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15-381 Midterm Exam

Be sure to put your name and login on EVERY page. There are 13 pages.

There are a total of 170 points and 10 problems on this exam (note that the last problem is only worth 2 points).

This exam is OPEN BOOK. You may use Russell and Norvig and your notes.

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1 Simple Answer (20pts)

No explanation necessary.

(a) True or False? If all moves are unit cost, Breadth First Search always finds the optimal path to the goal.

(b) True or False? Iterative Deepening Depth First Search always finds the optimal path to the goal.

(c) True or False? In a graph with no loops, Depth First Search always finds the goal quicker than Breadth First Search.

(d) True or False? All admissible heuristics are equal. That is, A* will search states in the same order for all admissible heuristics.

(e) True or False? In some problems, Constraint Propagation can completely solve the problem without any search.

(f) True or False? Alpha-Beta Pruning will always visit less nodes than simple Minimax search.

(g) Multiple Choice. The meaning of $P \wedge \sim P$ is:

A. All possible interpretations.

B. The empty set of interpretations.

C. Depends on what other propositional symbols exist in this world.

D. Any interpretation that does not assign P a value.

(h) True or False? "False $\models$ True"

(i) True or False? "False $\models$ False"

(j) True or False? "$(\sim a) \vee b \models a \Rightarrow b$"

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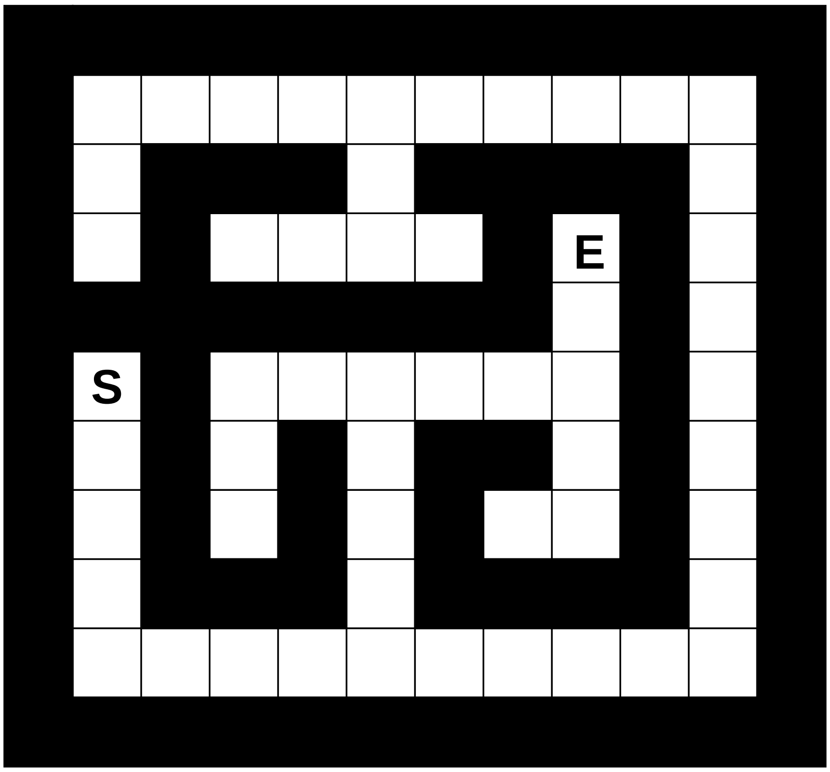
2 Basic Search (20pts)

Imagine a scenario with a robot trying to navigate in the following maze from the start position marked "S" to the end position marked "E". At each step the robot can move in one of the four compass directions.

