Data Representation

15-213/18-243: Introduction to Computer Systems

8th Lecture, 4 February 2010

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Last Time

- **For loops**
  - for loop → while loop → do-while loop → goto version
  - for loop → while loop → goto “jump to middle” version

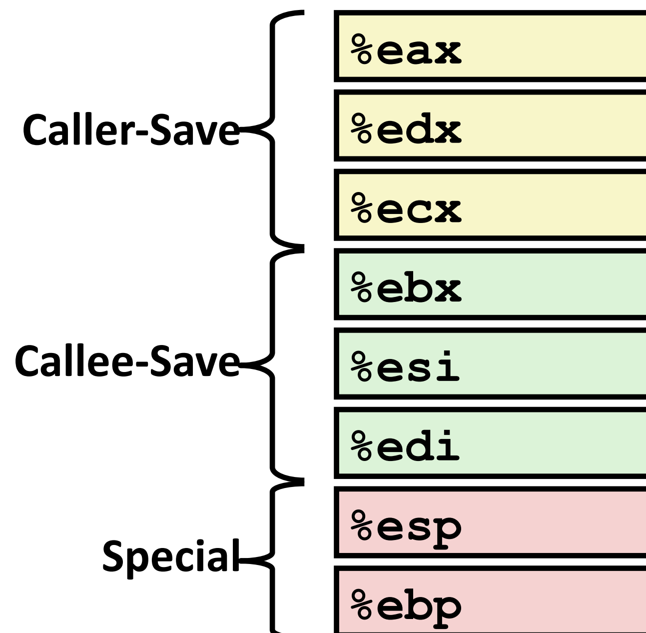
- **Switch statements**
  - Jump tables: `jmp *.L62(,%edx,4)`
  - Decision trees

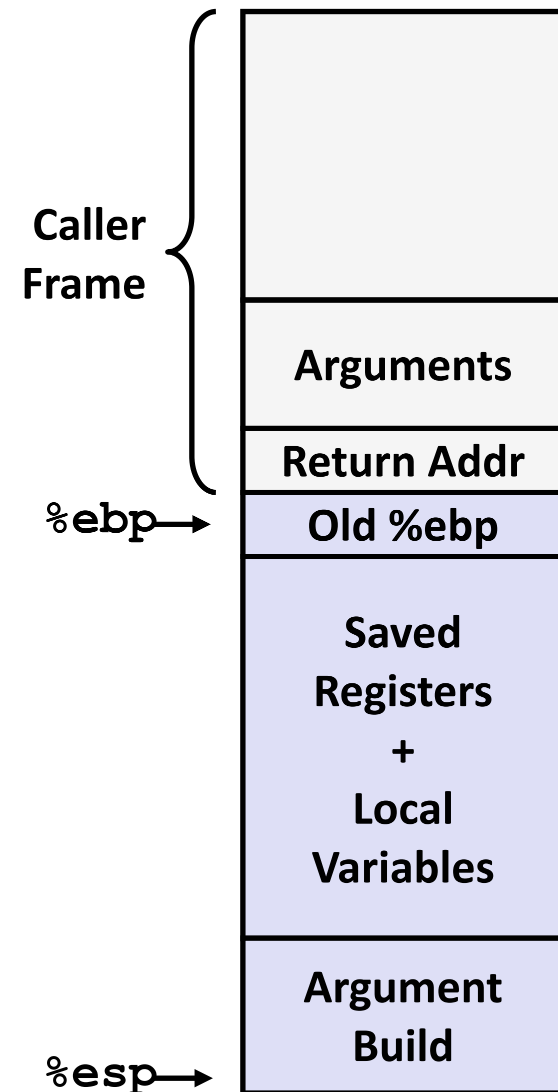
Jump table

```
.section .rodata
   .align 4
.L62:
  .long    .L61   # x = 0
  .long    .L56   # x = 1
  .long    .L57   # x = 2
  .long    .L58   # x = 3
  .long    .L61   # x = 4
  .long    .L60   # x = 5
  .long    .L60   # x = 6
```

Last Time

- **Procedures (IA32)**
  - call / return
  - `%esp`, `%ebp`
  - local variables
  - recursive functions





Today

- **Procedures (x86-64)**
- **Arrays**
  - One-dimensional
  - Multi-dimensional (nested)
  - Multi-level
- **Structures**

x86-64 Integer Registers

64-bit	32-bit	64-bit	32-bit
%rax	%eax	%r8	%r8d
%rbx	%ebx	%r9	%r9d
%rcx	%ecx	%r10	%r10d
%rdx	%edx	%r11	%r11d
%rsi	%esi	%r12	%r12d
%rdi	%edi	%r13	%r13d
%rsp	%esp	%r14	%r14d
%rbp	%ebp	%r15	%r15d

- Twice the number of registers as IA32
- Accessible as 64, 32, 16 or 8 bit objects

x86-64 Integer Registers

Register	Use	Register	Use
`%rax`	Return value	`%r8`	Argument #5
`%rbx`	Callee saved	`%r9`	Argument #6
`%rcx`	Argument #4	`%r10`	Reserved
`%rdx`	Argument #3	`%r11`	Used for linking
`%rsi`	Argument #2	`%r12`	Callee saved
`%rdi`	Argument #1	`%r13`	Callee saved
`%rsp`	Stack pointer	`%r14`	Callee saved
`%rbp`	Callee saved	`%r15`	Callee saved

x86-64 Registers

- **Arguments passed to functions via registers**
  - If more than 6 integral parameters, then pass rest on stack
  - These registers can be used as caller-saved as well
- **All references to stack frame via stack pointer**
  - Eliminates need to update `%ebp/%rbp`
- **Other Registers**
  - 6+1 callee saved
  - 2 or 3 have special uses

x86-64 Long Swap

```
void swap(long *xp, long *yp)
{
  long t0 = *xp;
  long t1 = *yp;
  *xp = t1;
  *yp = t0;
}
```

```
swap:
    movq    (%rdi), %rdx
    movq    (%rsi), %rax
    movq    %rax, (%rdi)
    movq    %rdx, (%rsi)
    ret
```

