```



**What is the problem with this implementation?**

**Low arithmetic intensity (does not exploit temporal locality in access to A and B)**

# Blocked dense matrix multiplication

```
float A[M][K];  
float B[K][N];  
float C[M][N];
```

```
// compute C += A * B
```

```
#pragma omp parallel for
```

```
for (int jblock=0; jblock<M; jblock+=BLOCKSIZE_J)
```

```
    for (int iblock=0; iblock<N; iblock+=BLOCKSIZE_I)
```

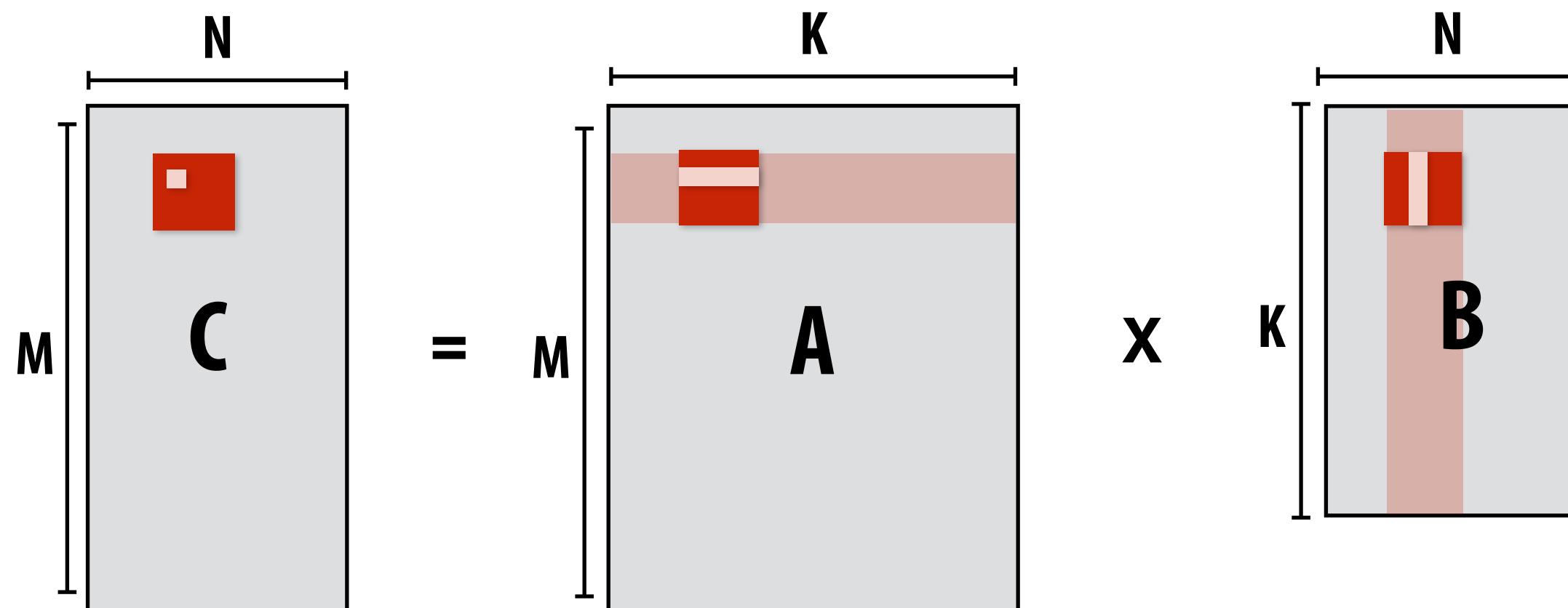
```
        for (int kblock=0; kblock<K; kblock+=BLOCKSIZE_K)
```