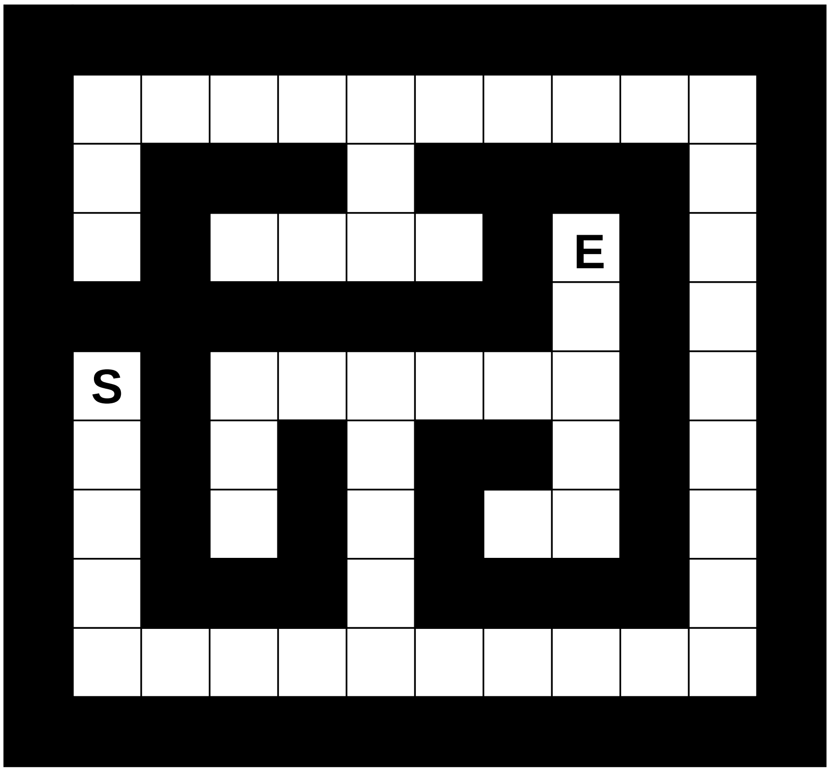
The robot contemplates alternatives in the following order:

1. Move South
2. Move East
3. Move North
4. Move West

(a) Mark the set of states that are expanded during the search, in the order they are expanded, by putting a 1 in the first state, a 2 in the second, and so forth. (Hint: put "1" in the cell marked "S"). Assume the search is Depth First Search (DFS). Use the version of DFS that avoids loops by never re-expanding a state that is on the current path.



(b) Using the same notation, mark the set of states expanded using Breadth First Search, in the order they are expanded.



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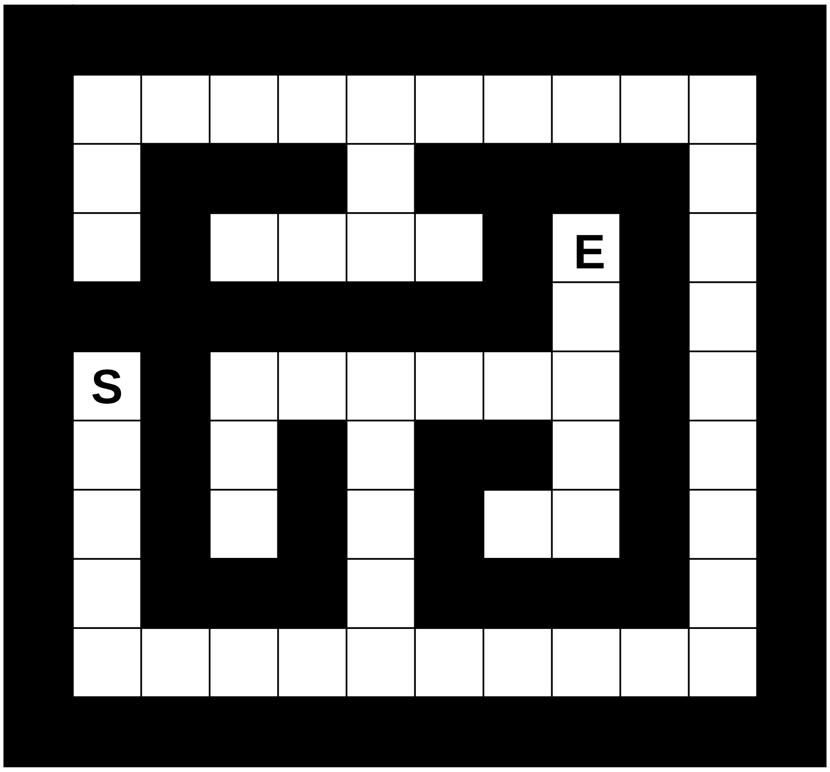
(c) (This part is harder—save it for the end: most of the rest of the exam is easier). Using the same notation, mark the set of states expanded using Best First Greedy Search, in the order they are expanded, assuming a heuristic