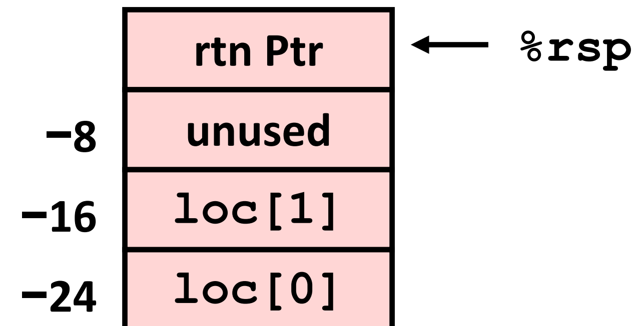
- **Operands passed in registers**
  - First (`xp`) in `%rdi`, second (`yp`) in `%rsi`
  - 64-bit pointers
- **No stack operations required (except `ret`)**
- **Avoiding stack**
  - Can hold all local information in registers

x86-64 Locals in the Red Zone

```
/* Swap, using local array */
void swap_a(long *xp, long *yp)
{
    volatile long loc[2];
    loc[0] = *xp;
    loc[1] = *yp;
    *xp = loc[1];
    *yp = loc[0];
}
```

```
swap_a:
   movq   (%rdi), %rax
   movq   %rax, -24(%rsp)
   movq   (%rsi), %rax
   movq   %rax, -16(%rsp)
   movq   -16(%rsp), %rax
   movq   %rax, (%rdi)
   movq   -24(%rsp), %rax
   movq   %rax, (%rsi)
   ret
```

- **Avoiding Stack Pointer Change**
  - Can hold all information within small window beyond stack pointer



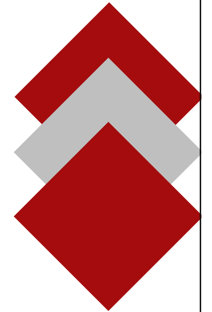
x86-64 NonLeaf without Stack Frame

```
long scount = 0;

/* Swap a[i] & a[i+1] */
void swap_ele_se(long a[], int i)
{
    swap(&a[i], &a[i+1]);
    scount++;
}
```

- No values held while swap being invoked
- No callee save registers needed

```
swap_ele_se:
  movslq %esi,%rsi              # Sign extend i
  leaq   (%rdi,%rsi,8), %rdi    # &a[i]
  leaq   8(%rdi), %rsi          # &a[i+1]
  call   swap                   # swap()
  incq   scount(%rip)           # scount++;
  ret
```



x86-64 Call using Jump

```
long scount = 0;

/* Swap a[i] & a[i+1] */
void swap_ele(long a[], int i)
{
    swap(&a[i], &a[i+1]);
}
```

```
swap_ele:
  movslq %esi,%rsi
  leaq   (%rdi,%rsi,8), %rdi
  leaq   8(%rdi), %rsi
  jmp   swap
```

x86-64 Call using Jump

```
long scount = 0;

/* Swap a[i] & a[i+1] */
void swap_ele(long a[], int i)
{
    swap(&a[i], &a[i+1]);
}
```

- When swap executes `ret`, it will return from `swap_ele`
- Possible since swap is a "tail call" (no instructions follow)
  - "Leaf" subroutine

```
swap_ele:
  movslq %esi,%rsi                # Sign extend i
  leaq   (%rdi,%rsi,8), %rdi      # &a[i]
  leaq   8(%rdi), %rsi            # &a[i+1]
  jmp    swap                     # swap()
```

x86-64 Stack Frame Example

```
long sum = 0;
/* Swap a[i] & a[i+1] */
void swap_ele_su
  (long a[], int i)
{
    swap(&a[i], &a[i+1]);
    sum += a[i];
}
```

```
swap_ele_su:
  movq     %rbx, -16(%rsp)
  movslq   %esi,%rbx
  movq     %r12, -8(%rsp)
  movq     %rdi, %r12
  leaq     (%rdi,%rbx,8), %rdi
  subq     $16, %rsp
  leaq     8(%rdi), %rsi
  call     swap
  movq     (%r12,%rbx,8), %rax
  addq     %rax, sum(%rip)
  movq     (%rsp), %rbx
  movq     8(%rsp), %r12
  addq     $16, %rsp
  ret
```

- Keeps values of a and i in callee save registers
- Must set up stack frame to save these registers

Understanding x86-64 Stack Frame

```
swap_ele_su:
   movq     %rbx, -16(%rsp)        # Save %rbx
   movslq   %esi,%rbx              # Extend & save i
   movq     %r12, -8(%rsp)         # Save %r12
   movq     %rdi, %r12             # Save a
   leaq     (%rdi,%rbx,8), %rdi    # &a[i]
   subq     $16, %rsp              # Allocate stack frame
   leaq     8(%rdi), %rsi          #     &a[i+1]
   call     swap                   # swap()
   movq     (%r12,%rbx,8), %rax    # a[i]
   addq     %rax, sum(%rip)        # sum += a[i]
   movq     (%rsp), %rbx           # Restore %rbx
   movq     8(%rsp), %r12          # Restore %r12
   addq     $16, %rsp              # Deallocate stack frame
   ret
```

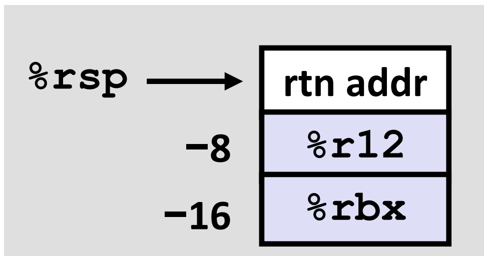
Understanding x86-64 Stack Frame

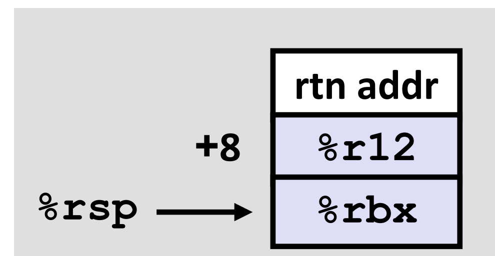
```
swap_ele_su:
   movq     %rbx, -16(%rsp)        # Save %rbx
   movq     %r12, -8(%rsp)         # Save %r12

   subq     $16, %rsp              # Allocate stack frame




   movq     (%rsp), %rbx           # Restore %rbx
   movq     8(%rsp), %r12          # Restore %r12
   addq     $16, %rsp              # Deallocate stack frame
```





Interesting Features of Stack Frame

- **Allocate entire frame at once**
  - All stack accesses can be relative to `%rsp`
  - Do by decrementing stack pointer
  - Can delay allocation, since safe to temporarily use red zone
- **Simple deallocation**
  - Increment stack pointer
  - No base/frame pointer needed

x86-64 Procedure Summary

- **Heavy use of registers**
  - Parameter passing
  - More temporaries since more registers
- **Minimal use of stack**
  - Sometimes none
  - Allocate/deallocate entire block
- **Many tricky optimizations**
  - What kind of stack frame to use
  - Calling with jump
  - Various allocation techniques