```
            for (int j=0; j<BLOCKSIZE_J; j++)
```

```
                for (int i=0; i<BLOCKSIZE_I; i++)
```

```
                    for (int k=0; k<BLOCKSIZE_K; k++)
```

```
                        C[jblock+j][iblock+i] += A[jblock+j][kblock+k] * B[kblock+k][iblock+i];
```



**Idea: compute partial result for block of C while required blocks of A and B remain in cache (Assumes BLOCKSIZE chosen to allow block of A, B, and C to remain resident)**

**Self check: do you want as big a BLOCKSIZE as possible? Why?**

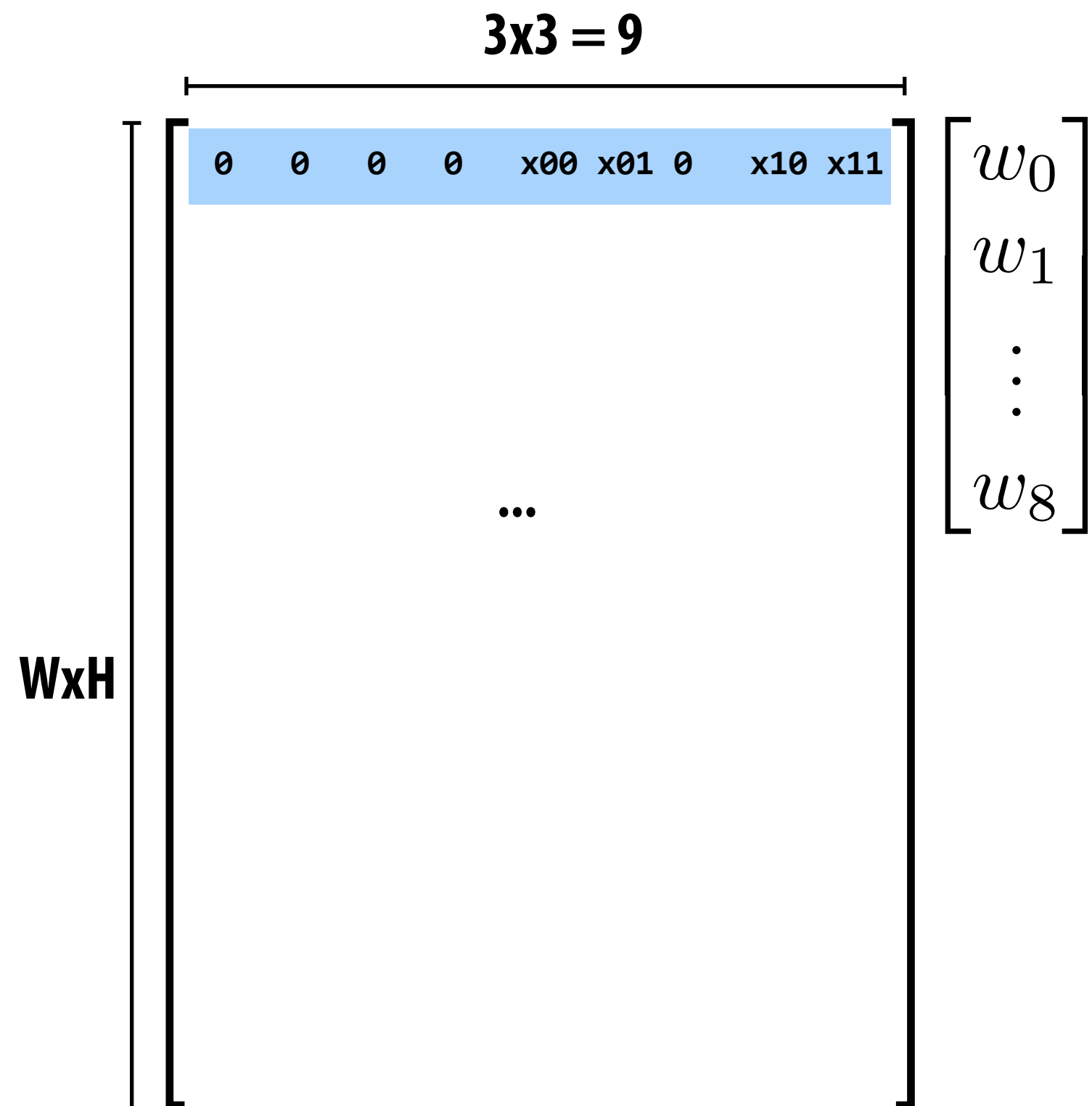


# Convolution as matrix-vector product

Construct matrix from elements of input image

$x_{00}$	$x_{01}$	$x_{02}$	$x_{03}$	...			
$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	...			
$x_{20}$	$x_{21}$	$x_{22}$	$x_{23}$	...			
$x_{30}$	$x_{31}$	$x_{32}$	$x_{33}$	...			
...	...	...	...				

$O(N)$  storage overhead for filter with  $N$  elements  
Must construct input data matrix



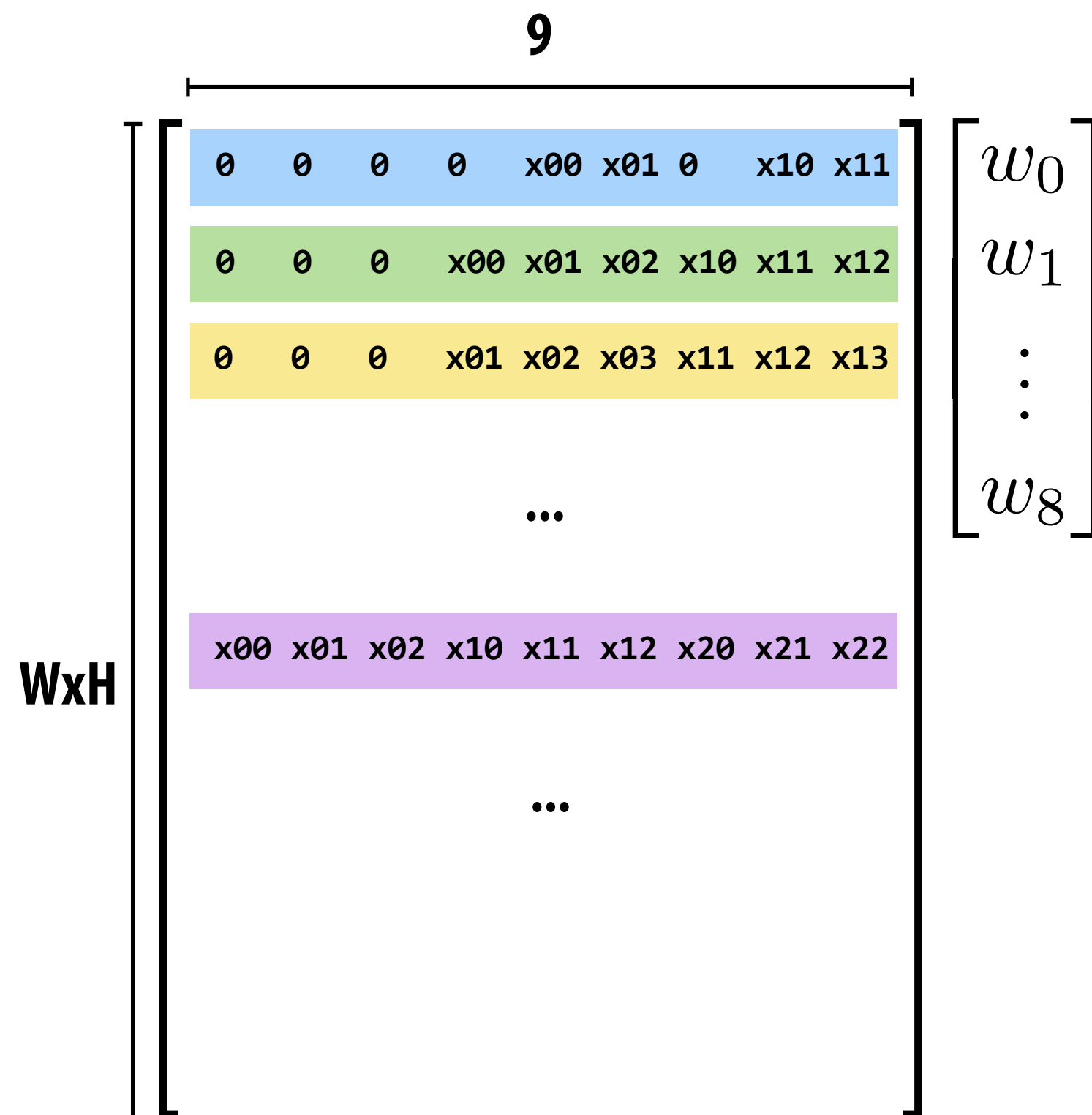
Note: 0-pad matrix

# 3x3 convolution as matrix-vector product

Construct matrix from elements of input image

	$x_{00}$	$x_{01}$	$x_{02}$	$x_{03}$	...			
	$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	...			
	$x_{20}$	$x_{21}$	$x_{22}$	$x_{23}$	...			
	$x_{30}$	$x_{31}$	$x_{32}$	$x_{33}$	...			
	...	...	...	...				

$O(N)$  storage overhead for filter with  $N$  elements  
Must construct input data matrix



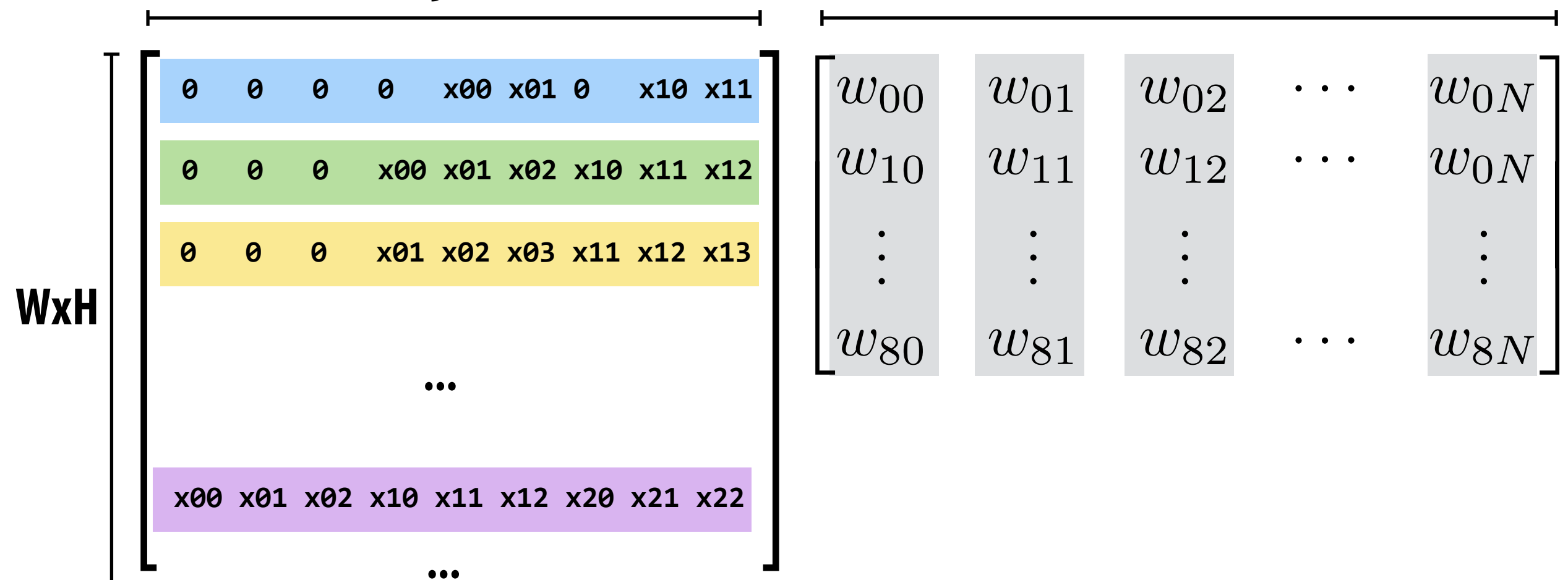
Note: 0-pad matrix

# Multiple convolutions as matrix-matrix mult

	$X_{00}$	$X_{01}$	$X_{02}$	$X_{03}$	...			
	$X_{10}$	$X_{11}$	$X_{12}$	$X_{13}$	...			
	$X_{20}$	$X_{21}$	$X_{22}$	$X_{23}$	...			
	$X_{30}$	$X_{31}$	$X_{32}$	$X_{33}$	...			
	...	...	...	...				

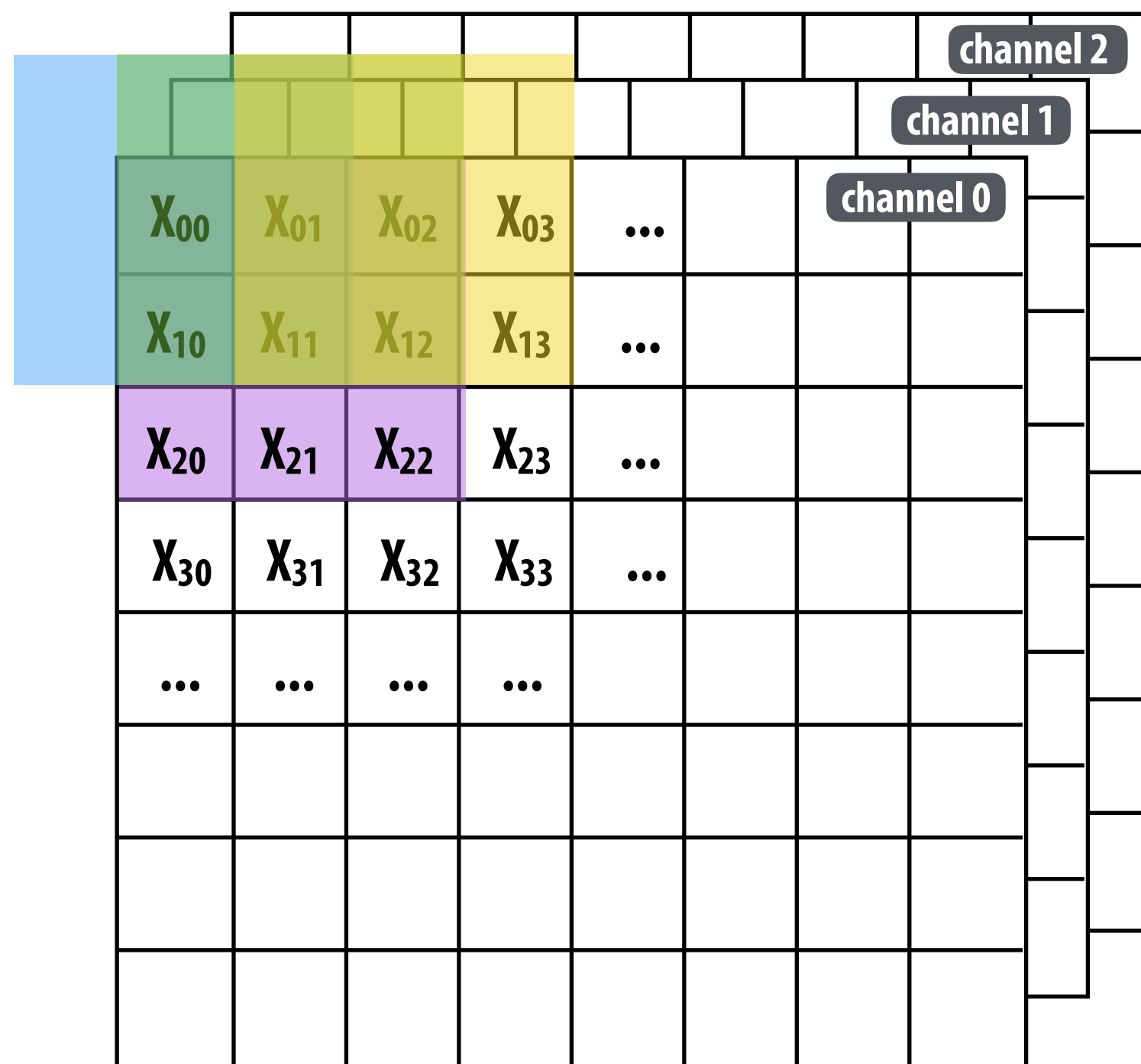
9

num filters



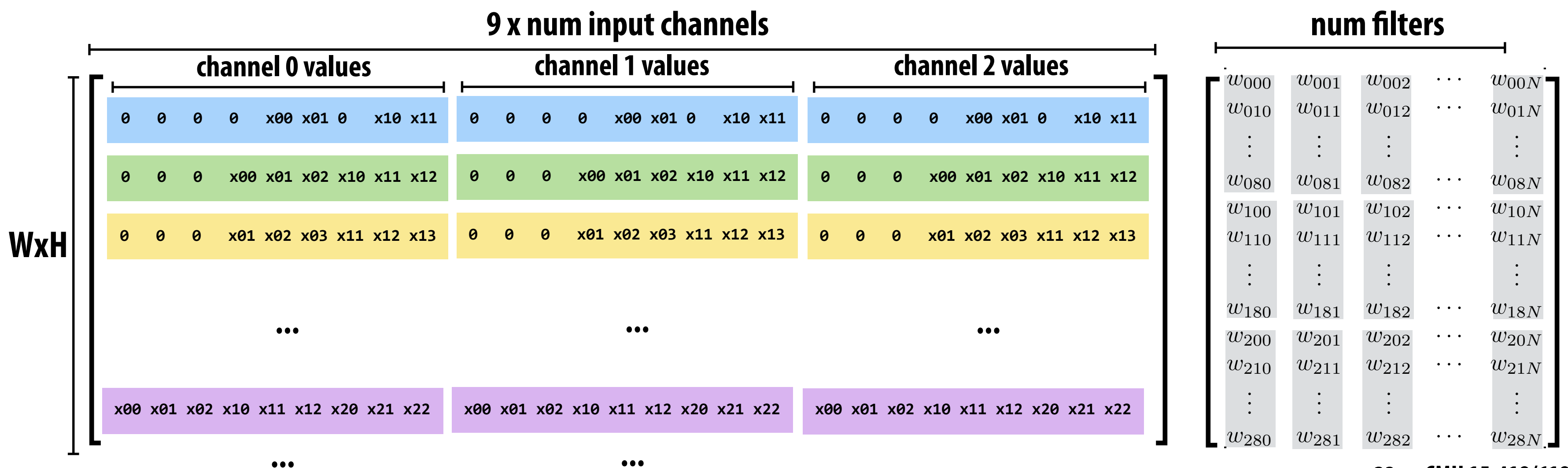


# Multiple convolutions on multiple input channels



For each filter, sum responses over input channels

Equivalent to  $(3 \times 3 \times \text{num\_channels})$  convolution on  $(W \times H \times \text{num\_channels})$  input data



# VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

inputs/outputs get multiplied by image batch size

multiply by next layer's conv window size to form input matrix to next conv layer!!! (for VGG, this is a 9x data amplification)

	weights mem:	output size (per image)	(mem)
input: 224 x 224 RGB image	—	224x224x3	150K
conv: (3x3x3) x 64	6.5 KB	224x224x64	12.3 MB
conv: (3x3x64) x 64	144 KB	224x224x64	12.3 MB
maxpool	—	112x112x64	3.1 MB
conv: (3x3x64) x 128	228 KB	112x112x128	6.2 MB
conv: (3x3x128) x 128	576 KB	112x112x128	6.2 MB
maxpool	—	56x56x128	1.5 MB
conv: (3x3x128) x 256	1.1 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
maxpool	—	28x28x256	766 KB
conv: (3x3x256) x 512	4.5 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
maxpool	—	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
maxpool	—	7x7x512	98 KB
fully-connected 4096	392 MB	4096	16 KB
fully-connected 4096	64 MB	4096	16 KB
fully-connected 1000	15.6 MB	1000	4 KB
soft-max		1000	4 KB

# Reducing network footprint

- **Large storage cost for model parameters**
  - AlexNet model: ~200 MB
  - VGG-16 model: ~500 MB
  - This doesn't even account for intermediates during evaluation
- **Footprint: cumbersome to store, download, etc.**
  - 500 MB app downloads make users unhappy!
- **Consider energy cost of 1B parameter network**
  - Running on input stream at 20 Hz
  - 640 pJ per 32-bit DRAM access
  - $(20 \times 1\text{B} \times 640\text{pJ}) = 12.8\text{W}$  for DRAM access  
**(more than power budget of any modern smartphone)**





# Compressing a network

**Step 1: prune low-weight links** (iteratively retrain network, then prune)

- Over 90% of weights can be removed without significant loss of accuracy
- Store weight matrices in compressed sparse row (CSR) format

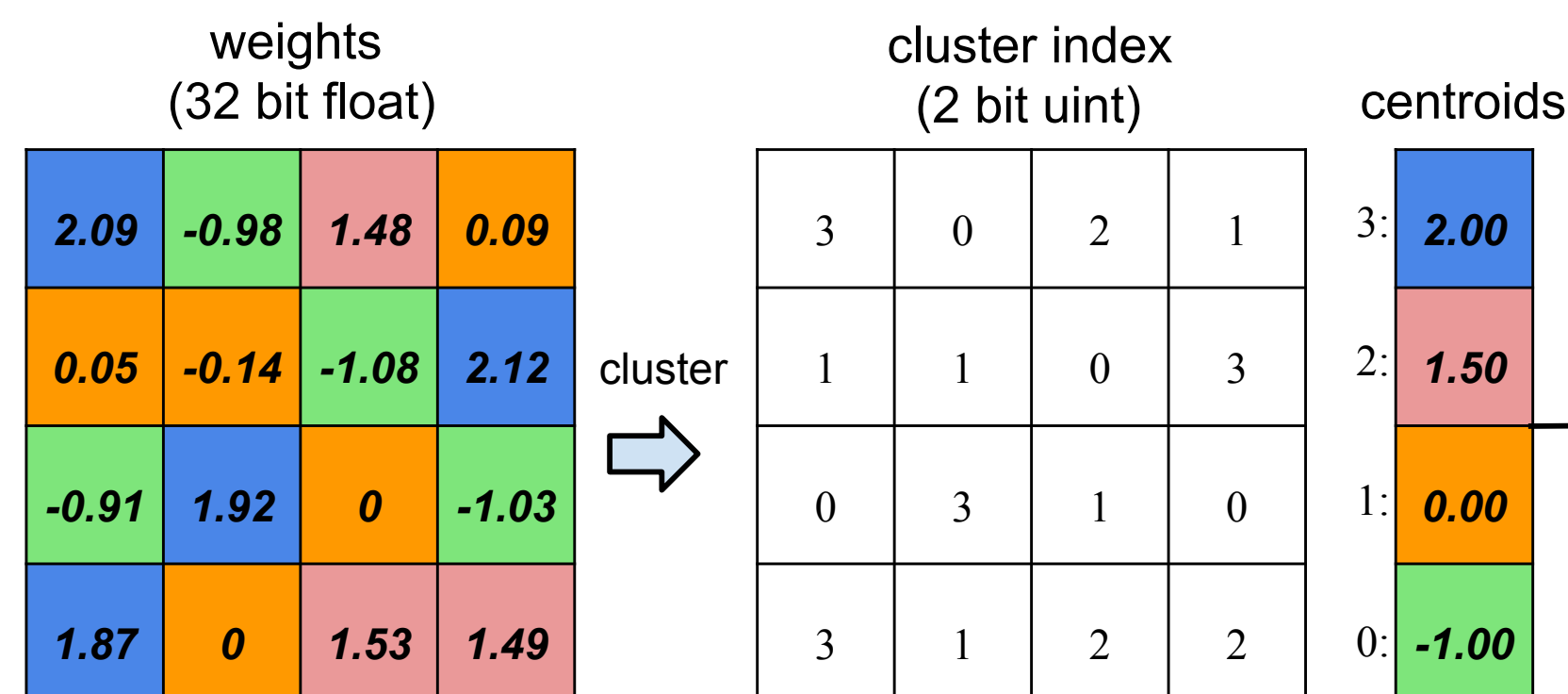
Indicies      1      4      9      ...  
Value        1.8   0.5   2.1

0	1.8	0	0	0.5	0	0	0	0	2.1	...
---	-----	---	---	-----	---	---	---	---	-----	-----

**Step 2: weight sharing:** make surviving connects share a small set of weights

- Cluster weights via k-means clustering (irregular (“learned”) quantization)
- Compress weights by only storing cluster index ( $\lg(k)$  bits)
- Retrain network to improve quality of cluster centroids

**Step 3: Huffman encode** quantized weights and CSR indices



# VGG-16 compression

Substantial savings due to combination of pruning, quantization, Huffman encoding

Layer	#Weights	Weights% (P)	Weigh bits (P+Q)	Weight bits (P+Q+H)	Index bits (P+Q)	Index bits (P+Q+H)	Compress rate (P+Q)	Compress rate (P+Q+H)
conv1_1	2K	58%	8	6.8	5	1.7	40.0%	29.97%
conv1_2	37K	22%	8	6.5	5	2.6	9.8%	6.99%
conv2_1	74K	34%	8	5.6	5	2.4	14.3%	8.91%
conv2_2	148K	36%	8	5.9	5	2.3	14.7%	9.31%
conv3_1	295K	53%	8	4.8	5	1.8	21.7%	11.15%
conv3_2	590K	24%	8	4.6	5	2.9	9.7%	5.67%
conv3_3	590K	42%	8	4.6	5	2.2	17.0%	8.96%
conv4_1	1M	32%	8	4.6	5	2.6	13.1%	7.29%
conv4_2	2M	27%	8	4.2	5	2.9	10.9%	5.93%
conv4_3	2M	34%	8	4.4	5	2.5	14.0%	7.47%
conv5_1	2M	35%	8	4.7	5	2.5	14.3%	8.00%
conv5_2	2M	29%	8	4.6	5	2.7	11.7%	6.52%
conv5_3	2M	36%	8	4.6	5	2.3	14.8%	7.79%
fc6	103M	4%	5	3.6	5	3.5	1.6%	1.10%
fc7	17M	4%	5	4	5	4.3	1.5%	1.25%
fc8	4M	23%	5	4	5	3.4	7.1%	5.24%
