MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

TEN POWERFUL IDEAS

and applications

QUIZ 3	Bruce Maggs	April 27, 1998

NAME :

ANDREW ID :

SECTION : A B C D

INSTRUCTIONS. For the first 7 problems you do not need to give any reason or justification for your answers. For the remaining problems you must give complete explanations. WRITE CLEARLY. WRITE YOUR NAME AND ANDREW ID ABOVE AND CIRCLE YOUR SECTION.

The easy stuff.

This part is to test your ability to study material that has been pre-digested and labeled as useful or likely to appear on exams. If you do not do well on this section, your study habits could use adjustment.

1. [5 points]
Suppose that A and B are events in a probability space, and that $\Pr[A] = 0.5$, $\Pr[B] = 0.2$, and $\Pr[A \cup B] = 0.6$. What is $\Pr[A \cap B]$?

2. [5 points]
Suppose that X is a non-negative integer random variable that takes on the value i with probability $\Pr[X = i]$. What is the definition of $E[X]$, the expectation of X?

3. [5 points]
Suppose that each of the 112 students in 15-299 is assigned uniformly and independently at random to one of four recitation sections. What is the probability that all six students named "Pat" are assigned to the same recitation section?

4. [5 points]
Draw all of the 5-node undirected graphs (with no self-loops and no multi-edges) that are not planar.

5. [5 points]
Suppose that an N-node planar graph is drawn in the plane (with no edges crossing) with F faces. How many edges does the graph have?

Basic Techniques.

This part will test your ability to apply techniques for which I have given explicit recipes. If you do not do well on this section, your study habits could use adjustment.

6. [10 points]
Suppose that A and B are events in a probability space. Using the fact that $A = (A \cap B) \cup (A \cap \overline{B})$, prove that $\Pr[A] = \Pr[A|B] \cdot \Pr[B] + \Pr[A|\overline{B}] \cdot \Pr[\overline{B}]$.

7. [10 points]
Suppose that a resident of Pittsburgh chosen at random is a Steelers fan with probability 0.90, that a resident chosen at random is a Penguins fan with probability 0.75, and that a resident Steelers fan chosen at random is a Penguins fan with probability 0.80. Now suppose that a resident is chosen at random from among those that are *not* Steelers fans. What is the probability that such a resident is a Penguins fan?

8. [10 points]
Suppose that a fair coin is tossed 100 times. What is the expected number of pairs of consecutive flips in which the coin takes on the same value?

A Moment's Thought!
This section tests your ability to think a little bit more insightfully. You must give complete explanations of your answers.

The *degree* of a node in a graph is the number of edges incident on that node.

9. [10 points]
Prove that the average node degree in a finite tree is less than two.

10. [15 points]
Prove that if the maximum degree in a finite tree is d, then the tree must have at least d nodes of degree one.

11. [20 points]
Suppose that an n-node random undirected graph is formed as follows. For each pair of distinct vertices u and v, the edge $\{u, v\}$ is present with probability p and not present with probability $1 - p$, independent of whether any other edge is or is not in the graph. What should the probability p be set to so that the expected number of cliques of size k is one?