Today

- Procedures (x86-64)
- **Arrays**
  - One-dimensional
  - Multi-dimensional (nested)
  - Multi-level
- Structures

Basic Data Types

- **Integral**
  - Stored and operated on in general (integer) registers
  - Signed vs. unsigned depends on instructions used

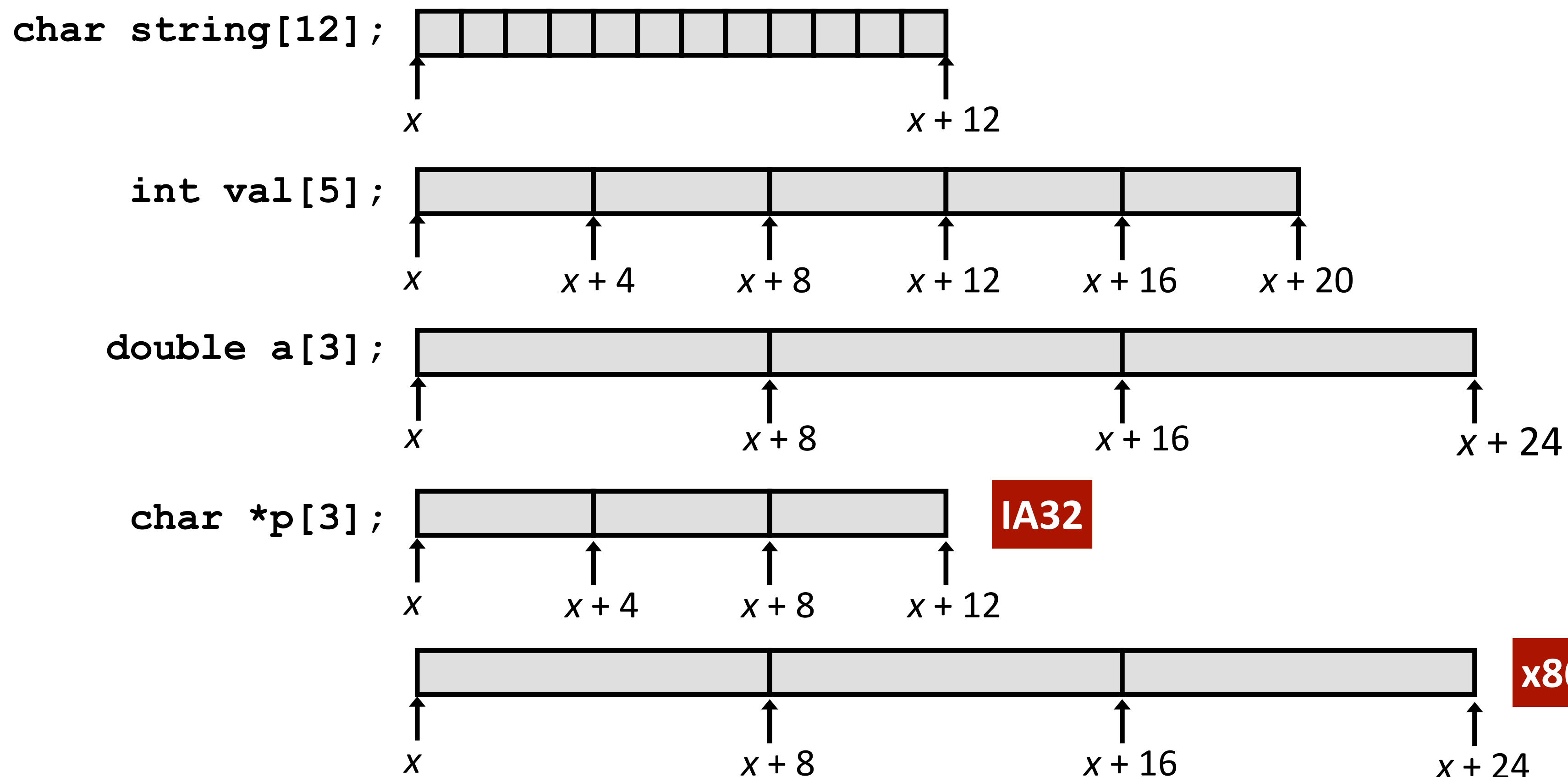
Intel	GAS	Bytes	C
byte	`b`	1	`[unsigned] char`
word	`w`	2	`[unsigned] short`
double word	`l`	4	`[unsigned] int`
quad word	`q`	8	`[unsigned] long int (x86-64)`

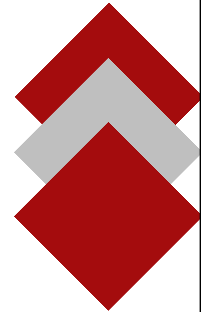
- **Floating Point**
  - Stored & operated on in floating point registers

Intel	GAS	Bytes	C
Single	`s`	4	`float`
Double	`l`	8	`double`
Extended	`t`	10/12/16	`long double`

Array Allocation

- **Basic Principle**
  - *T* **`A[`***L***`];`**
  - Array of data type ***T*** and length ***L***
  - Contiguously allocated region of ***L*** **`* sizeof(`*****T*****`)`** bytes

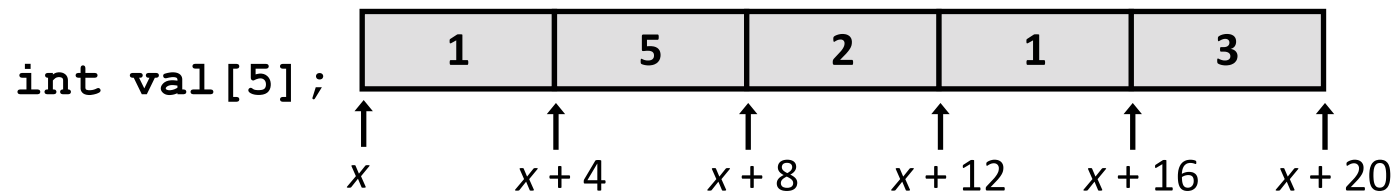




Array Access

- **Basic Principle**
  - *T* **A**[*L*];
  - Array of data type ***T*** and length ***L***
  - Identifier **A** can be used as a pointer to array element 0: Type ***T****