Total	138M	7.5%(13×)	6.4	4.1	5	3.1	3.2% (31×)	2.05% (49×)

**P = connection pruning (prune low weight connections)**

**Q = quantize surviving weights (using shared weights)**

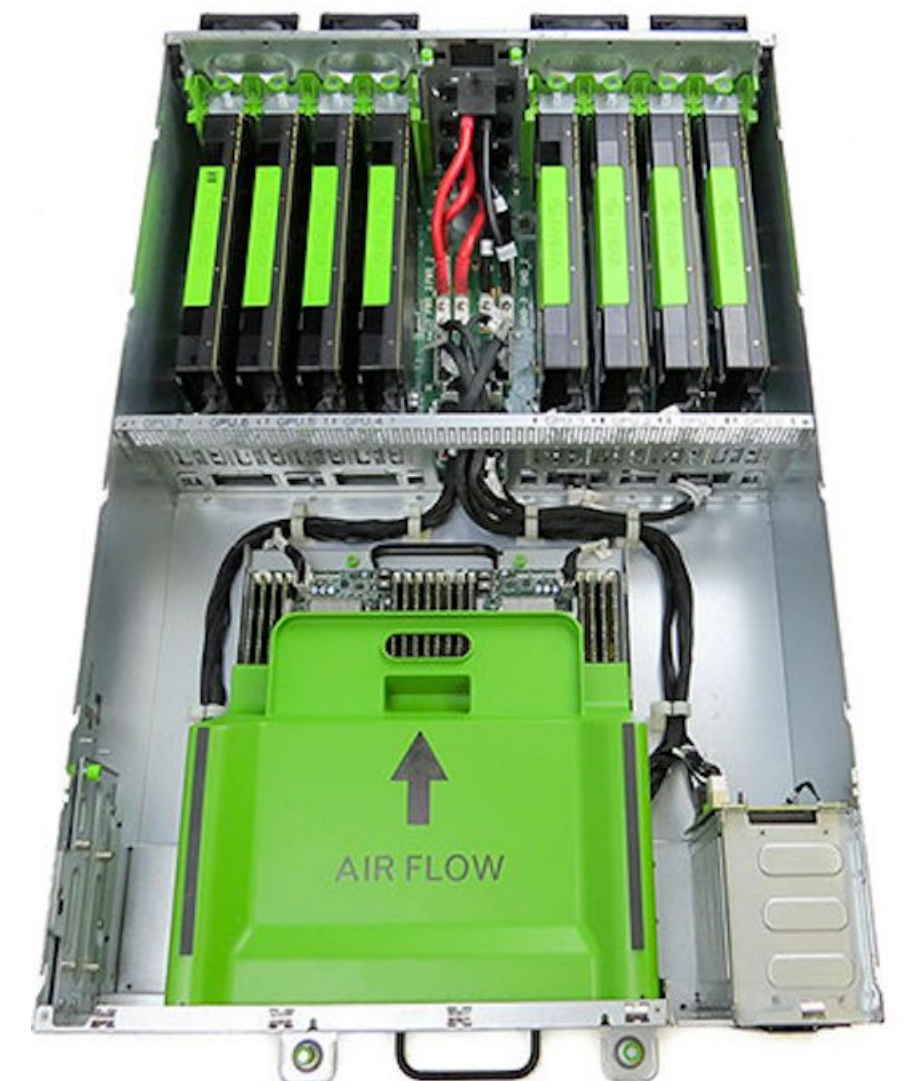
**H = Huffman encode**

## ImageNet Image Classification Performance

	Top-1 Error	Top-5 Error	Model size
VGG-16 Ref	31.50%	11.32%	552 MB
VGG-16 Compressed	31.17%	10.91%	<b>11.3 MB</b> (49×)

# Deep neural networks on GPUs

- **Today, best performing DNN implementations target GPUs**
  - **High arithmetic intensity computations** (computational characteristics similar to dense matrix-matrix multiplication)
  - **Benefit from flop-rich architectures**
  - **Highly-optimized library of kernels exist for GPUs (cuDNN)**
    - **Most CPU-based implementations use basic matrix-multiplication-based formulation (good implementations could run faster!)**



Facebook's Big Sur



# Summary: Efficiently Evaluating DNNs

## ■ Computational structure

- **Convlayers: high arithmetic intensity, significant portion of cost of evaluating a network**
- **Similar data access patterns to dense-matrix multiplication** (exploiting temporal reuse is key)
- **But straight reduction to matrix-matrix multiplication is often sub-optimal**
- **Work-efficient techniques for convolutional layers (FFT-based, Winograd convolutions)**

## ■ Large numbers of parameters: significant interest in reducing size of networks for both training and evaluation

- **Pruning: remove least important network links**
- **Quantization: low-precision parameter representations often suffice**

## ■ Many ongoing studies of specialized hardware architectures for efficient evaluation

- **Future CPUs/GPUs, ASICs, FPGAs, ...**
- **Specialization will be important to achieving “always on” applications**

# Two Distinct Issues with Deep Networks

## ■ Evaluation

- often takes milliseconds

## ■ Training

- often takes hours, days, weeks

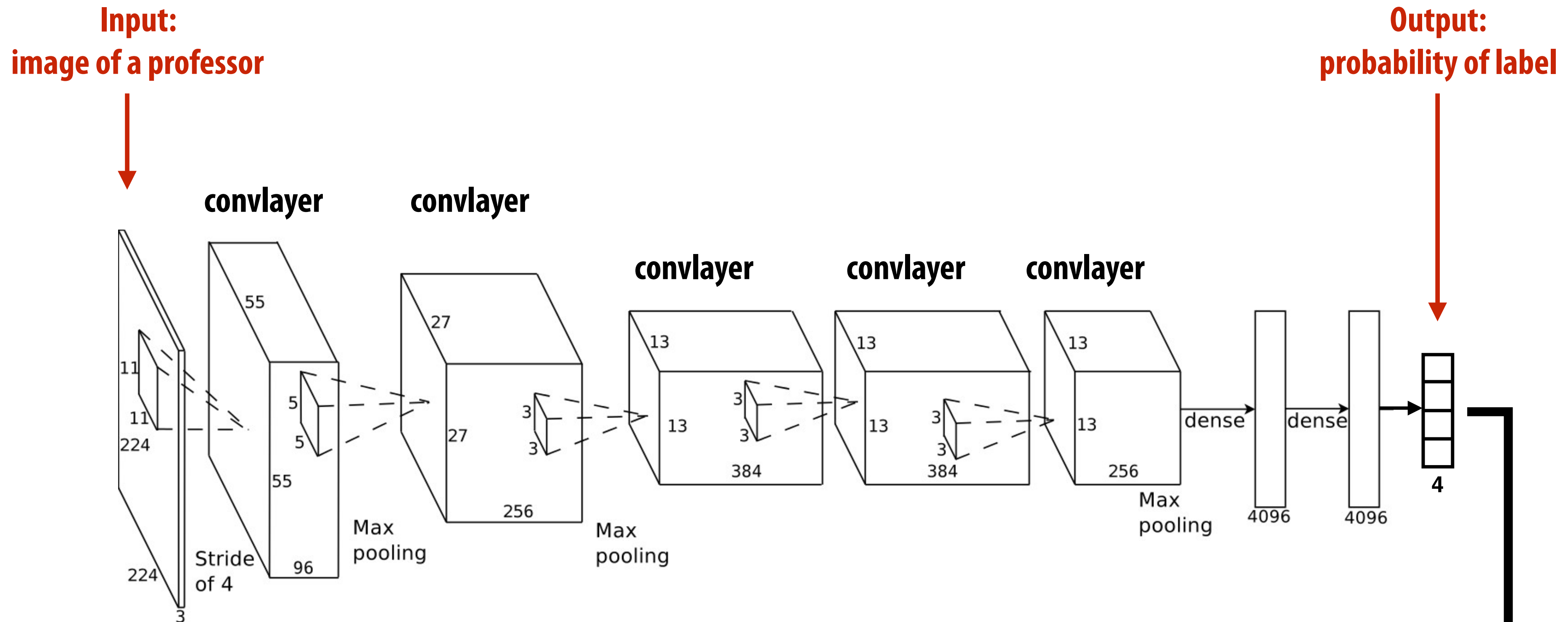
# “Training a network”

- Training a network is the process of **learning the value of network parameters** so that output of the network provides the desired result for a task
  - [Krizhevsky12] task = object classification
    - input 224 x 224 x 3 RGB image
    - output probability of 1000 ImageNet object classes: “dog”, “cat”, etc...
    - ~ 60M weights



# Professor classification network

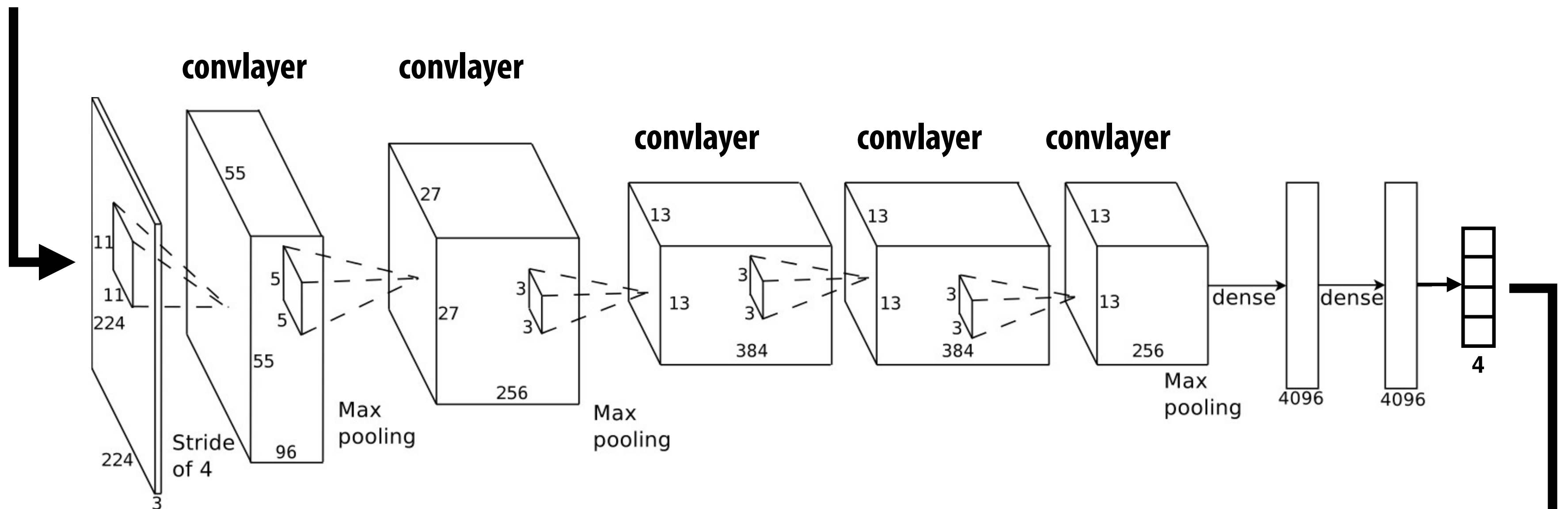
Classifies professors as easy, mean, boring, or nerdy based on their appearance.



Recall from last time:  
10's-100's of millions of parameters

Easy: ??  
Mean: ??  
Boring: ??  
Nerdy: ??

# Professor classification network



**Easy: 0.26**  
**Mean: 0.08**  
**Boring: 0.14**  
**Nerdy: 0.52**

**Where did the parameters come from?**



# Training data (ground truth answers)



[label omitted]



[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



[label omitted]



[label omitted]



[label omitted]



Nerdy



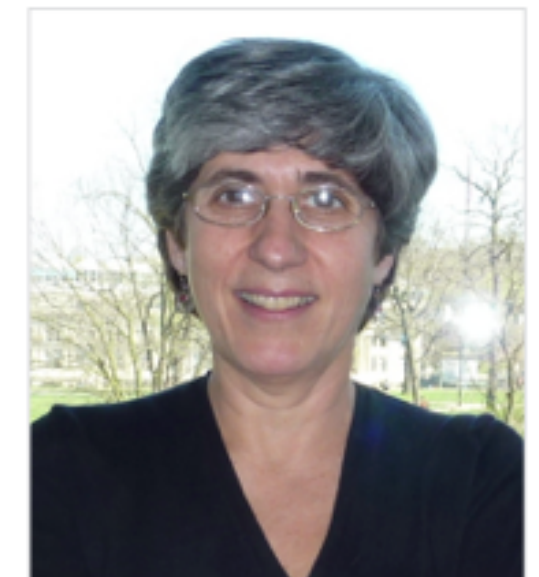
[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



[label omitted]



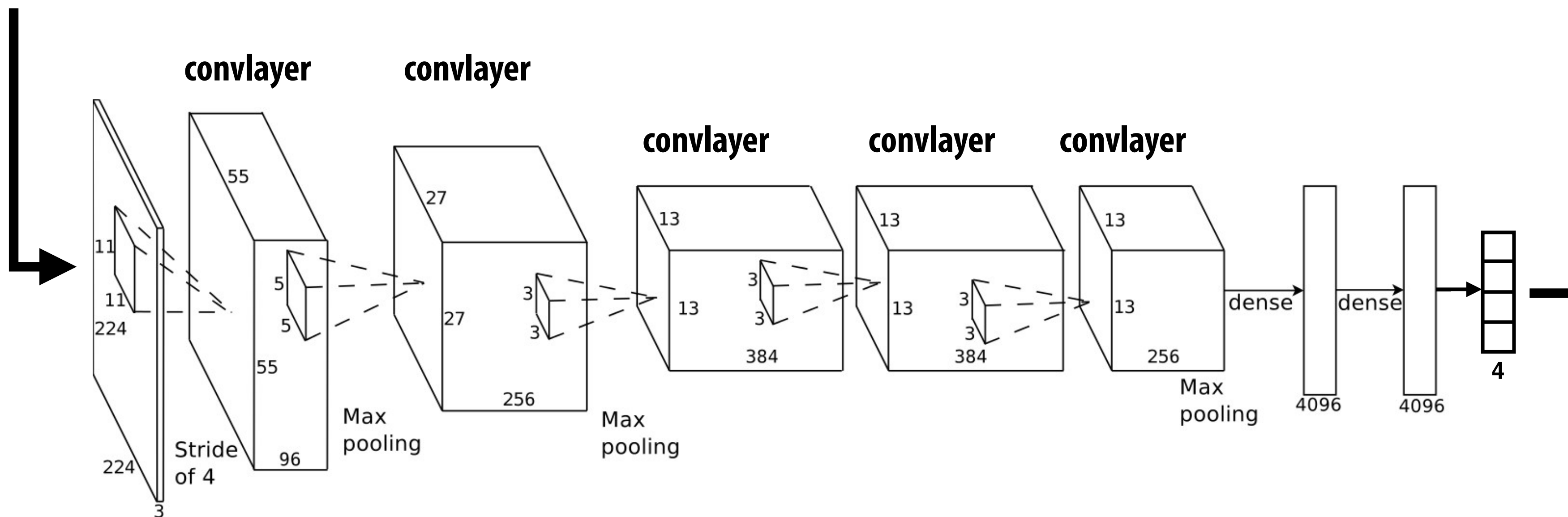
Nerdy



# Professor classification network



New image of Kayvon  
(not in training set)



**Easy: 0.0**  
**Mean: 0.0**  
**Boring: 0.0**  
**Nerdy: 1.0**

**Ground truth**  
**(what the answer should be)**

**Easy: 0.26**  
**Mean: 0.08**  
**Boring: 0.14**  
**Nerdy: 0.52**

**Network output**

# Error (loss)

**Ground truth:**  
**(what the answer should be)**

**Easy: 0.0**  
**Mean: 0.0**  
**Boring: 0.0**  
**Nerdy: 1.0**

**Network output: \***

**Easy: 0.26**  
**Mean: 0.08**  
**Boring: 0.14**  
**Nerdy: 0.52**

**Common example: softmax loss:**

$$L = -\log \left( \frac{e^{f_c}}{\sum_j e^{f_j}} \right)$$

Output of network for correct category

Output of network for all categories

\* In practice a network using a softmax classifier outputs unnormalized, log probabilities ( $f_j$ ), but I'm showing a probability distribution above for clarity

# Training

**Goal of training: learning good values of network parameters so that network outputs the correct classification result for any input image**

**Idea: minimize loss for all the training examples (for which the correct answer is known)**

$$L = \sum_i L_i \quad (\text{total loss for entire training set is sum of losses } L_i \text{ for each training example } x_i)$$

**Intuition: if the network gets the answer correct for a wide range of training examples, then hopefully it has learned parameter values that yield the correct answer for future images as well.**

# Intuition: gradient descent

Say you had a function  $f$  that contained a hidden parameters  $p_1$  and  $p_2$ :  $f(x_i)$

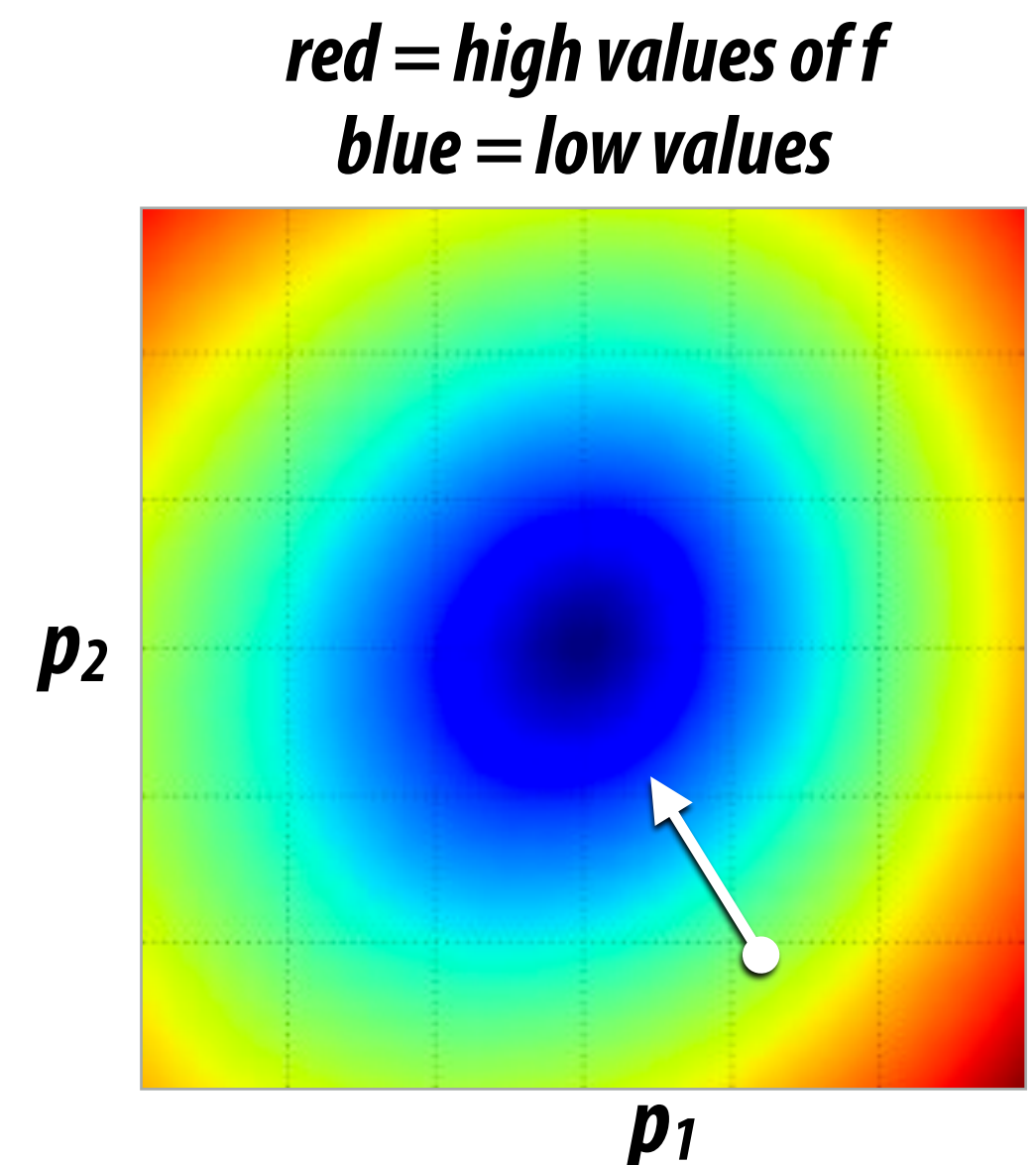
And for some input  $x_i$ , your training data says the function should output 0.

But for the current values of  $p_1$  and  $p_2$ , it currently outputs 10.

$$f(x_i, p_1, p_2) = 10$$

And say I also gave you expressions for the derivative of  $f$  with respect to  $p_1$  and  $p_2$  so you could compute their value at  $x_i$ .

$$\frac{df}{dp_1} = 2 \quad \frac{df}{dp_2} = -5 \quad \nabla f = [2, -5]$$



How might you adjust the values  $p_1$  and  $p_2$  to reduce the error for this training example?



# Basic gradient descent