$$\begin{aligned} h(x) &= \text{(difference in horizontal position between } x \text{ and End)} + \\ &\quad 0.999 \times \text{(difference in vertical position between } x \text{ and End)} \end{aligned}$$

(Example: $h(\text{Start}) = 7 + 2 \times 0.999 = 8.998$). Assume that Best First Greedy Search never re-expands a state that has been previously expanded.



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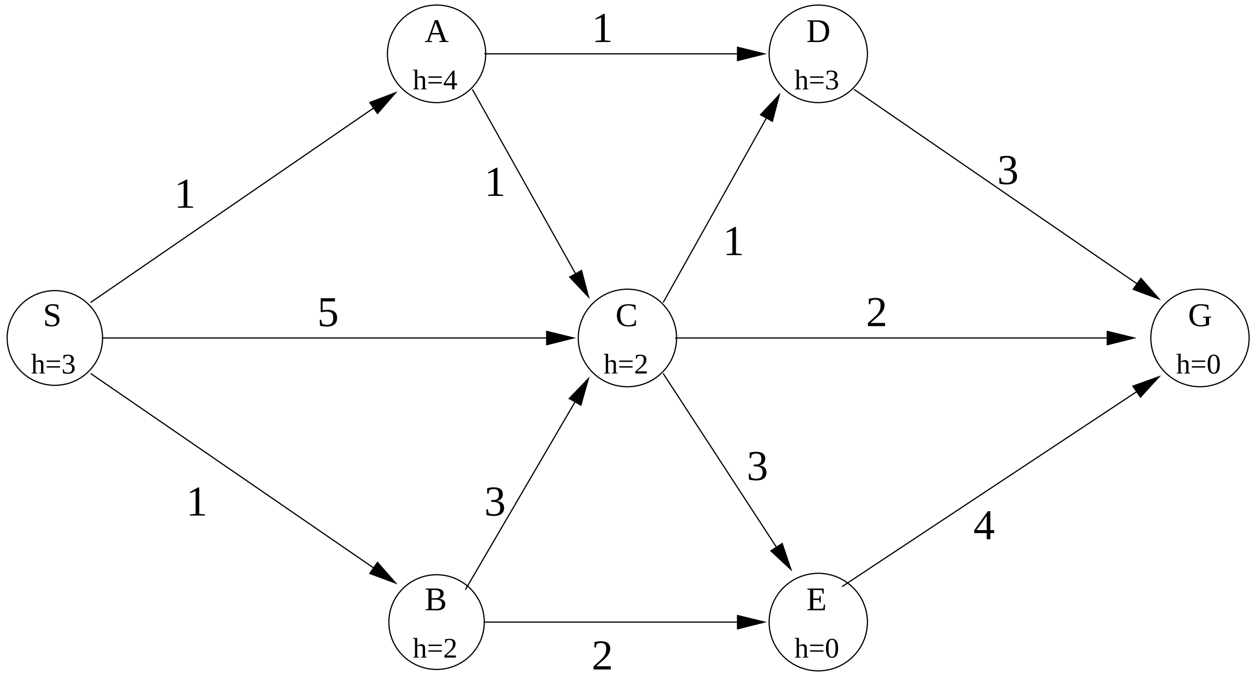
3 A* Search (20pts)

Suppose we ask A* search to compute the least-cost path from S to G in the following graph. The numbers on the arrows indicate transition costs and the "h=" numbers show the (admissible) heuristic values used for each state.