`int val[5];`



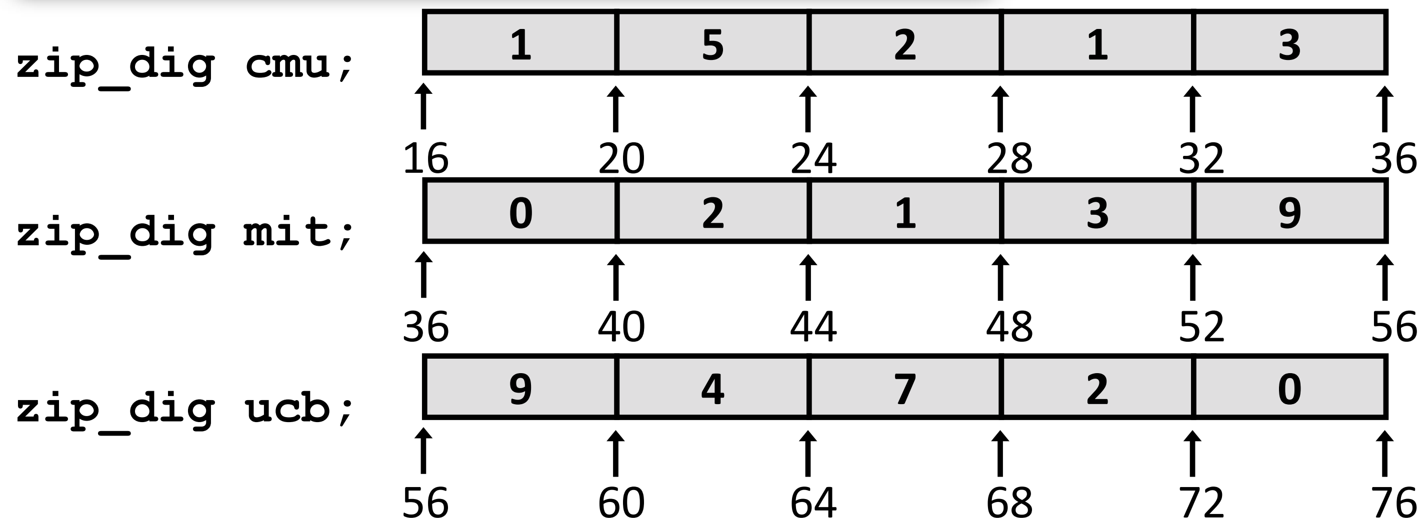
- **Reference** **Type** **Value**

```
val[4]
val
val+1
&val[2]
val[5]
*(val+1)
val + i
```

Array Example

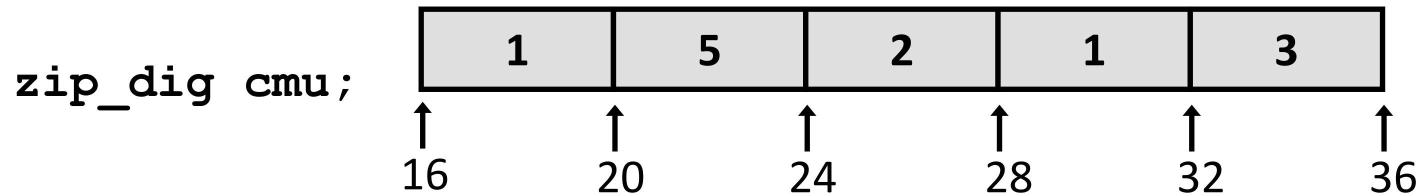
```
typedef int zip_dig[5];

zip_dig cmu = { 1, 5, 2, 1, 3 };
zip_dig mit = { 0, 2, 1, 3, 9 };
zip_dig ucb = { 9, 4, 7, 2, 0 };
```



- **Declaration "`zip_dig cmu`" equivalent to "`int cmu[5]`"**
- **Example arrays were allocated in successive 20 byte blocks**
  - Not guaranteed to happen in general

Array Accessing Example

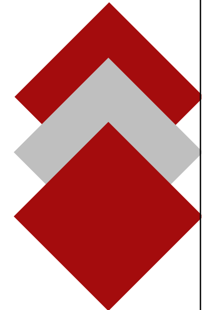



```
int get_digit(zip_dig z, int dig)
{
  return z[dig];
}
```

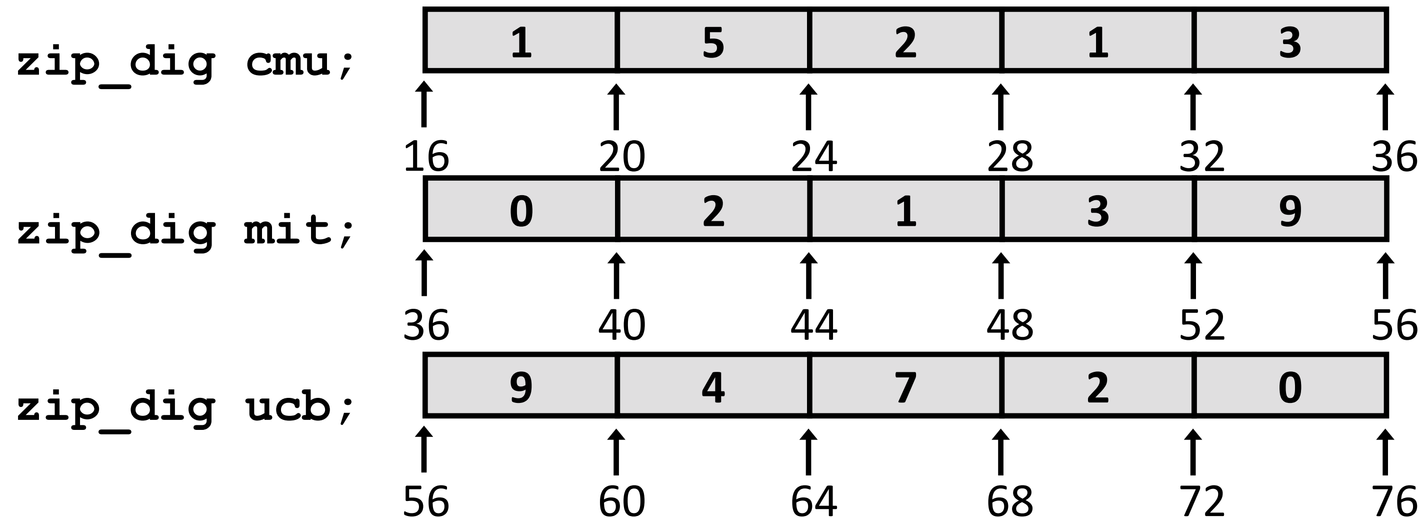
IA32

```
  # %edx = z
  # %eax = dig
  movl (%edx,%eax,4),%eax  # z[dig]
```

- Register `%edx` contains starting address of array
- Register `%eax` contains array index
- Desired digit at `4*%eax + %edx`
- Use memory reference `(%edx, %eax, 4)`