```
while (loss too high):  
    for each item  $x_i$  in training set:  
        grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )  
  
    params += -grad * step_size;
```

**Mini-batch stochastic gradient descent** (mini-batch SGD): choose a random (small) subset of the training examples to compute gradient in each iteration of the while loop

How to compute  $df/dp$  for a complex neural network with millions of parameters?

# Derivatives using the chain rule

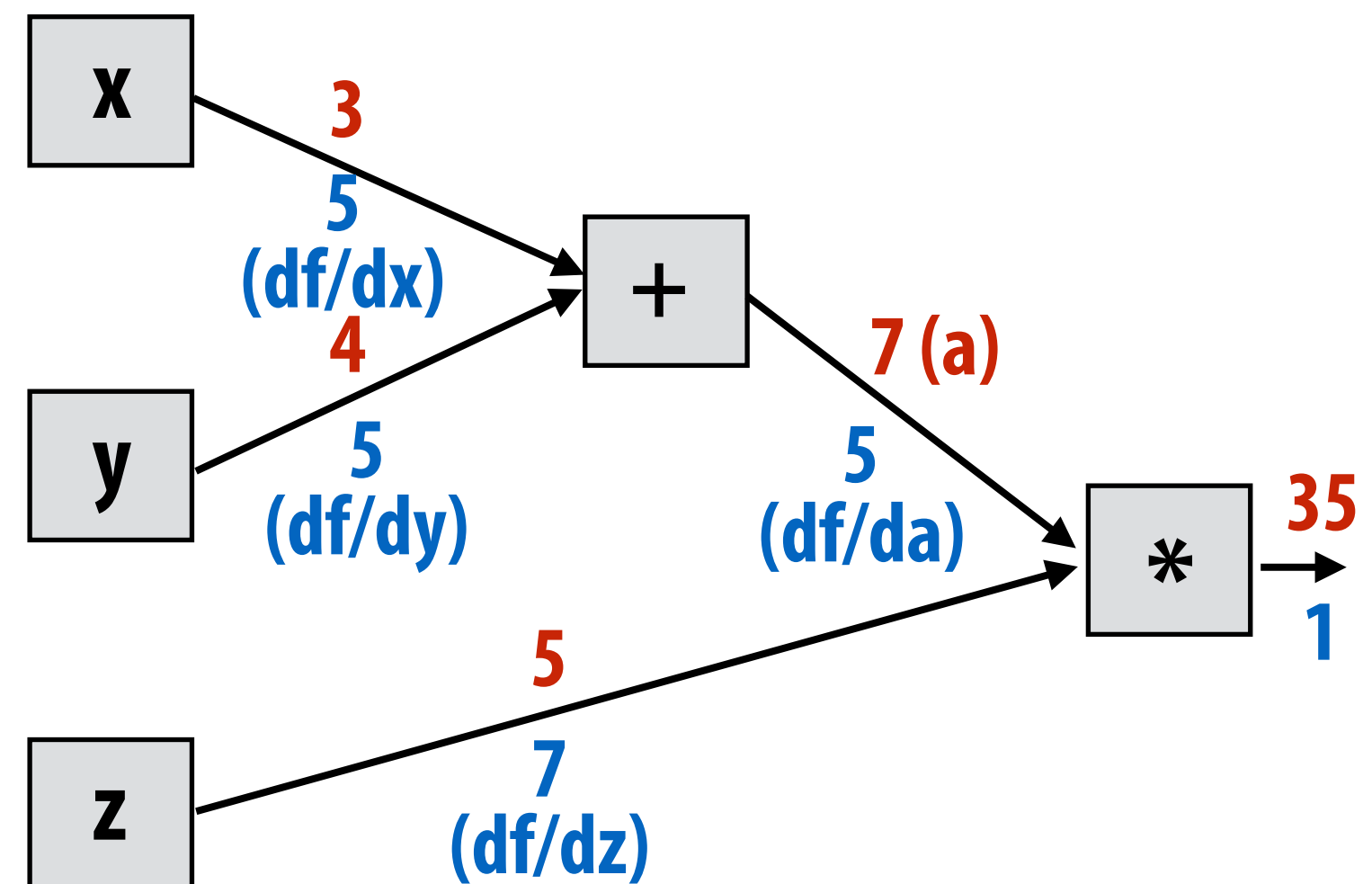
$$f(x, y, z) = (x + y)z = az$$

Where:  $a = x + y$

$$\frac{df}{da} = z \quad \frac{da}{dx} = 1 \quad \frac{da}{dy} = 1$$

So, by the derivative chain rule:

$$\frac{df}{dx} = \frac{df}{da} \frac{da}{dx} = z$$



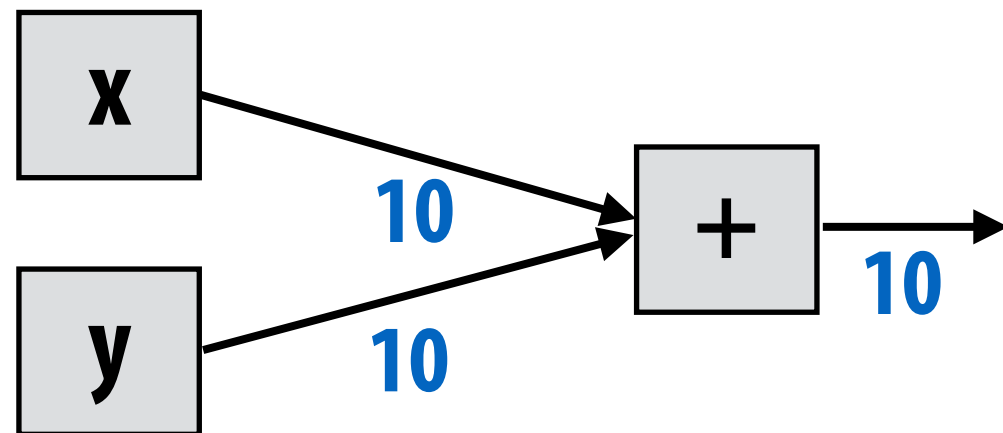
Red = output of node  
Blue =  $df/dnode$

# Backpropagation

Red = output of node

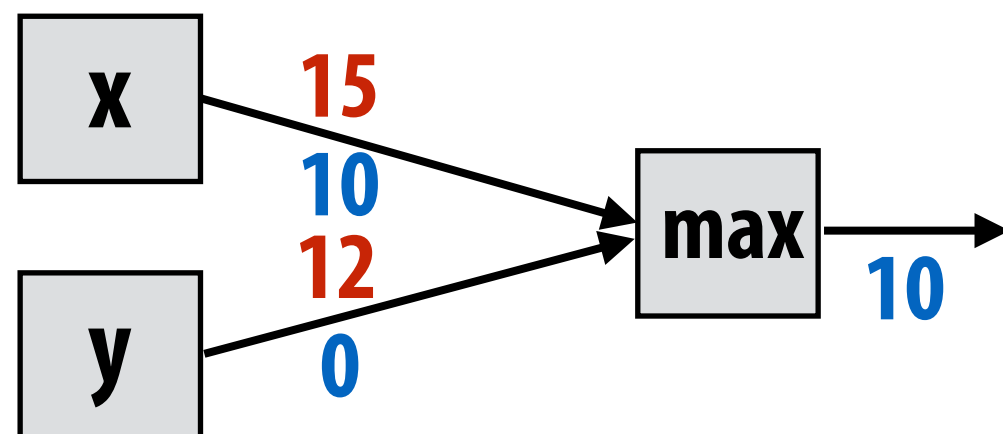
Blue =  $df/dnode$

Recall:  $\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx}$



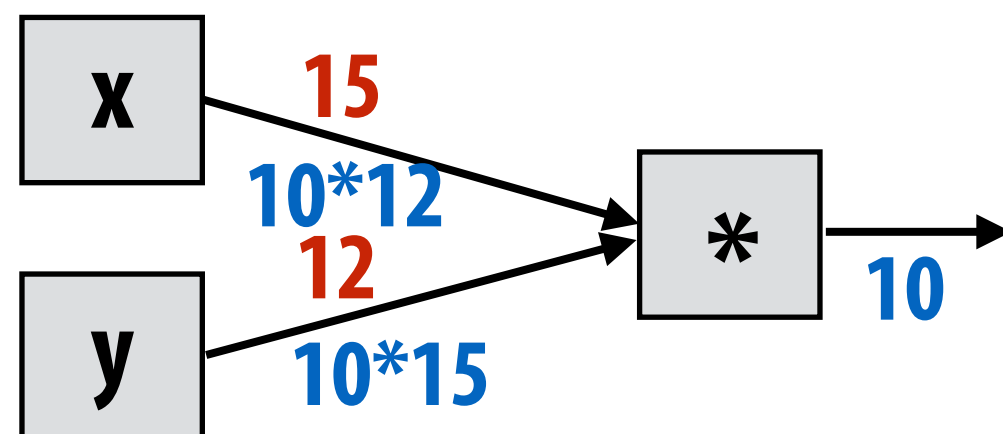
$$g(x, y) = x + y$$

$$\frac{dg}{dx} = 1, \frac{dg}{dy} = 1$$



$$g(x, y) = \max(x, y)$$

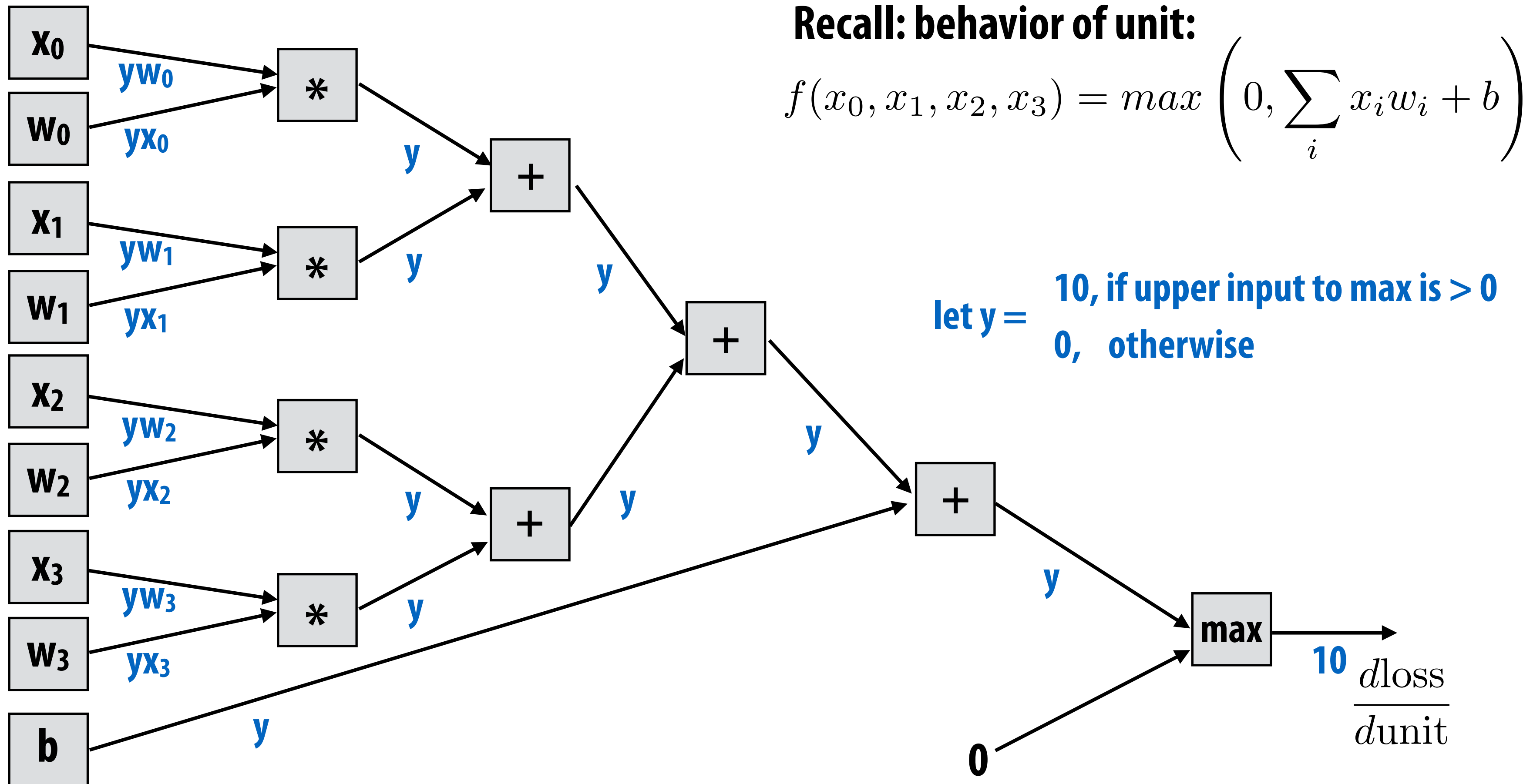
$$\frac{dg}{dx} = \begin{cases} 1, & \text{if } x > y \\ 0, & \text{otherwise} \end{cases}$$



$$g(x, y) = xy$$

$$\frac{dg}{dx} = y, \frac{dg}{dy} = x$$

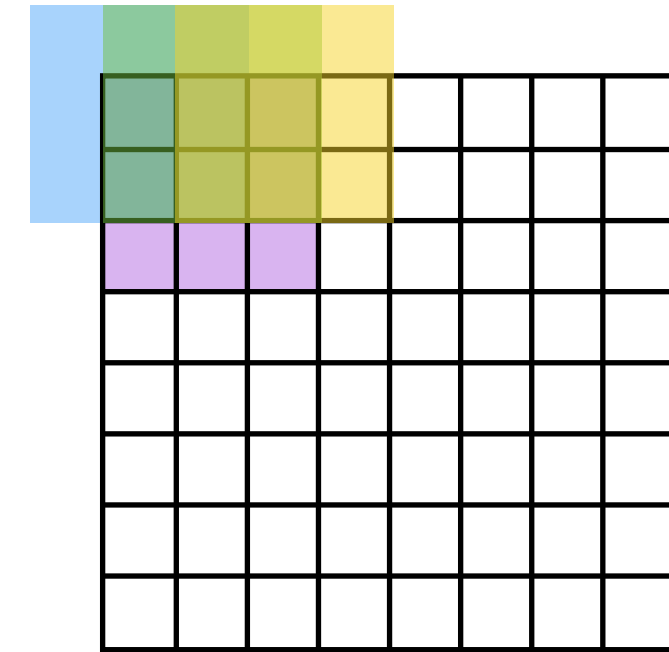
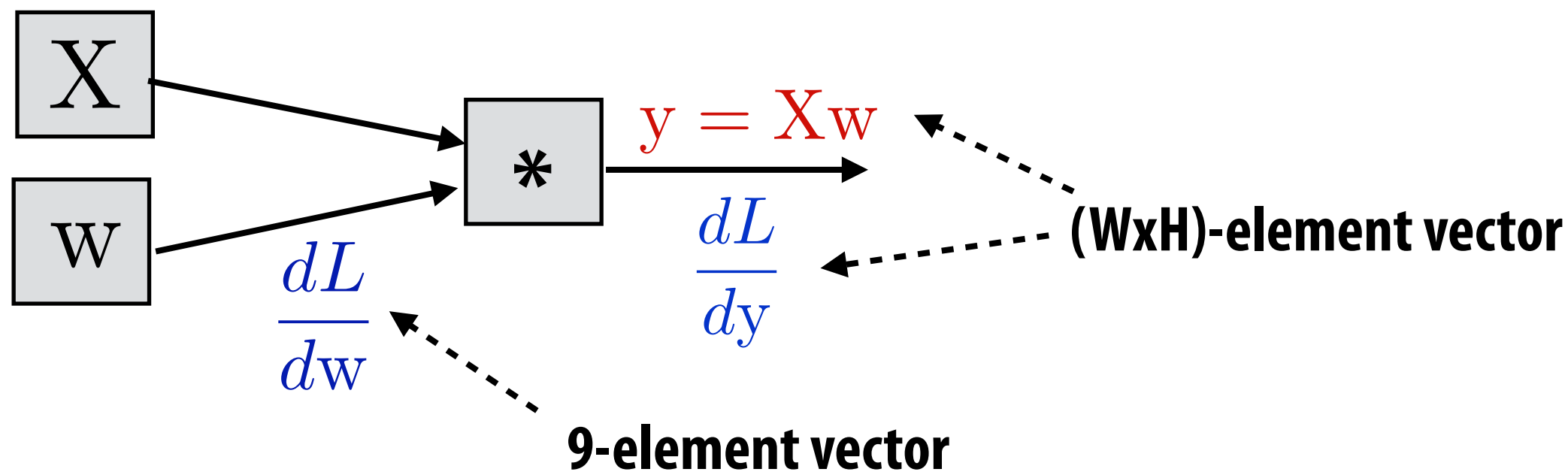
# Backpropagating through single unit



**Observe: output of prior layer ( $x_i$ 's) and output of this unit must be retained in order to compute weight gradients for this unit during backprop.**



# Backpropagation: matrix form



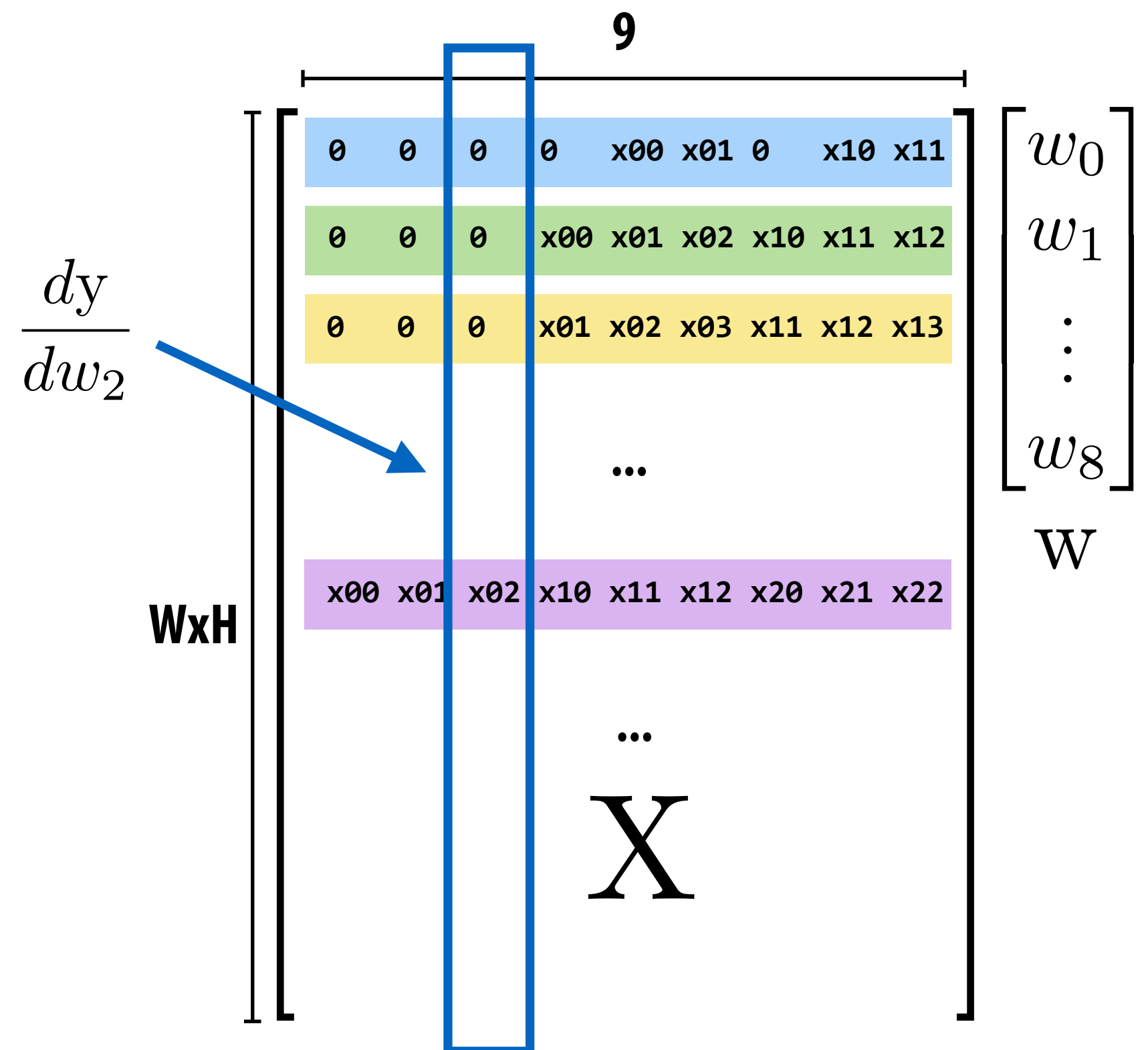
$$\frac{dy_j}{dw_i} = X_{ji}$$

$$\frac{dL}{dw_i} = \sum_j \frac{dL}{dy_j} \frac{dy_j}{dw_i}$$

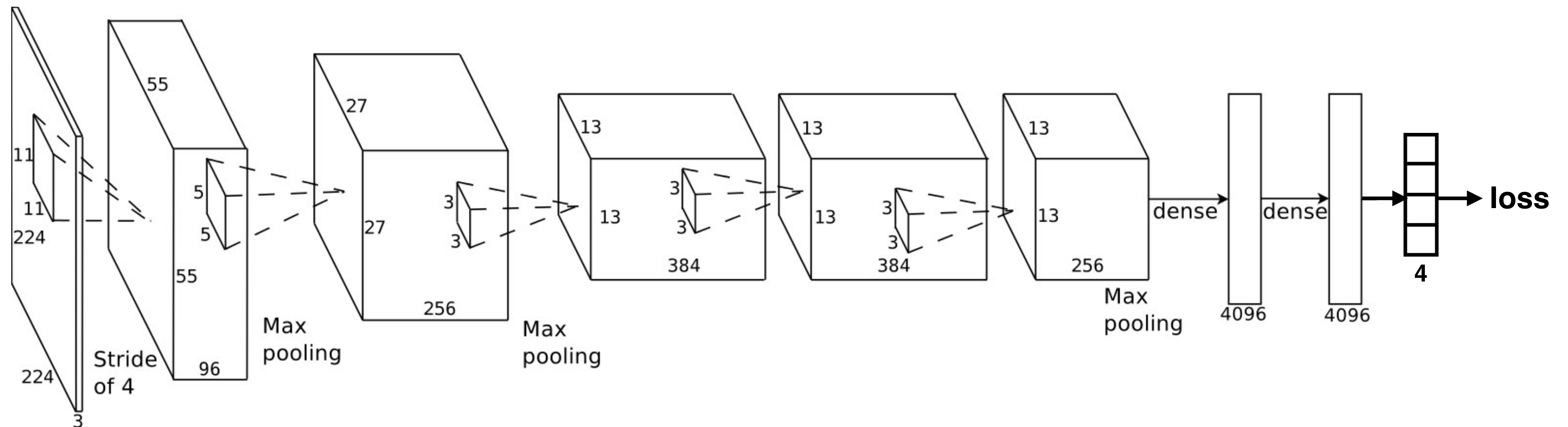
$$= \sum_j \frac{dL}{dy_j} X_{ji}$$

Therefore:

$$\frac{dL}{dw} = X^T \frac{dL}{dy}$$



# Back-propagation through the entire professor classification network



For each training example  $x_i$  in mini-batch:

**Perform forward evaluation to compute loss for  $x_i$**

**Note: must retain all layer outputs + output gradients (needed to compute weight gradients during backpropagation)**

**Compute gradient of loss w.r.t. final layer's outputs**

**Backpropagate gradient to compute gradient of loss w.r.t. all network parameters**

**Accumulate gradients (over all images in batch)**

**Update all parameter values:  $w_{i\_new} = w_{i\_old} - step\_size * gradi$**

# VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

inputs/outputs get multiplied by mini-batch size

Unlike forward evaluation:  
 1. must store outputs and gradient of outputs  
 2. cannot immediately free outputs once consumed by next level of network

	weights mem:	output size (per image)	(mem)
input: 224 x 224 RGB image	—	224x224x3	150K
conv: (3x3x3) x 64	6.5 KB	224x224x64	12.3 MB
conv: (3x3x64) x 64	144 KB	224x224x64	12.3 MB
maxpool	—	112x112x64	3.1 MB
conv: (3x3x64) x 128	228 KB	112x112x128	6.2 MB
conv: (3x3x128) x 128	576 KB	112x112x128	6.2 MB
maxpool	—	56x56x128	1.5 MB
conv: (3x3x128) x 256	1.1 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
maxpool	—	28x28x256	766 KB
conv: (3x3x256) x 512	4.5 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
maxpool	—	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
maxpool	—	7x7x512	98 KB
fully-connected 4096	<b>392 MB</b>	4096	16 KB
fully-connected 4096	<b>64 MB</b>	4096	16 KB
fully-connected 1000	15.6 MB	1000	4 KB
soft-max		1000	4 KB

Must also store per-weight gradients

Many implementations also store gradient "momentum" as well (multiply by 3)

# SGD workload

`while (loss too high):` ← **At first glance, this loop is sequential (each step of “walking downhill” depends on previous)**

`for each item  $x_i$  in mini-batch:` ← **Parallel across images**  
`grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )`

↑  
**sum reduction**

← **large computation with its own parallelism  
(but working set may not fit on single machine)**

`params += -grad * step_size;`

← **trivial data-parallel over parameters**



# Deep network training workload

## ■ Huge computational expense

- Must evaluate the network (forward and backward) for millions of training images
- Must iterate for many iterations of gradient descent (100's of thousands)
- Training modern networks takes days

## ■ Large memory footprint

- Must maintain network layer outputs from forward pass
- Additional memory to store gradients for each parameter
- Recall parameters for popular VGG-16 network require ~500 MB of memory (training requires GBs of memory for academic networks)
- Scaling to larger networks requires partitioning network across nodes to keep network + intermediates in memory

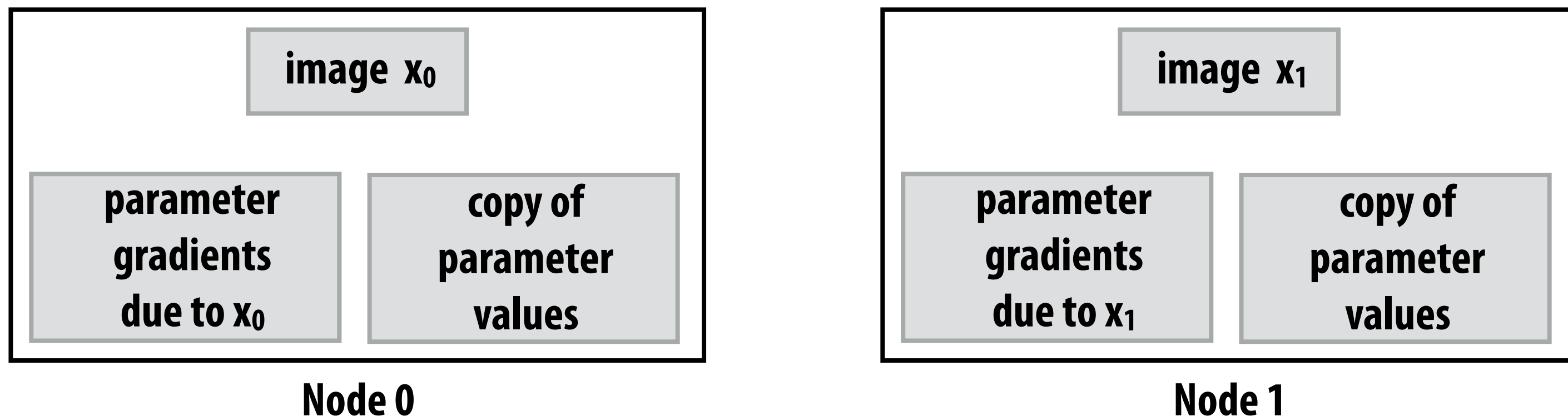
## ■ Dependencies /synchronization (not embarrassingly parallel)

- Each parameter update step depends on previous
- Many units contribute to same parameter gradients (fine-scale reduction)
- Different images in mini batch contribute to same parameter gradients

# Data-parallel training (across images)