Fill in the following table with the contents of the priority queue after each iteration. After the iteration that A* decides to terminate, write "Terminated".

After Iteration	The priority Queue contains
Initialization	(S, 3)
1	(B, 3), (A, 5), (C, 7)
2	
3	
4	
5	
6	
7	
8	
9	

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4 Constraint Propagation (20pts)

Consider the "Minesweeper" game. Each square contains either a zero (touching no bombs); a number n (touching exactly n bombs); or nothing (unknown). As discussed in class, the problem of determining which squares contain bombs can be formulated as a constraint satisfaction problem, with a variable for each "unknown" square touching a numbered square and a constraint for each numbered square. The possible values for the unknown squares are B (bomb) and E (empty).

For each of the following configurations, state whether performing constraint propagation will uniquely discover the values of all unknown squares without requiring depth first search:

(a)

0	0	1	
0	0	2	
1	2	3	
B			

(b)

0	0	1	E
0	0	2	
1	2	3	

(c)

0	0	1	B
0	0	2	
1	2	3	
B			

(d) In the following configuration, assume you were to do depth-first search with forward checking but without constraint propagation starting from the square marked *. Which value would you choose under the least-constraining value heuristic?

1	1	1	1	1	1
		*			

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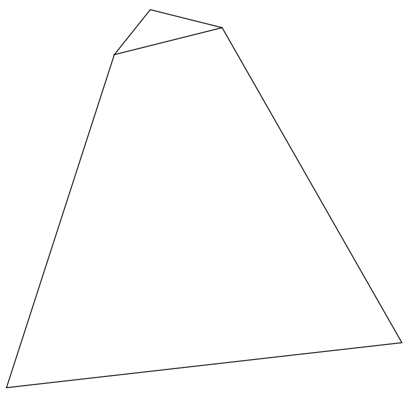
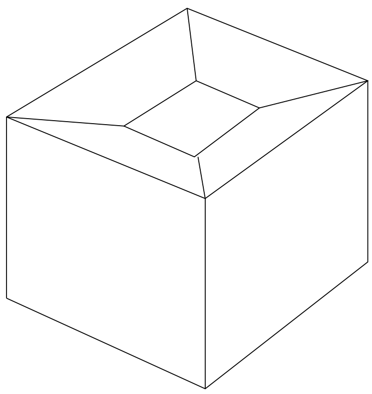
5 The Waltz Algorithm (15pts)

(a) Execute the Waltz algorithm on the following figure, producing a labeling for every edge.



(b) Is it valid to execute the Waltz algorithm on the following figure? Why or why not? If you claim that it is, label each edge according to it. If your claim is that it is not, indicate at least one source of difficulty on the figure.