Referencing Examples



- **Reference Address Value Guaranteed?**

  `mit[3]`

  `mit[5]`

  `mit[-1]`

  `cmu[15]`

- No bounds checking
- Out of range behavior implementation-dependent
- No guaranteed relative allocation of different arrays

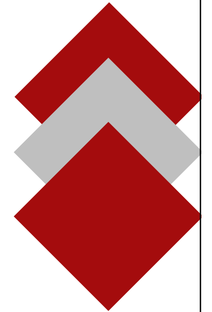
Array Loop Example

- **Original**
  - Convert 5 individual digits (held in an array, zip_dig format) into a decimal number

```
int zd2int(zip_dig z)
{
  int i;
  int zi = 0;
  for (i = 0; i < 5; i++) {
    zi = 10 * zi + z[i];
  }
  return zi;
}
```

- **Transformed**
  - As generated by GCC
  - Eliminate loop variable `i`
  - Convert array code to pointer code
  - Express in do-while form (no test at entrance)

```
int zd2int(zip_dig z)
{
  int zi = 0;
  int *zend = z + 4;
  do {
    zi = 10 * zi + *z;
    z++;
  } while (z <= zend);
  return zi;
}
```



Array Loop Implementation (IA32)

```
int zd2int(zip_dig z)
{
  int zi = 0;
  int *zend = z + 4;
  do {
    zi = 10 * zi + *z;
    z++;
  } while(z <= zend);
  return zi;
}
```

```
  # %ecx = z
  xorl %eax,%eax
  leal 16(%ecx),%ebx
.L59:
  leal (%eax,%eax,4),%edx
  movl (%ecx),%eax
  addl $4,%ecx
  leal (%eax,%edx,2),%eax
  cmpl %ebx,%ecx
  jle .L59
```

Array Loop Implementation (IA32)

- **Registers**

```
%ecx      z
%eax      zi
%ebx      zend
```

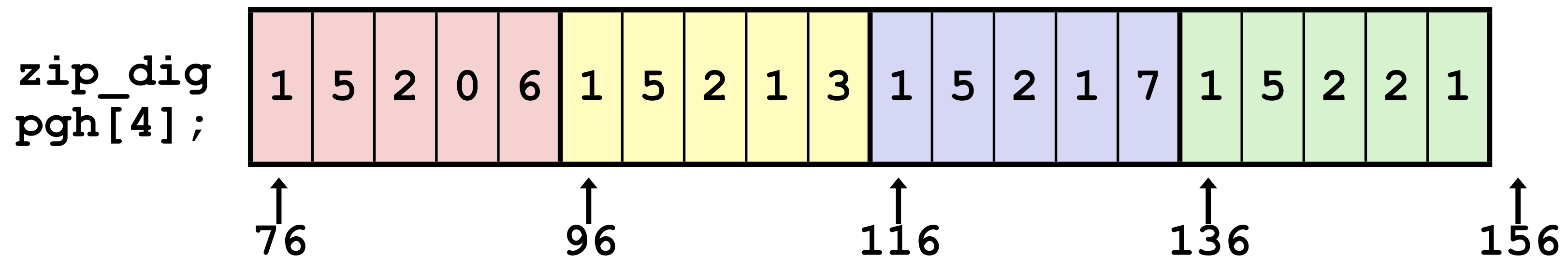
- **Computations**
  - `10*zi + *z` implemented as `*z + 2*(zi+4*zi)`
  - `z++` increments by 4

```
int zd2int(zip_dig z)
{
  int zi = 0;
  int *zend = z + 4;
  do {
    zi = 10 * zi + *z;
    z++;
  } while(z <= zend);
  return zi;
}
```

```
  # %ecx = z
  xorl %eax,%eax              # zi = 0
  leal 16(%ecx),%ebx          # zend  = z+4
.L59:
  leal (%eax,%eax,4),%edx # 5*zi
  movl (%ecx),%eax            # *z
  addl $4,%ecx                # z++
  leal (%eax,%edx,2),%eax # zi = *z + 2*(5*zi)
  cmpl  %ebx,%ecx             # z : zend
  jle  .L59                   # if <= goto loop
```

Nested Array Example

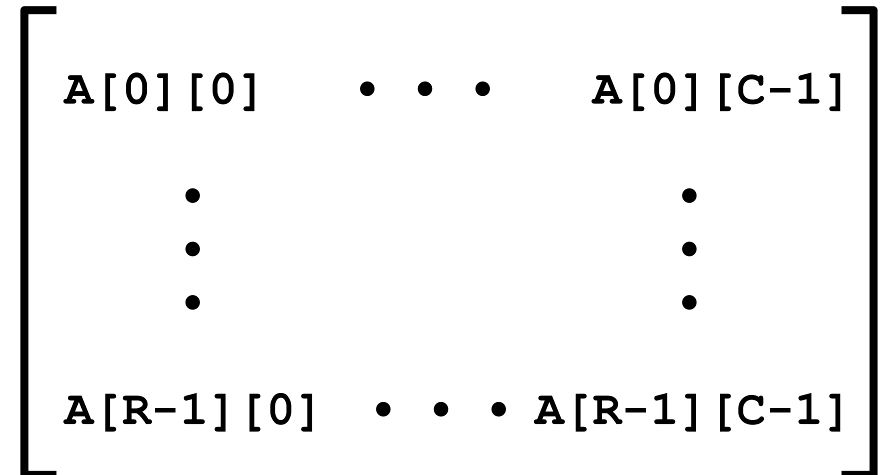
```
#define PCOUNT 4
zip_dig pgh[PCOUNT] =
  {{1, 5, 2, 0, 6},
   {1, 5, 2, 1, 3 },
   {1, 5, 2, 1, 7 },
   {1, 5, 2, 2, 1 }};
```



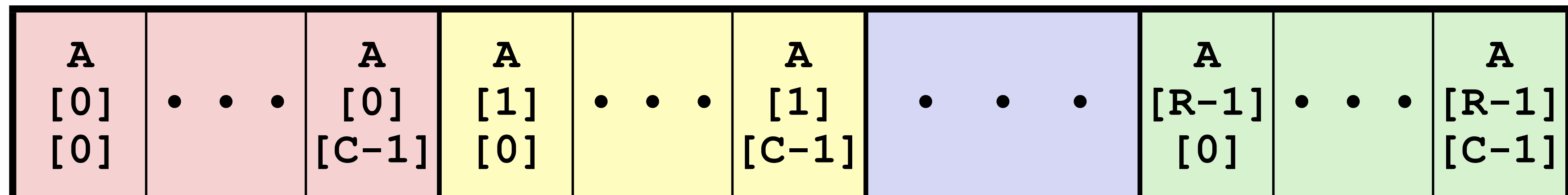
- "**`zip_dig pgh[4]`" equivalent to "`int pgh[4][5]`"**
  - Variable **`pgh`**: array of 4 elements, allocated contiguously
  - Each element is an array of 5 **`int`**'s, allocated contiguously
- **"Row-Major" ordering of all elements guaranteed**