```
for each item  $x_i$  in mini-batch:  
    grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )  
params += -grad * step_size;
```

Consider parallelization of the outer for loop across machines in a cluster



```
partition mini-batch across nodes  
for each item  $x_i$  in mini-batch assigned to local node:  
    // just like single node training  
    grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )  
barrier();  
sum reduce gradients, communicate results to all nodes  
barrier();  
update copy of parameter values
```

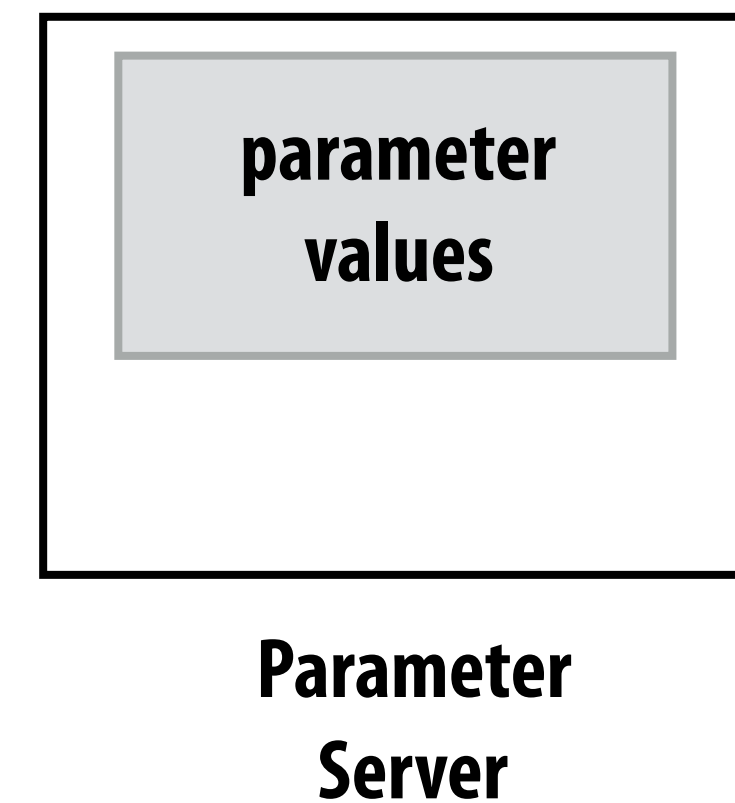
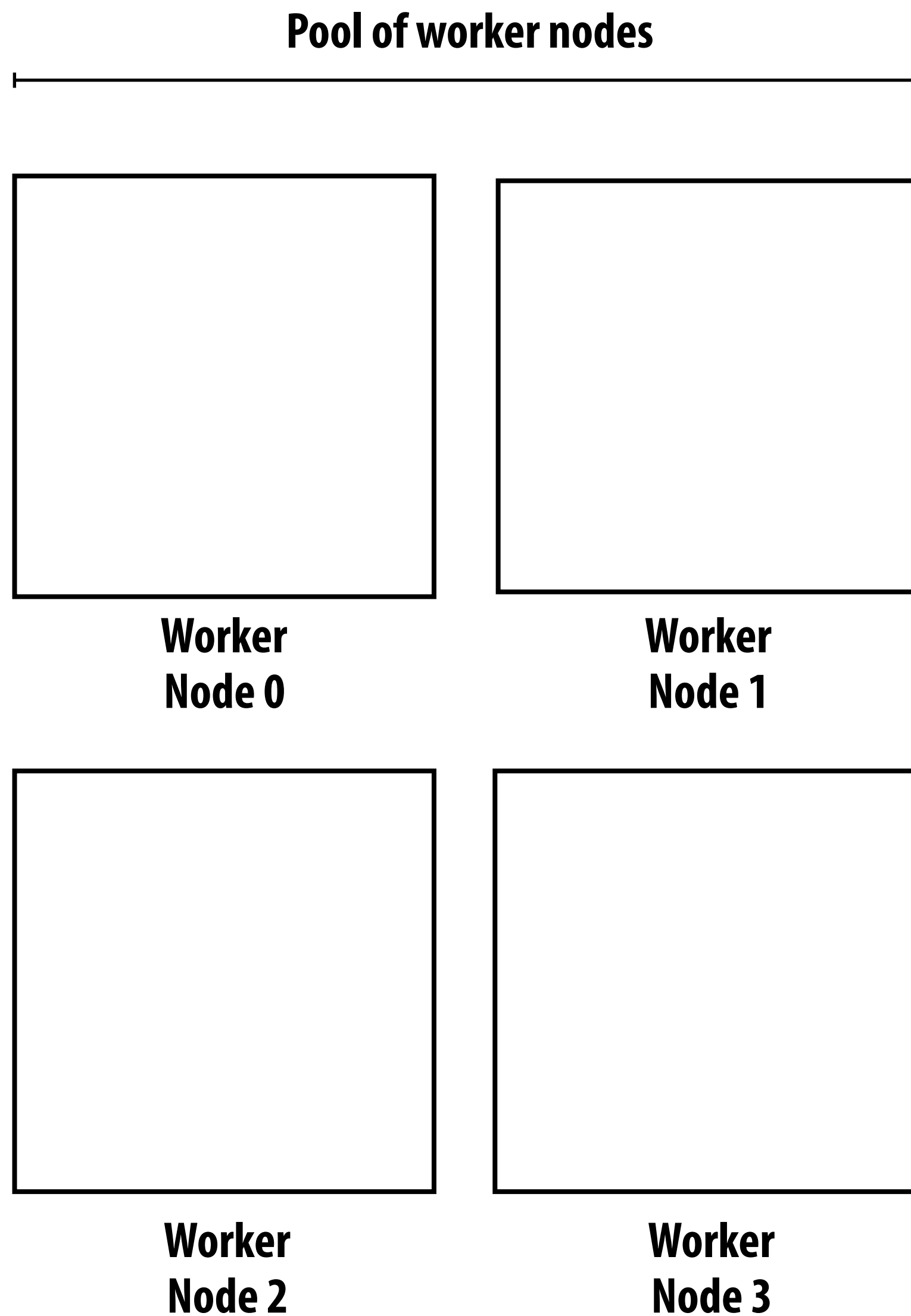
# Challenges of computing at cluster scale

- **Slow communication between nodes**
  - **Clusters do not feature high-performance interconnects typical of supercomputers**
- **Nodes with different performance (even if machines are the same)**
  - **Workload imbalance at barriers (sync points between nodes)**

**Modern solution: exploit characteristics of SGD using asynchronous execution!**

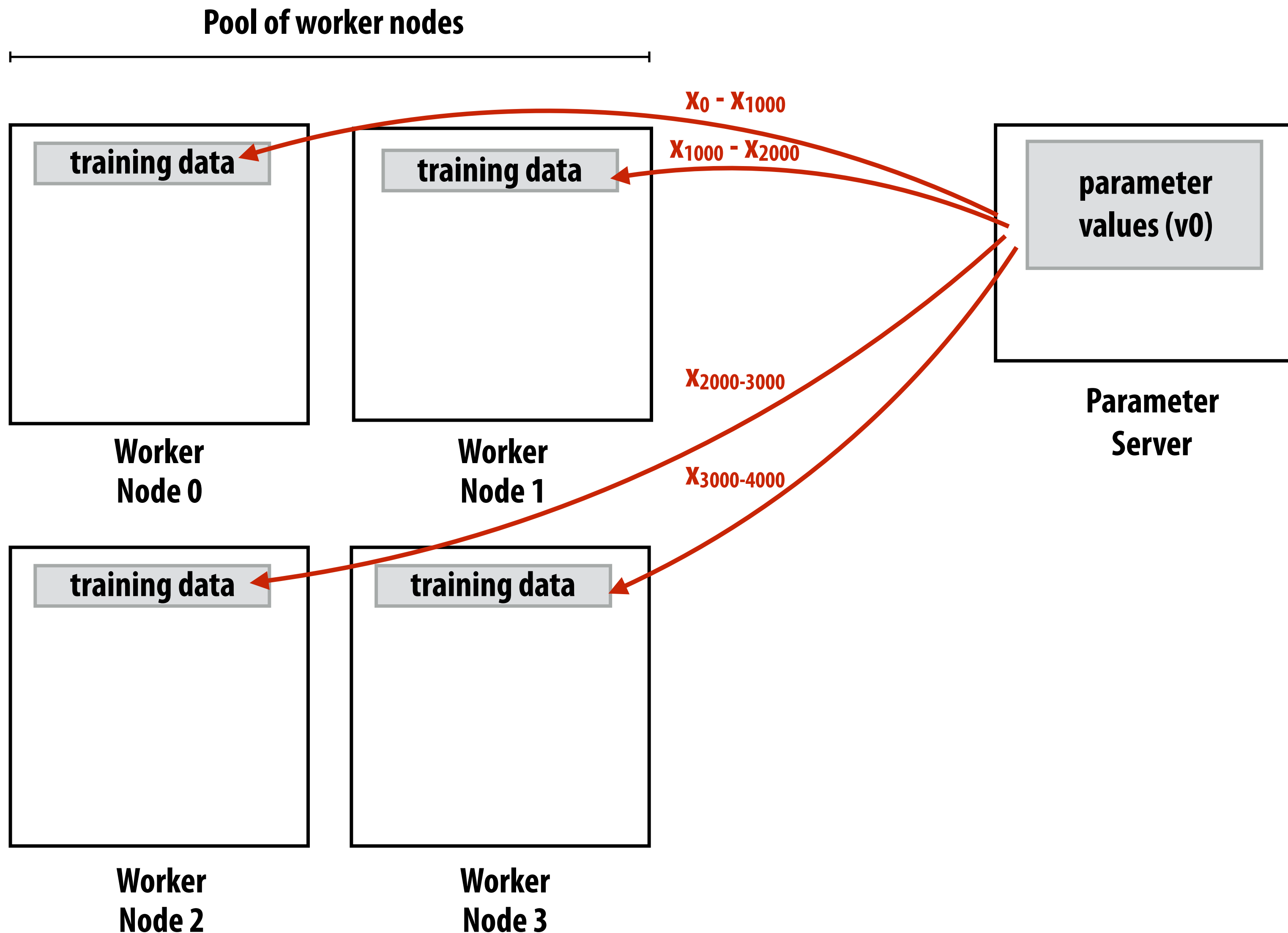
# Parameter server design

Parameter Server [Li OSDI14]  
Google's DistBelief [Dean NIPS12]  
Microsoft's Project Adam [Chilimbi OSDI14]

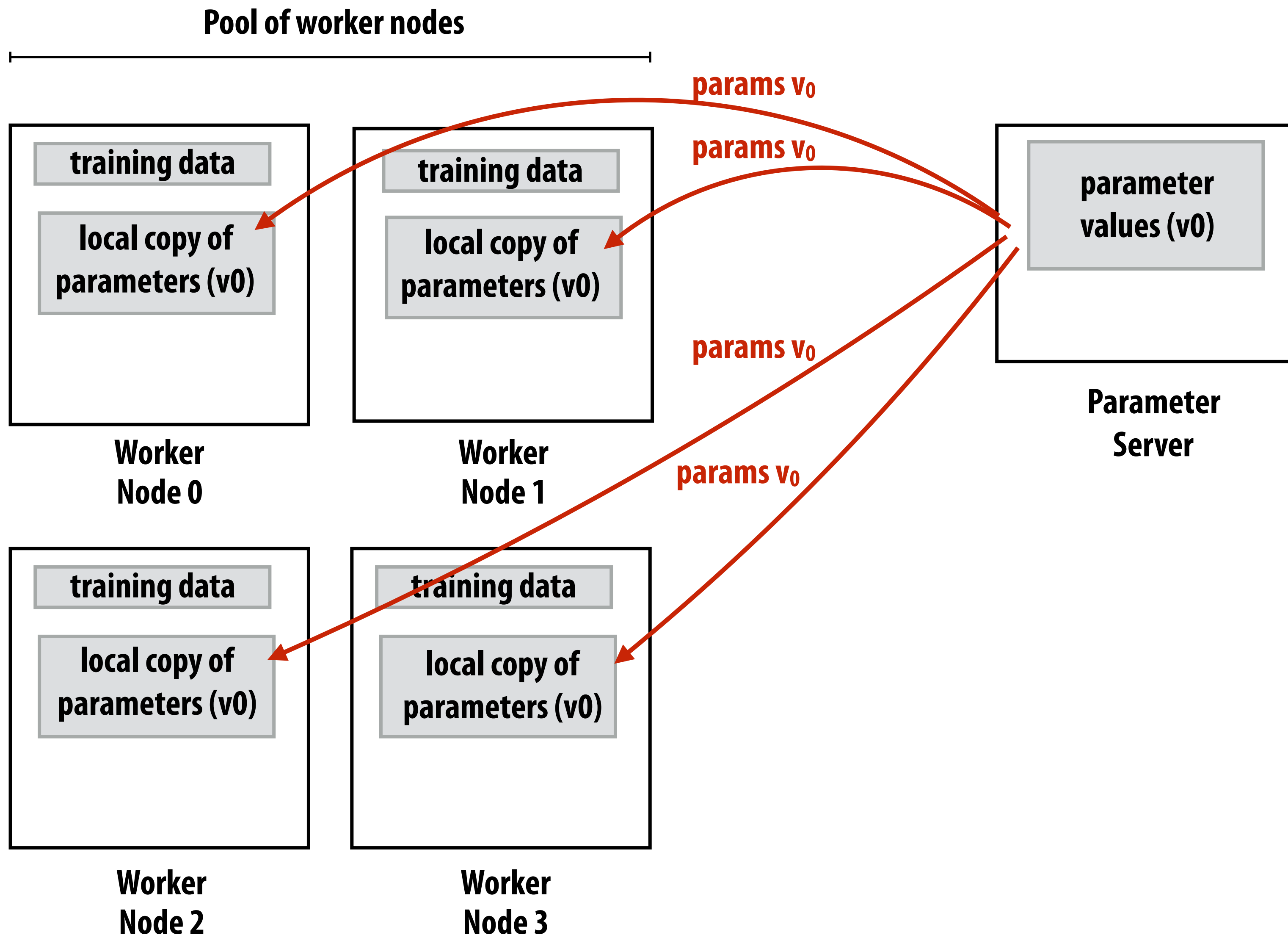




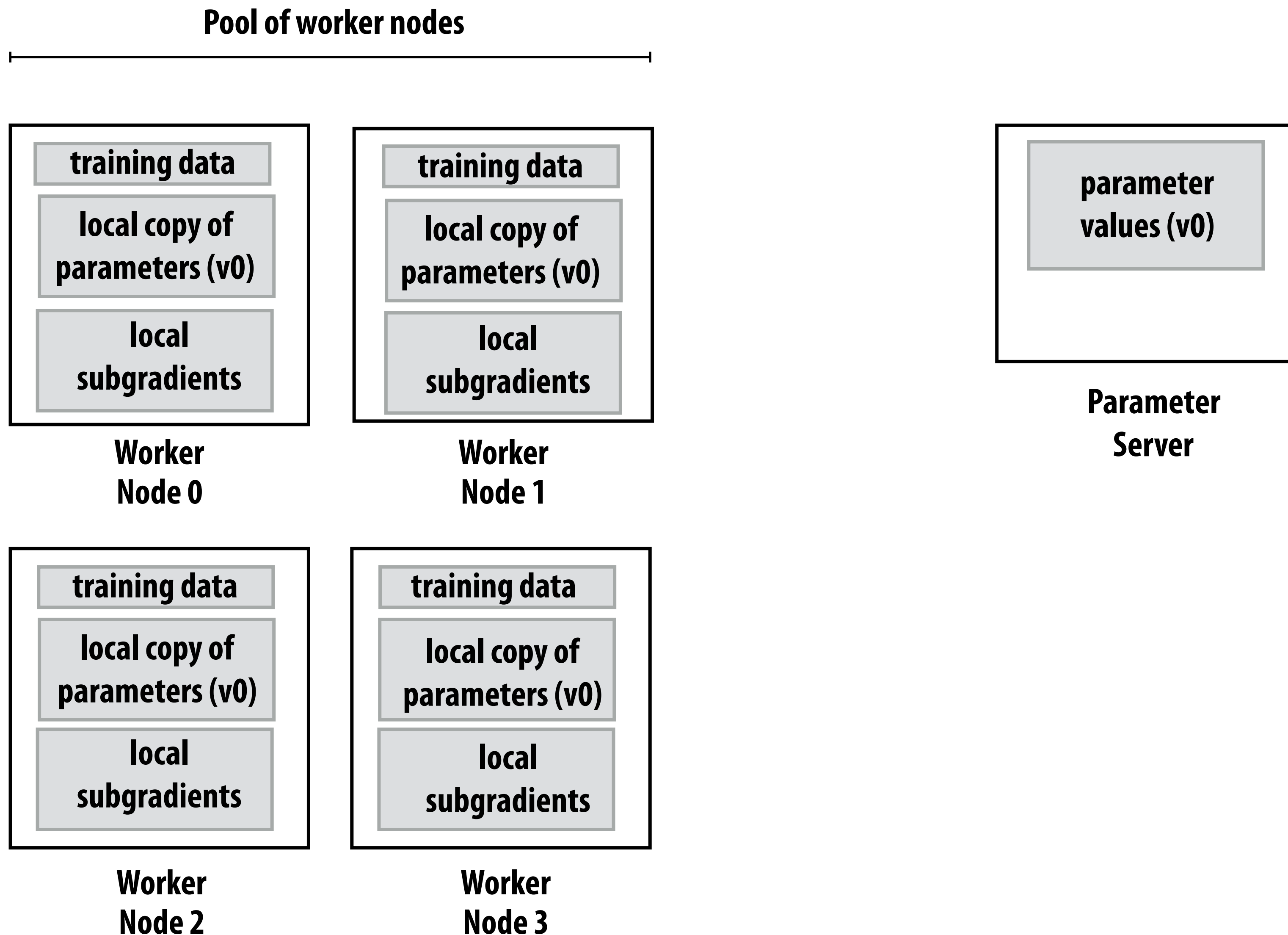
# Training data partitioned among workers



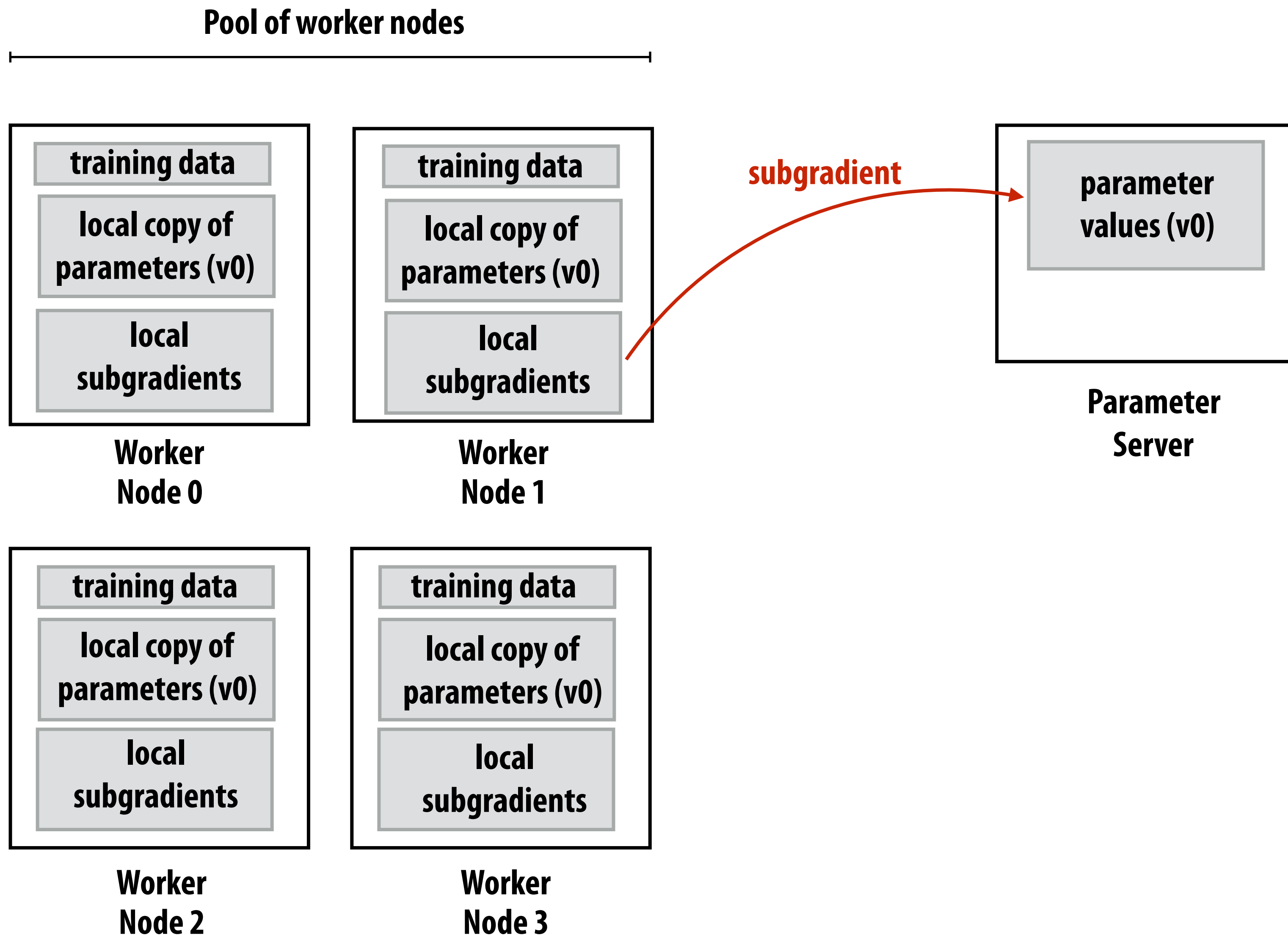
# Copy of parameters sent to workers



# Workers independently compute local "subgradients"

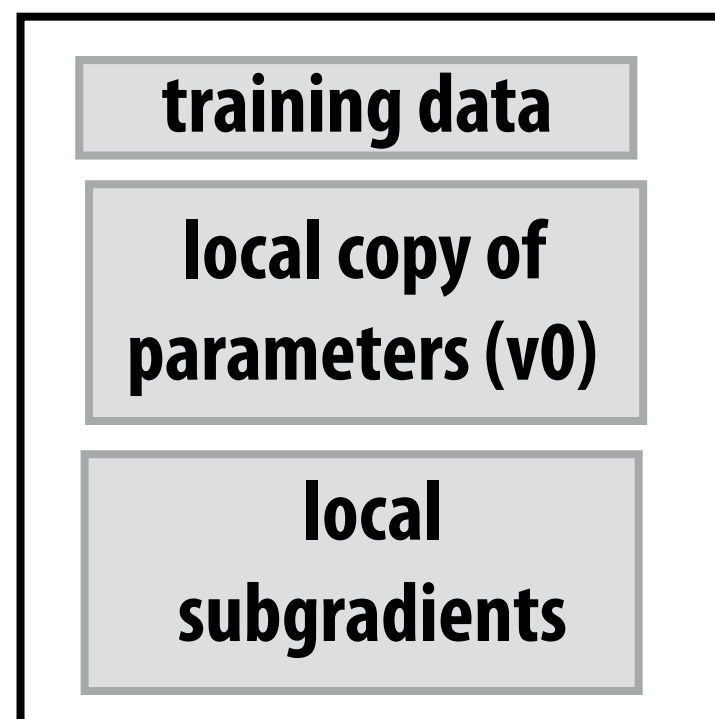


# Worker sends subgradient to parameter server

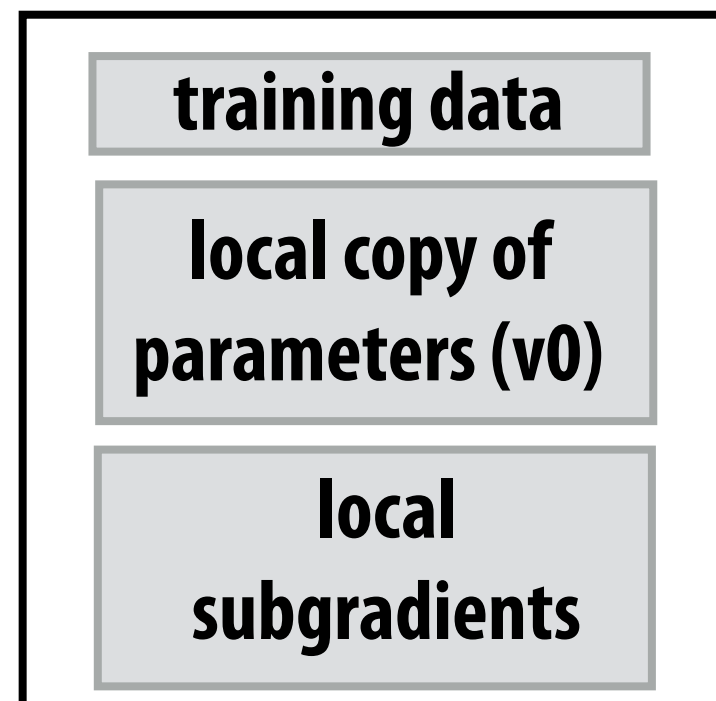




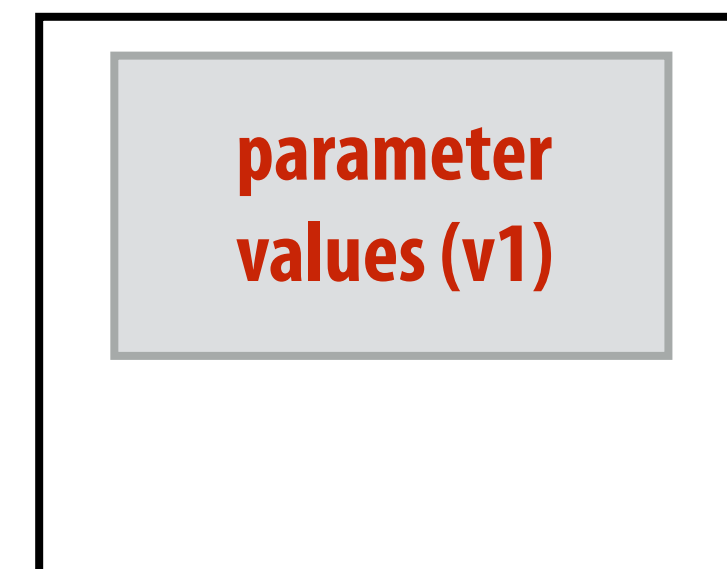
# Server updates global parameter values based on subgradient



Worker  
Node 0

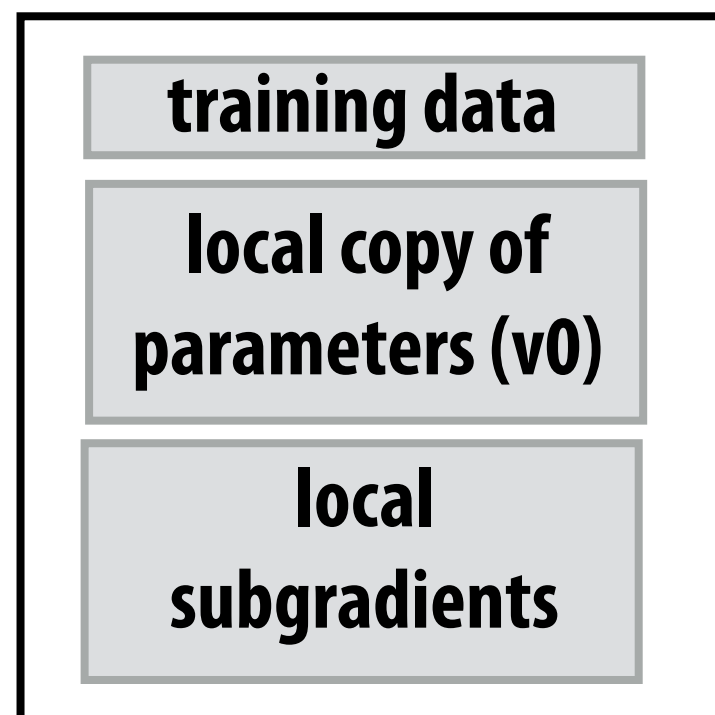


Worker  
Node 1

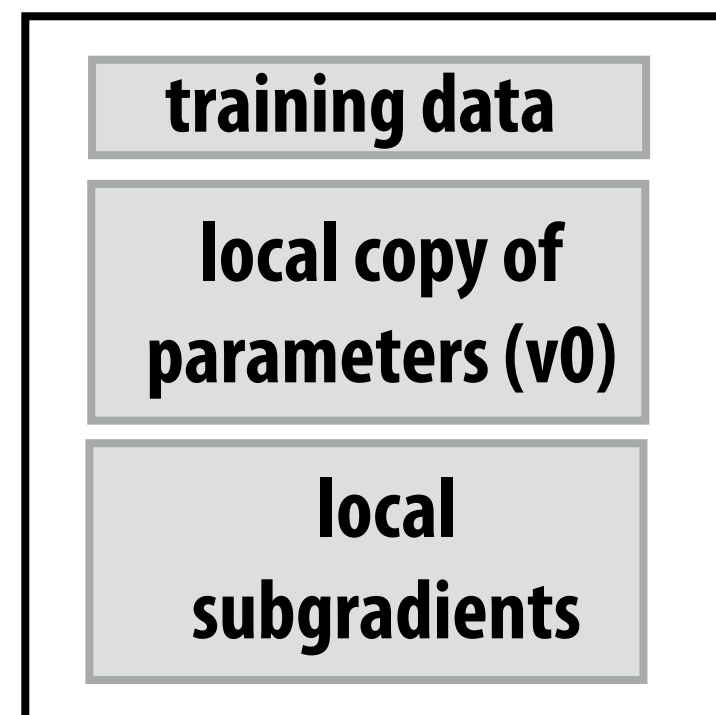


Parameter  
Server