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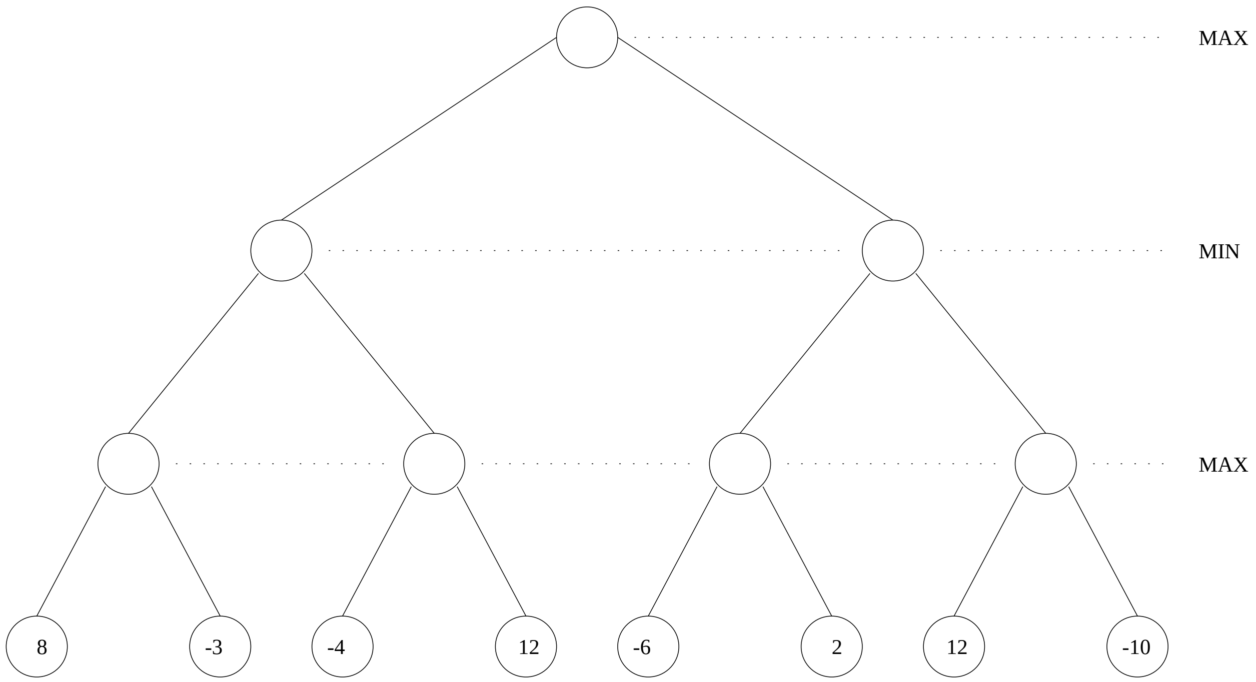
6 Minimax Search (15pts)

Perform the minimax search algorithm on the following tree (you may use alpha-beta pruning if you choose, but you aren't required to do so):

- Write in the game-theoretic value of the root node in the circle.
- Draw arrows along the path that would be taken if both players made optimal decisions.



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7 Game Theory (20pts)

After making it rich from your AI start-up company, you and your friend Sam decide to play the following game. You choose to hide either a \$100 bill or a \$10 bill, and your friend decides to hide either a \$50 bill or a \$20 bill. Then, both of you simultaneously reveal what both of you were hiding. If the sum of the money is evenly divisible by \$20, then you win the money Sam hid, but if the sum is not divisible by \$20, Sam wins the money you hid.

(a) Write the Matrix Normal Form for the game from your perspective.

(b) What is your optimal mixed strategy? Be sure to show your work.

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8 Propositional Logic (18pts)

(a) In a world with only three propositional symbols, P, Q and R, give the formal meaning of

$$P \vee (Q \Rightarrow R)$$

(b) Consider the following knowledge base, KB:

- StudyHard $\wedge$ DoHomework $\Rightarrow$ GoodGradeOnMidterm
- StudyHard $\vee$ DoHomework
- Lazy $\iff$ $\sim$ StudyHard
- DoHomework

Then answer the following:

A. True or False? KB $\models$ Lazy

B. True or False? KB $\models$ DoHomework

C. True or False? KB $\models$ GoodGradeOnMidterm

D. True or False? KB $\models$ StudyHard $\vee$ DoHomework

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9 Knowledge Engineering (20pts)

We are constructing a knowledge base in which we intend to give the following function symbols and predicates the following informal meanings.

- The predicate symbol $Dog(x)$ returns true iff x is a dog.
- The predicate symbol $Cat(x)$ returns true iff x is a cat.
- The predicate symbol $Owns(x, y)$ returns true iff x owns y.
- The predicate symbol $Loves(x, y)$ returns true iff x loves y.
- The predicate symbol $AnimalLover(x)$ returns true iff x is an animal lover.
- The function symbol $Owner(x)$ always returns the owner of x.

In the following problems, circle the single answer that expresses the given English statement in formal logic.

(a) Jack owns a dog.

A. $\exists x.Owns(Jack, x)$
B. $\exists x.(Owner(x) = Jack) \wedge Dog(x)$
C. $\forall x.Dog(x) \Rightarrow Owns(Jack, x)$
D. $Dog(x) \iff (Owner(x) = Jack)$

(b) Either Kelly owns a dog or Kelly is not an animal lover.