Multidimensional (Nested) Arrays

- **Declaration**
  - ***T*** `A`[*R*][*C*];
  - 2D array of data type ***T***
  - ***R*** rows, ***C*** columns
  - Type ***T*** element requires ***K*** bytes
- **Array Size**
  - ***R* * *C* * *K*** bytes
- **Arrangement**
  - Row-Major Ordering



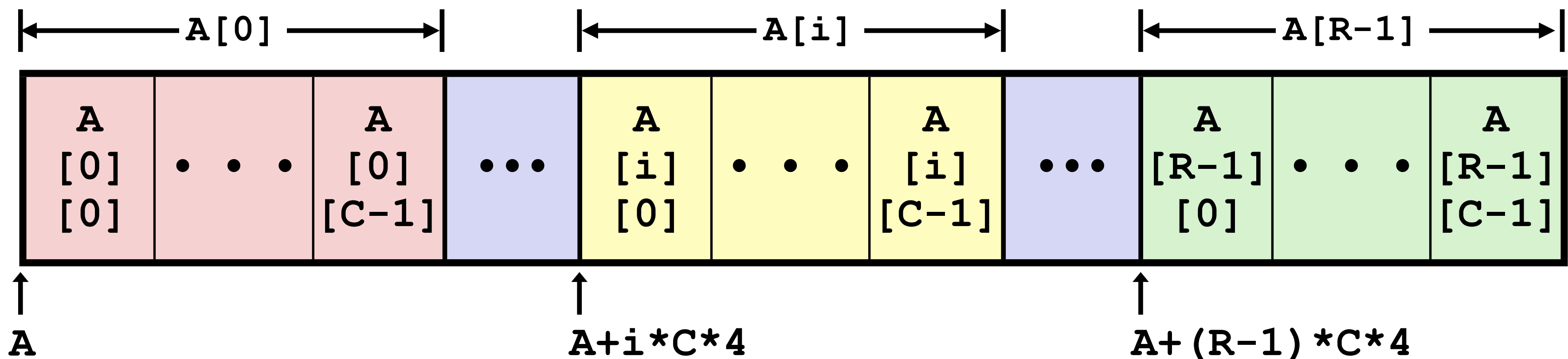

```
int A[R][C];
```

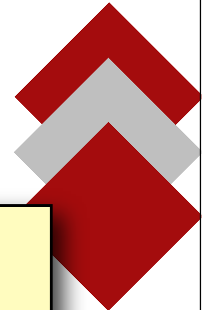


Nested Array Row Access

- **Row Vectors**
  - `A[i]` is array of *C* elements
  - Each element of type *T* requires *K* bytes
  - Starting address `A` + ***i* * (*C* * *K*)**

```
int A[R][C];
```





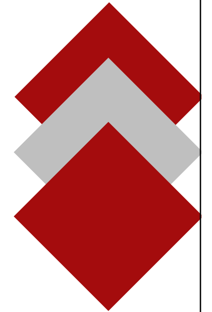
Nested Array Row Access Code

```
int *get_pgh_zip(int index)
{
  return pgh[index];
}
```

```
#define PCOUNT 4
zip_dig pgh[PCOUNT] =
  {{1, 5, 2, 0, 6},
   {1, 5, 2, 1, 3 },
   {1, 5, 2, 1, 7 },
   {1, 5, 2, 2, 1 }};
```

- What data type is `pgh[index]`?
- What is its starting address?

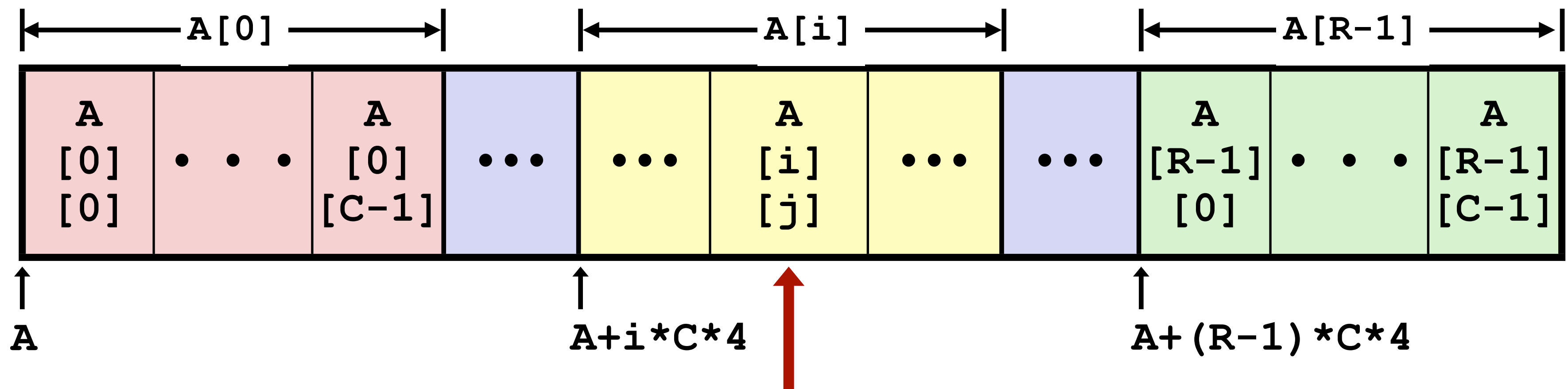
```
  # %eax = index
  leal (%eax,%eax,4),%eax
  leal pgh(,%eax,4),%eax
```



Nested Array Row Access

- **Array Elements**
  - **`A[i][j]`** is element of type ***T***, which requires *K* bytes
  - Address **`A`** $+ i * (C * K) + j * K = A + (i * C + j) * K$

```
int A[R][C];
```