```
params += -subgrad * step_size;
```



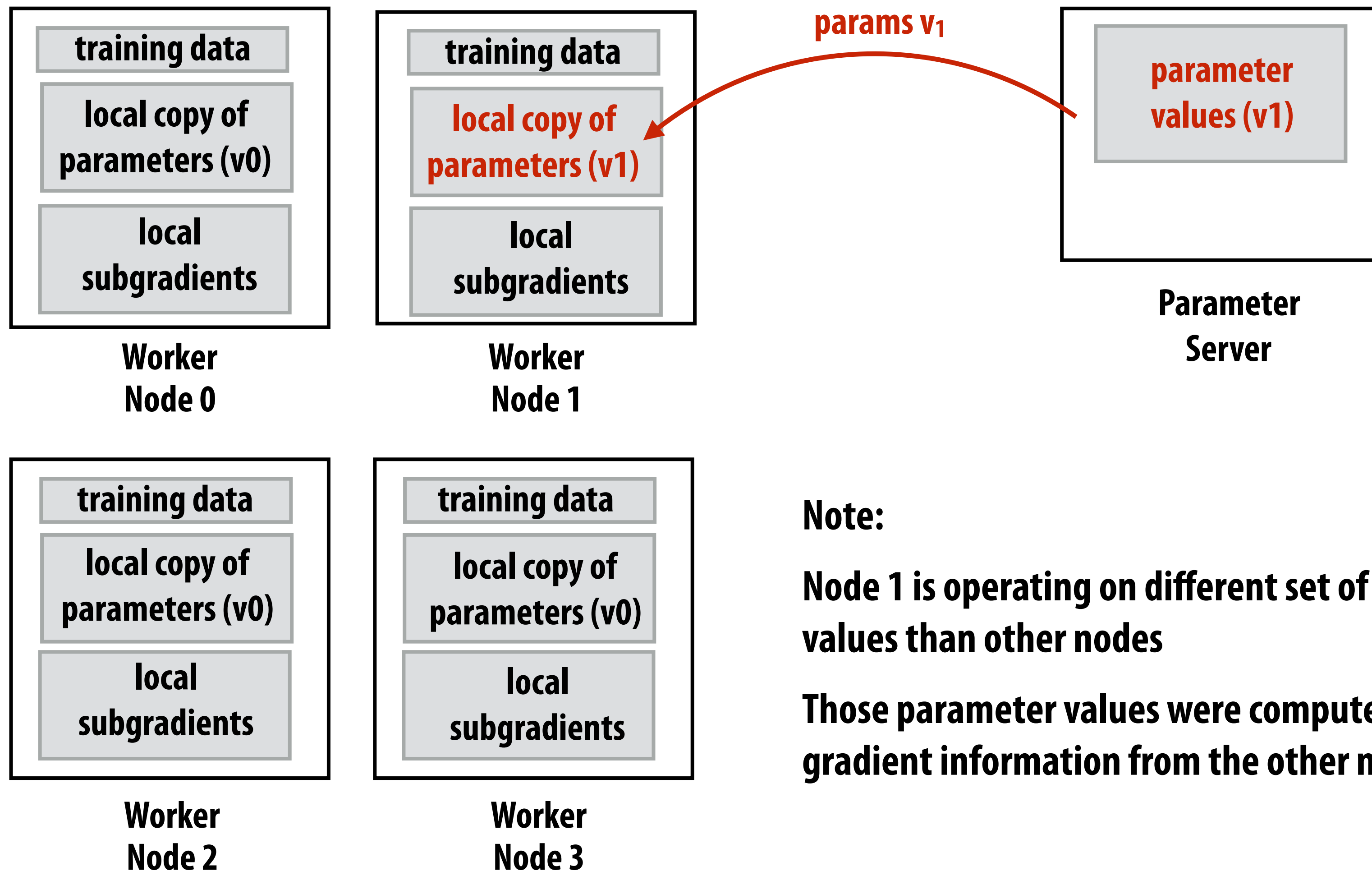
Worker  
Node 2



Worker  
Node 3

# Updated parameters sent to worker

Worker proceeds with another gradient computation step

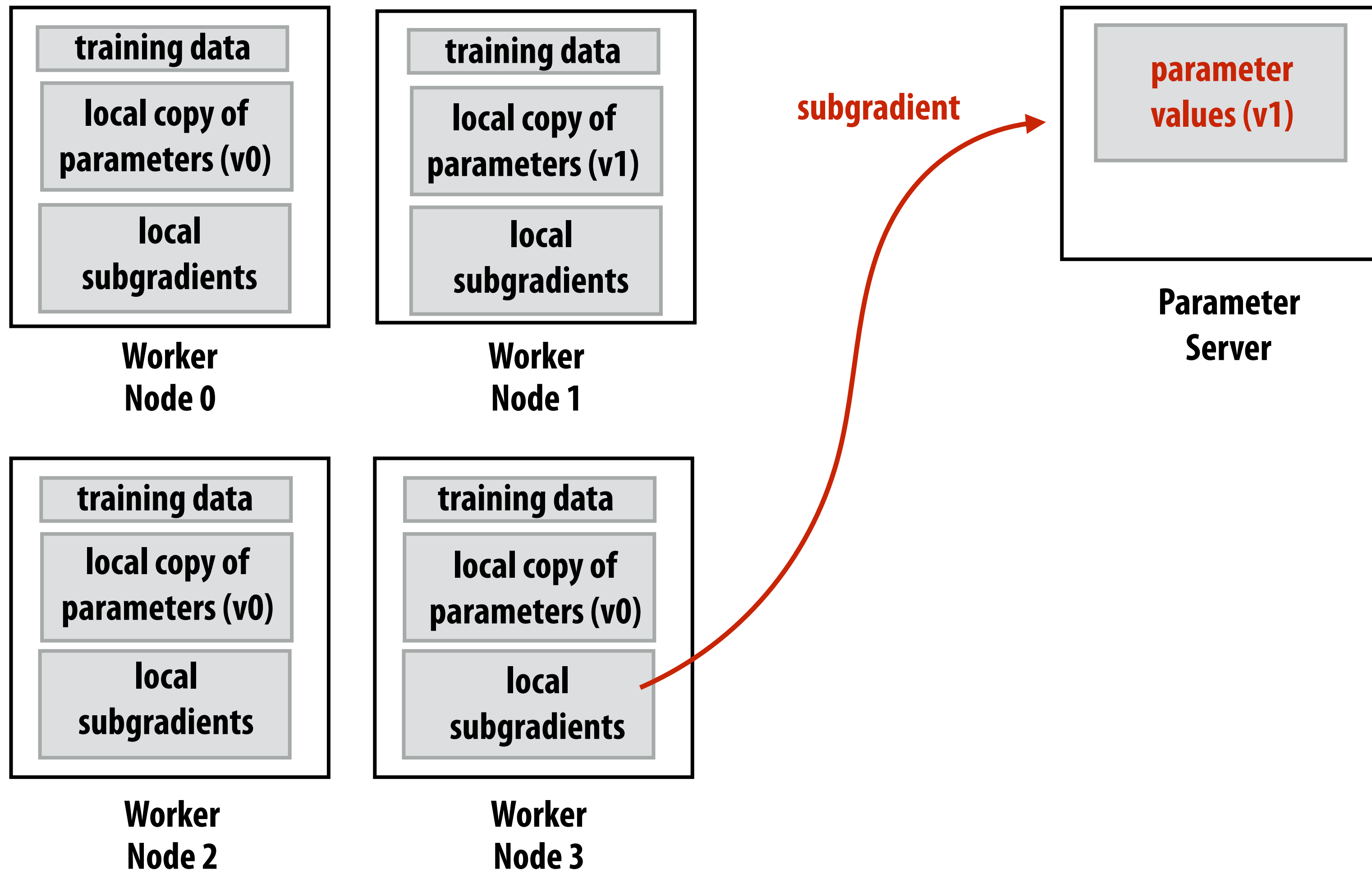


**Note:**

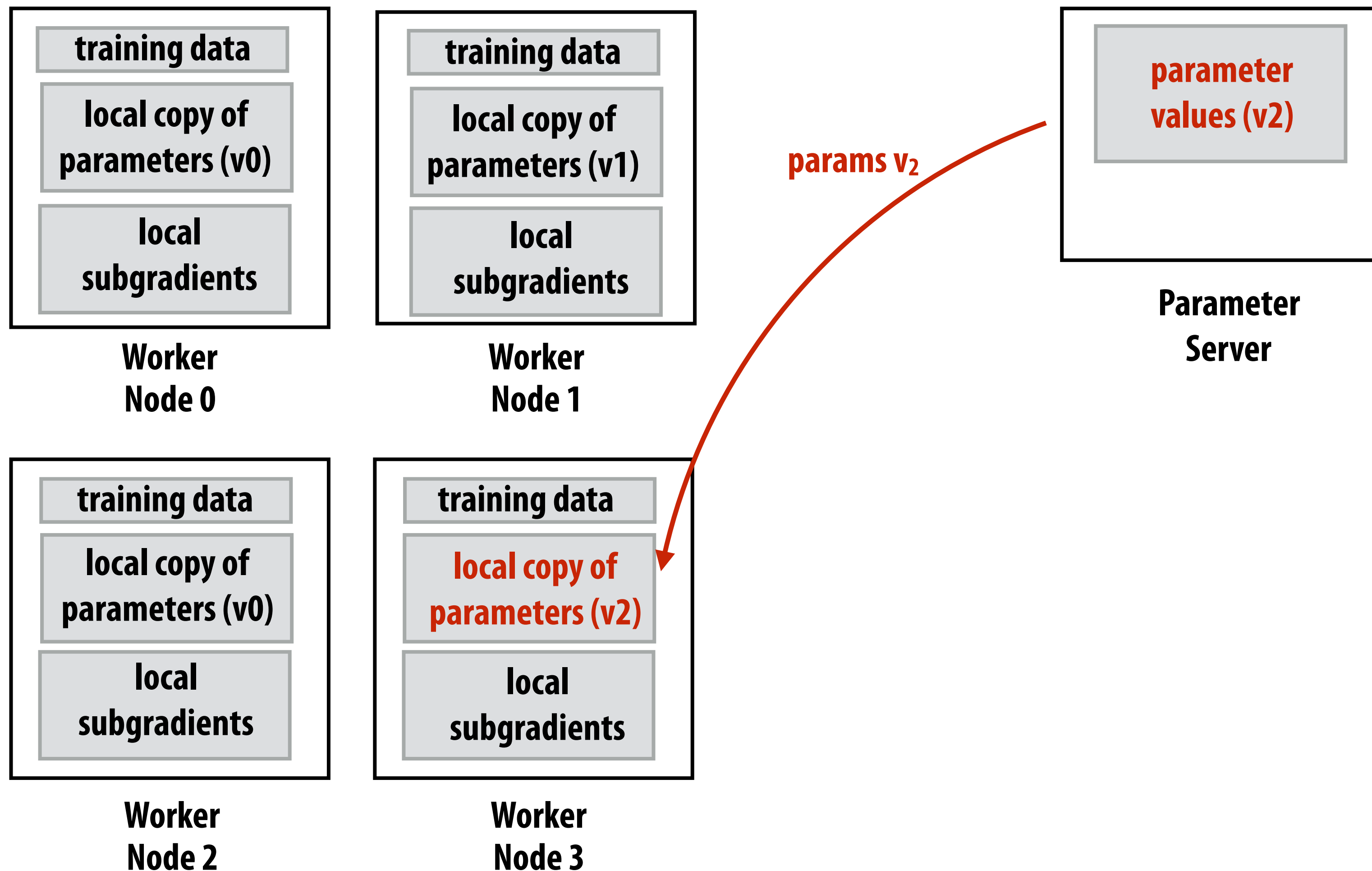
**Node 1 is operating on different set of parameter values than other nodes**

**Those parameter values were computed without gradient information from the other nodes**

# Updated parameters sent to worker (again)



# Worker continues with updated parameters



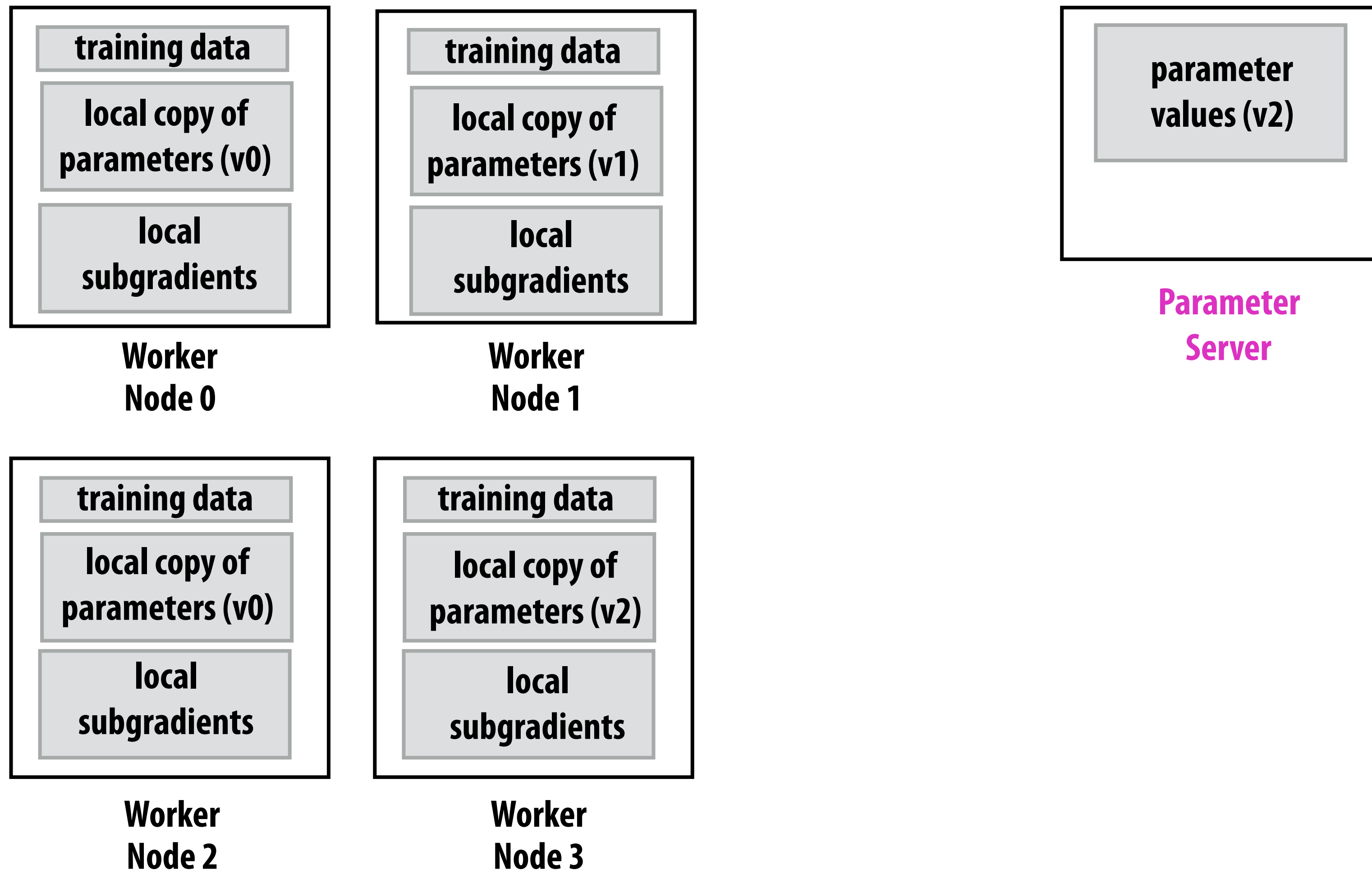


# Summary: asynchronous parameter update

- **Idea: avoid global synchronization on all parameter updates between each SGD iteration**
  - Design reflects realities of cluster computing:
    - Slow interconnects
    - Unpredictable machine performance
- **Solution: asynchronous (and partial) subgradient updates**
- **Will impact convergence of SGD**
  - Node  $N$  working on iteration  $i$  may not have parameter values that result the results of the  $i-1$  prior SGD iterations

# Bottleneck?

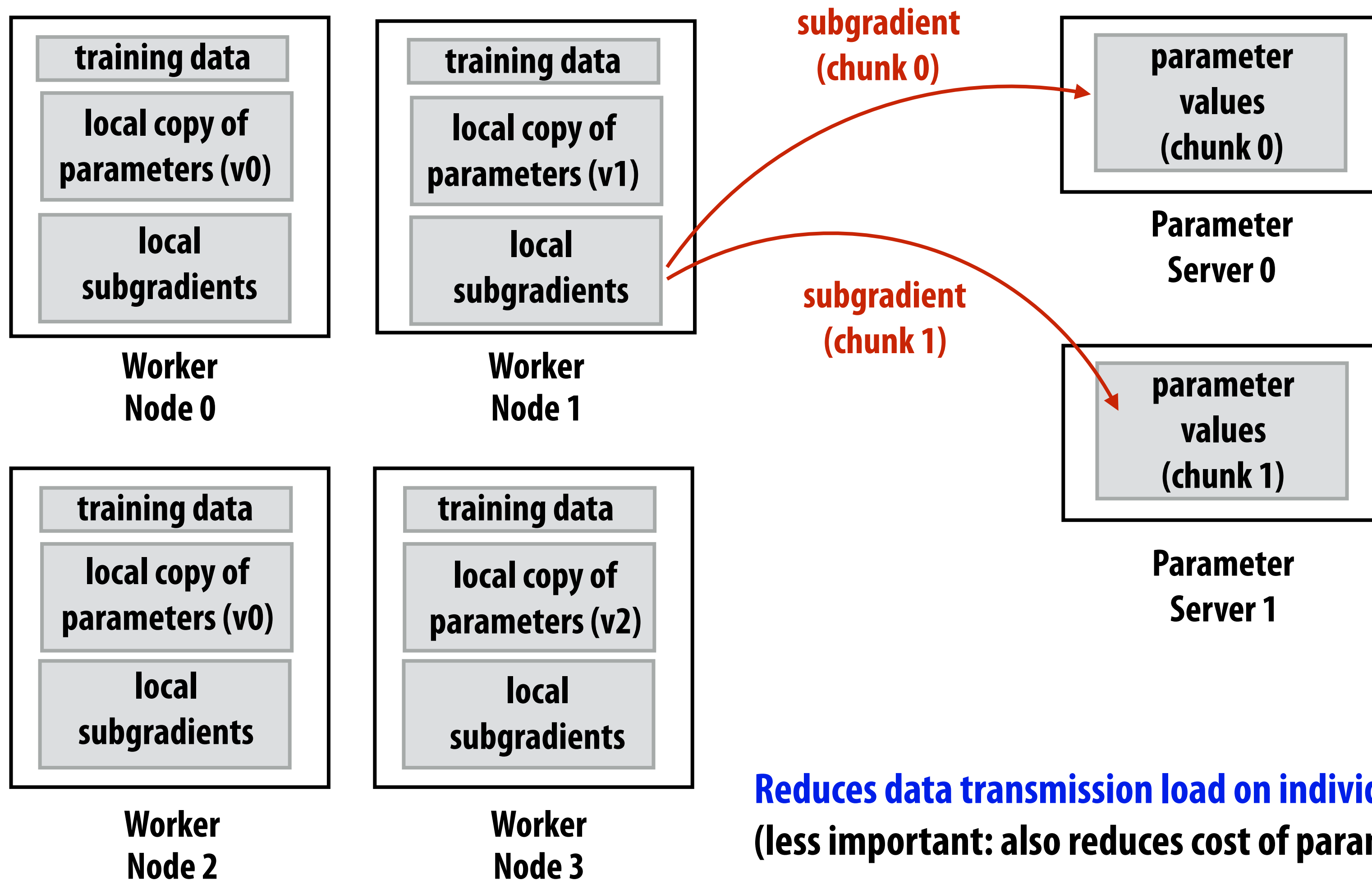
What if there is **heavy contention for parameter server?**



# Shard the parameter server

## Partition parameters across servers

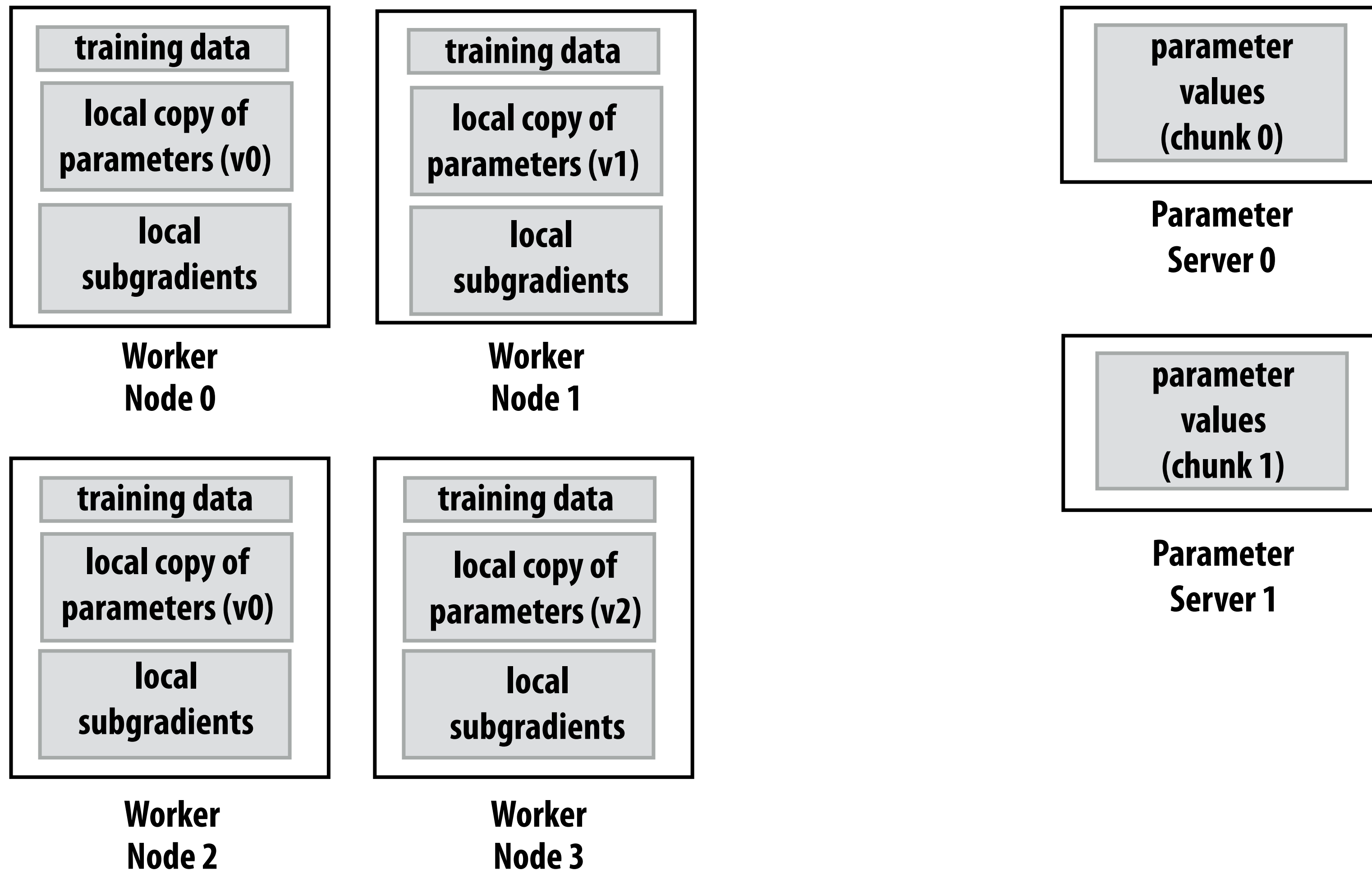
Worker sends chunk of subgradients to owning parameter server