A. $(\forall x.Owns(Kelly, x) \wedge Dog(x)) \vee \sim AnimalLover(Kelly)$
B. $AnimalLover(Kelly) \Rightarrow (\forall x.Owns(Kelly, x) \wedge Dog(x))$
C. $AnimalLover(Kelly) \Rightarrow (\exists x.Owns(Kelly, x) \wedge Dog(x))$
D. $(\exists x.Owns(Kelly, x) \wedge Dog(x)) \Rightarrow AnimalLover(Kelly)$

(c) Every cat owner is an animal lover.

A. $\forall x.(\exists y.Cat(y) \Rightarrow Owns(x, y)) \wedge AnimalLover(x)$
B. $\forall x.(\exists y.Cat(y) \wedge Owns(x, y)) \Rightarrow AnimalLover(x)$
C. $\exists x.(\forall y.Cat(y) \wedge Owns(x, y)) \Rightarrow AnimalLover(x)$
D. $\forall x.(\exists y.Cat(y) \wedge Owns(x, y)) \iff AnimalLover(x)$

(d) Every animal lover loves all cats and dogs.

A. $\forall x.AnimalLover(x) \Rightarrow (\forall y.(Cat(y) \vee Dog(y)) \wedge Loves(x, y))$
B. $\forall x.AnimalLover(x) \Rightarrow (\exists y.(Cat(y) \vee Dog(y)) \Rightarrow Loves(x, y))$
C. $\forall x.AnimalLover(x) \Rightarrow (\forall y.Loves(x, y) \Rightarrow (Cat(y) \vee Dog(y)))$
D. $\forall x.AnimalLover(x) \Rightarrow (\forall y.(Cat(y) \vee Dog(y)) \Rightarrow Loves(x, y))$

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10 Functionally Complete Operators (2pts)

This problem is only worth 2 points; so, we recommend you only work on this problem after you have completed rest of the exam.

In propositional logic, a set of operators is said to be *functionally complete* if it is capable of representing all logical functions — that is all truth tables in any number of variables. For instance, the set of operators introduced in class ($\wedge$, $\vee$, $\sim$, $\Rightarrow$, $\iff$) is functionally complete.

As an example, the XOR function ...

X	Y	X XOR Y
T	T	F
T	F	T
F	T	T
F	F	F

... can be expressed as a disjunction of conjunctions, one for each "True" row in the table:

$$(X \wedge \sim Y) \vee ((\sim X) \wedge Y).$$

In fact, this example demonstrates that the smaller set of operators $\wedge$, $\vee$, $\sim$ is itself functionally complete, since we didn't need to use the conditional or biconditional to carry out this process. (Alternatively, notice that the biconditional $X \iff Y$ can be equivalently expressed as $(X \Rightarrow Y) \wedge (Y \Rightarrow X)$, and in turn the conditional $X \Rightarrow Y$ can be expressed as $(\sim X) \vee Y$, so any expression using these symbols can be rewritten without them.)

In many cases — for instance, when designing microchips or automated reasoning systems — it turns out to be useful to reason using a different set of operators than the full set given in class.

(a) Which one of the following sets of operators is *not* functionally complete?

A. $\wedge$, $\vee$, $\sim$, XOR

B. NAND (where X NAND Y is defined $\sim (X \wedge Y)$)

C. $\wedge$, $\vee$

D. $\wedge$, $\sim$

(b) Imagine we wished to reason using the reduced (but still functionally complete) set of operators $\wedge$, $\vee$, $\sim$. Which of the following inference rules is a correct translation of the Modus Ponens rule ($X \Rightarrow Y, X \vdash Y$) into this language:

A. $(\sim X) \wedge Y, \sim Y \vdash X$

B. $(\sim X) \vee Y, X \vdash Y$

C. $(\sim X) \vee Y, Y \vdash X$

D. $(\sim X) \wedge Y, X \vdash Y$