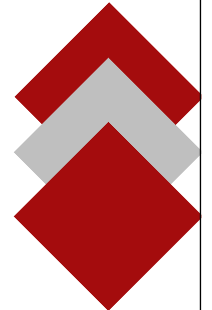


Nested Array Element Access Code

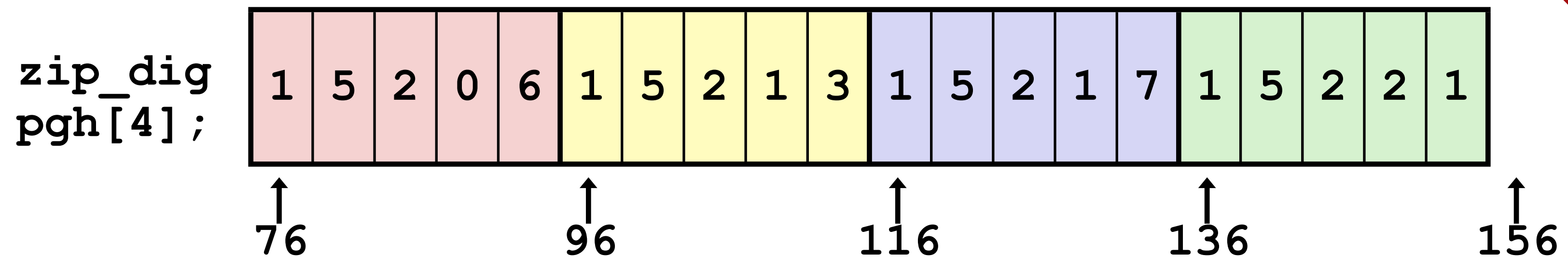
```
int get_pgh_digit(int index, int dig)
{
  return pgh[index][dig];
}
```

```
# %ecx = dig
# %eax = index
leal 0(,%ecx,4),%edx          # 4*dig
leal (%eax,%eax,4),%eax       # 5*index
movl pgh(%edx,%eax,4),%eax    # *(pgh + 4*dig + 20*index)
```

- **Array Elements**
  - `pgh[index][dig]` is `int`
  - Address: `pgh + 20*index + 4*dig`
- **IA32 Code**
  - Computes address `pgh + 4*dig + 4*(index+4*index)`
  - `movl` performs memory reference



Strange Referencing Examples



Reference	Address	Value	Guaranteed?
`pgh[3][3]`	`76+20*3+4*3 = 148`	2	
`pgh[2][5]`	`76+20*2+4*5 = 136`	1	
`pgh[2][-1]`	`76+20*2+4*-1 = 112`	3	
`pgh[4][-1]`	`76+20*4+4*-1 = 152`	1	
`pgh[0][19]`	`76+20*0+4*19 = 152`	1	
`pgh[0][-1]`	`76+20*0+4*-1 = 72`	??	

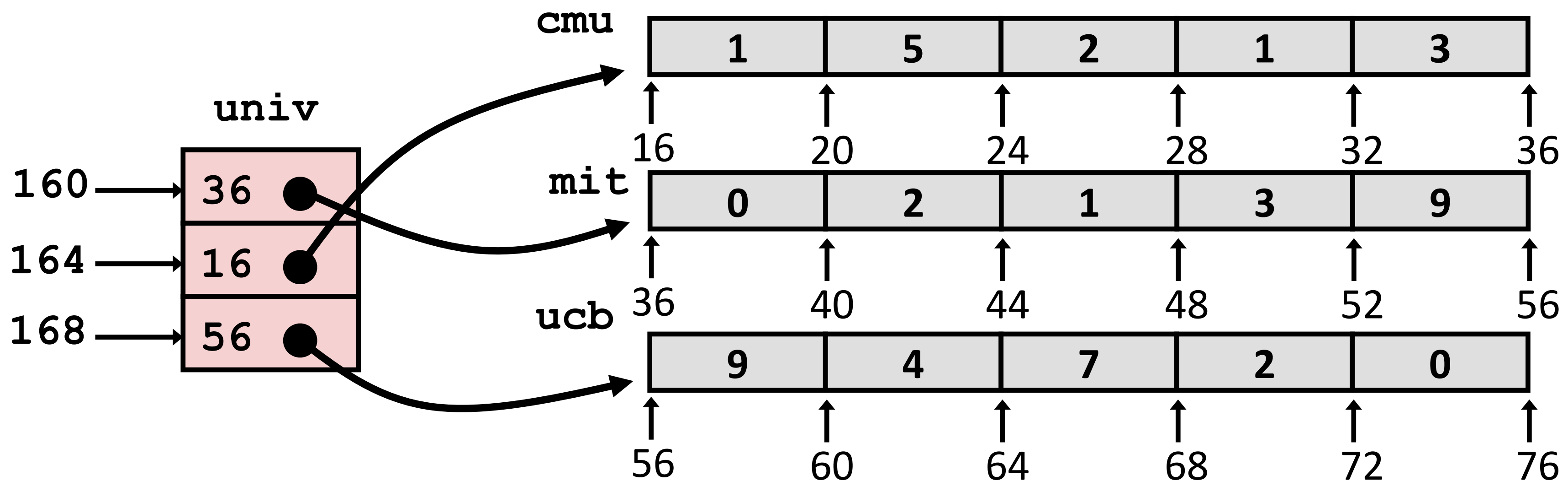
- Code does not do any bounds checking
- Ordering of elements within array guaranteed

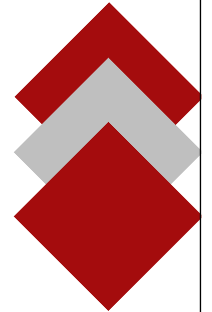
Multi-Level Array Example

```
zip_dig cmu = { 1, 5, 2, 1, 3 };
zip_dig mit = { 0, 2, 1, 3, 9 };
zip_dig ucb = { 9, 4, 7, 2, 0 };
```

```
#define UCOUNT 3
int *univ[UCOUNT] = {mit, cmu, ucb};
```

- Variable `univ` denotes array of 3 elements
- Each element is a pointer
  - 4 bytes
- ...which points to array of `int`'s





Element Access in Multi-Level Array

```
int get_univ_digit
   (int index, int dig)
{
   return univ[index][dig];
}
```

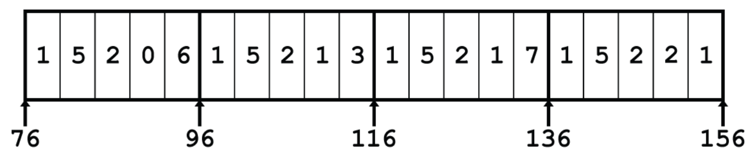
```
   # %ecx = index
   # %eax = dig
   leal 0(,%ecx,4),%edx
   movl univ(%edx),%edx
   movl (%edx,%eax,4),%eax
```

- **Computation (IA32)**
  - Element access **`Mem[Mem[univ+4*index]+4*dig]`**
  - Must do two memory reads
    - First get pointer to row array
    - Then access element within array