**Reduces data transmission load on individual servers**  
(less important: also reduces cost of parameter update)

# What if model parameters do not fit on one worker?

Recall high footprint of training large networks  
(particularly with large mini-batch sizes)



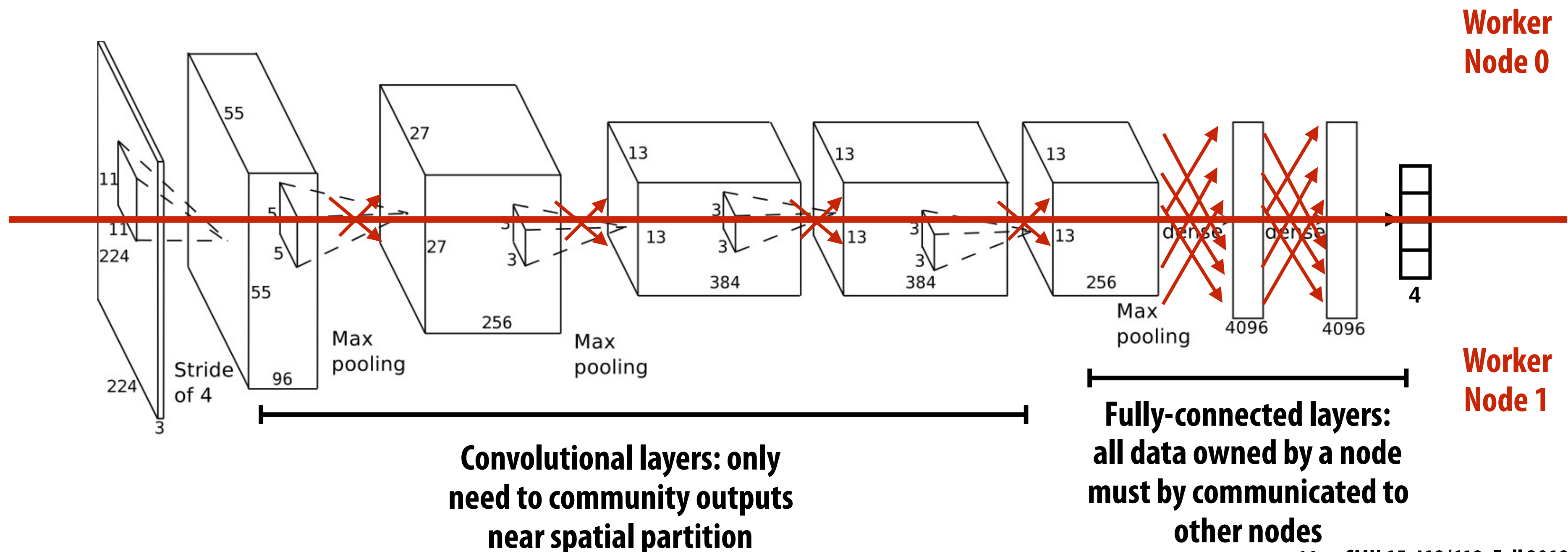


# Model parallelism

Partition network parameters across nodes  
(spatial partitioning to reduce communication)

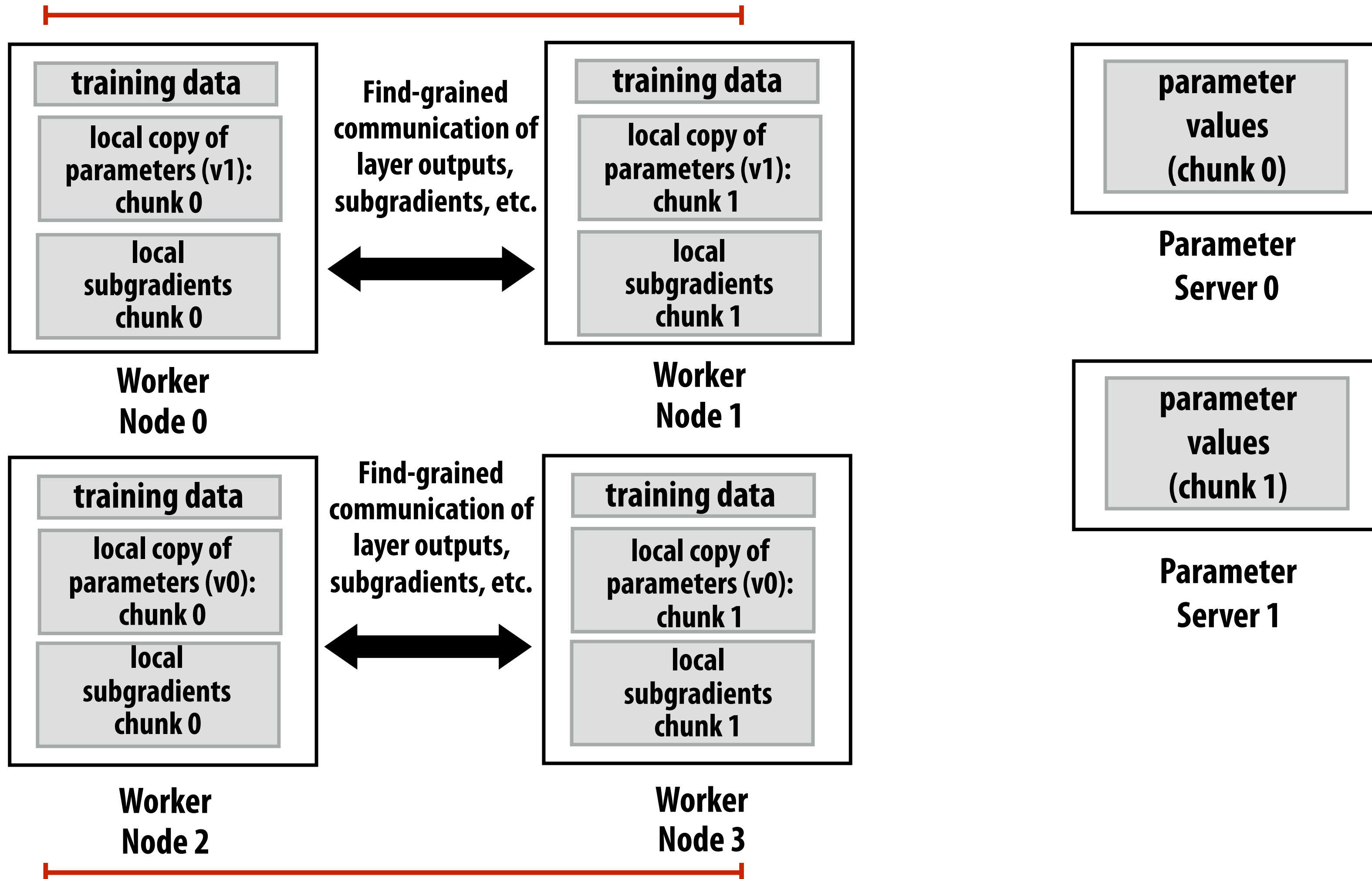
Reduce internode communication through network design:

- Use small spatial convolutions (1x1 convolutions)
- Reduce/shrink fully-connected layers



# Training data-parallel and model-parallel execution

Working on subgradient computation  
for a single copy of the model



Working on subgradient computation  
for a single copy of the model

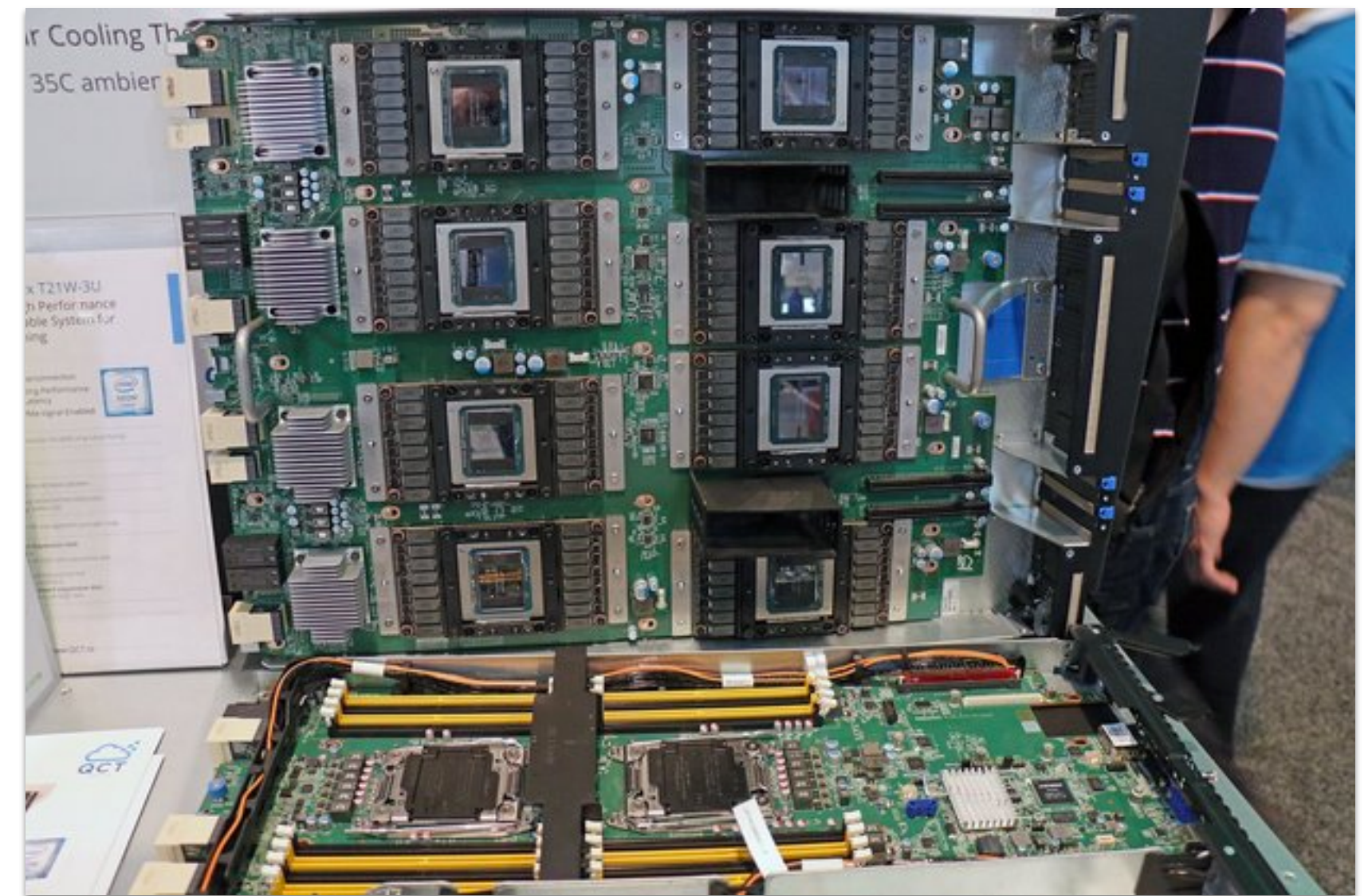


# Using supercomputers for training?

- **Fast interconnects critical for model-parallel training**
  - Fine-grained communication of outputs and gradients
- **Fast interconnect diminishes need for async training algorithms**
  - Avoid randomness in training due to computation schedule (there remains randomness due to SGD algorithm)



**OakRidge Titan Supercomputer**



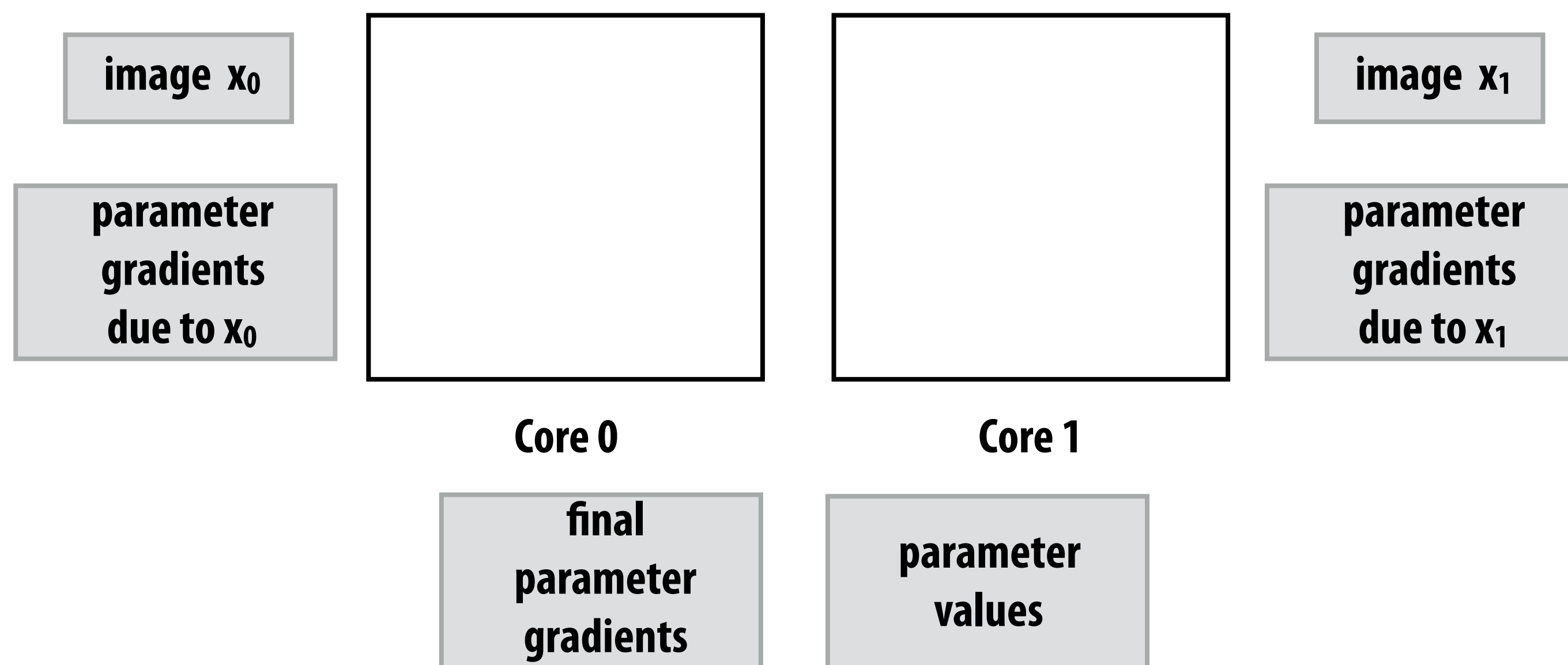
**NVIDIA DGX-1: 8 Pascal GPUs connected via high speed NV-Link interconnect**



# Parallelizing mini-batch on one machine

```
for each item  $x_i$  in mini-batch:  
    grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )  
params += -grad * step_size;
```

Consider parallelization of the outer for loop across cores



**Good: completely independent computations** (until gradient reduction)

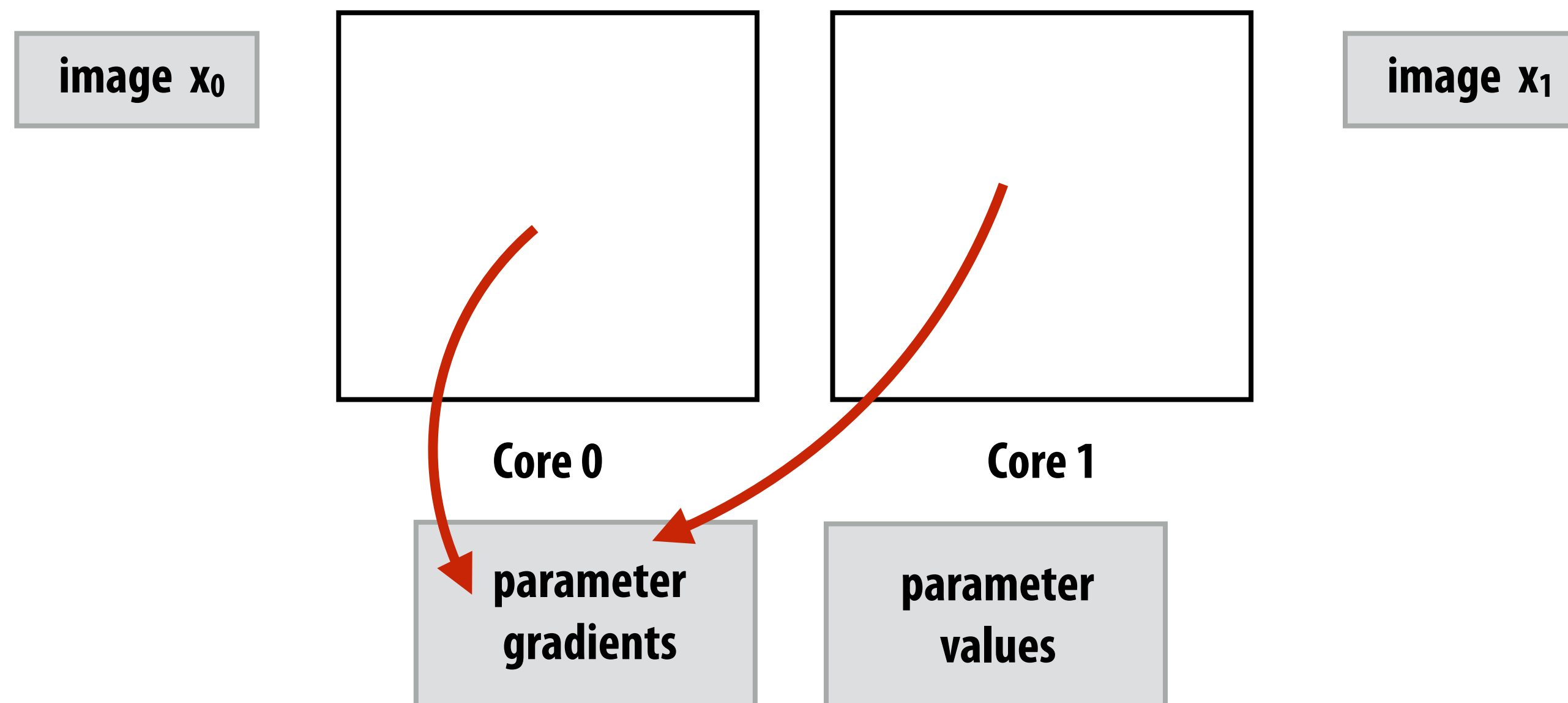
**Bad: complete duplication of parameter gradient state** (100's MB per core)

# Asynchronous update on one node

```
for each item  $x_i$  in mini-batch:  
    grad += evaluate_loss_gradient(f, loss_func, params,  $x_i$ )  
params += -grad * step_size;
```

**Cores update shared set of gradients.**

**Skip taking locks / synchronizing across cores: perform “approximate reduction”**





# Summary: training large networks in parallel

- **Most systems rely on asynchronous update to efficiently used clusters of commodity machines**
  - Modification of SGD algorithm to meet constraints of modern parallel systems
  - Open question: effects on convergence are problem dependent and not particularly well understood
  - Tighter integration / faster interconnects may provide alternative to these methods (facilitate tightly orchestrated solutions much like supercomputing applications)
- **Open question: how big of networks are needed?**
  - >90% of connections could be removed without significant impact on quality of network
  - High-performance training of deep networks is an interesting example of constant iteration of algorithm design and parallelization strategy (a key theme of this course! recall the original grid solver example!)