Array Element Accesses

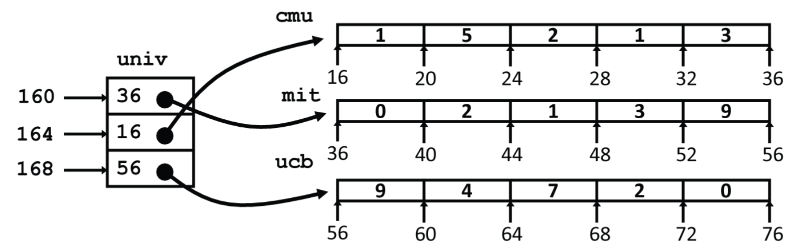
Nested array

```
int get_pgh_digit
   (int index, int dig)
{
  return pgh[index][dig];
}
```



Multi-level array

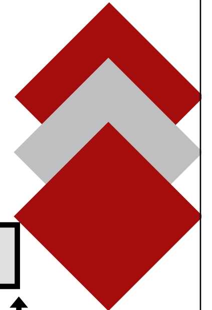
```
int get_univ_digit
   (int index, int dig)
{
  return univ[index][dig];
}
```



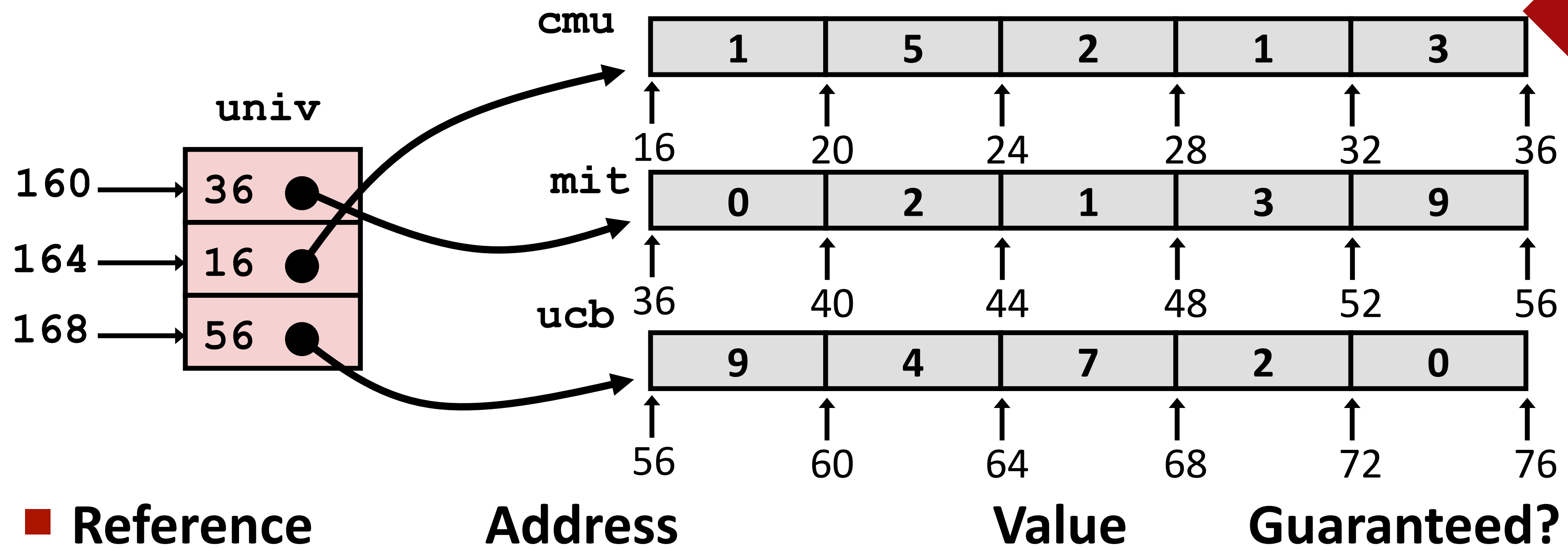
Access looks similar, but isn't:

`Mem[pgh+20*index+4*dig]`

`Mem[Mem[univ+4*index]+4*dig]`



Strange Referencing Examples



- **Reference** **Address** **Value** **Guaranteed?**

  `univ[2][3]`

  `univ[1][5]`

  `univ[2][-1]`

  `univ[3][-1]`

  `univ[1][12]`

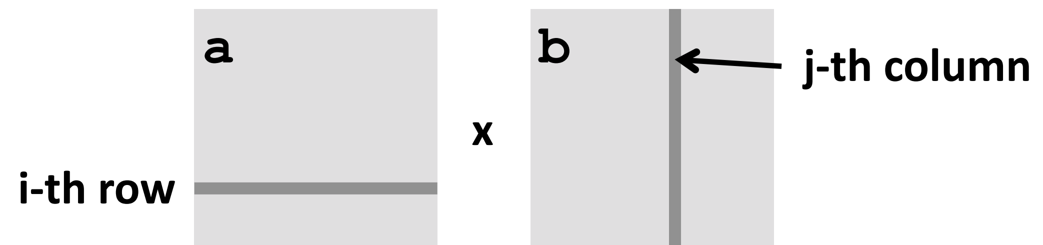
  - Code does not do any bounds checking
  - Ordering of elements in different arrays not guaranteed

Using Nested Arrays

- **Strengths**
  - C compiler handles doubly subscripted arrays
  - Generates very efficient code
  - Avoids multiply in index computation
- **Limitation**
  - Only works for fixed array size

```
#define N 16
typedef int fix_matrix[N][N];
```

```
/* Compute element i,k of
   fixed matrix product */
int fix_prod_ele
(fix_matrix a, fix_matrix b,
 int i, int k)
{
  int j;
  int result = 0;
  for (j = 0; j < N; j++)
    result += a[i][j]*b[j][k];
  return result;
}
```



Dynamic Nested Arrays

- **Strength**
  - Can create matrix of any size
- **Programming**
  - Must do index computation explicitly
- **Performance**
  - Accessing single element costly
  - Must do multiplication

```
int * new_var_matrix(int n)
{
  return (int *)
    calloc(sizeof(int), n*n);
}
```

```
int var_ele
  (int *a, int i, int j, int n)
{
  return a[i*n+j];
}
```

```
movl  12(%ebp),%eax        # i
movl  8(%ebp),%edx         # a
imull 20(%ebp),%eax        # n*i
addl  16(%ebp),%eax        # n*i+j
movl  (%edx,%eax,4),%eax   # Mem[a+4*(i*n+j)]
```

Dynamic Array Multiplication

- **Without Optimizations**
  - Multiplies: 3
    - 2 for subscripts
    - 1 for data
  - Adds: 4
    - 2 for array indexing
    - 1 for loop index
    - 1 for data

```
/* Compute element i,k of
   variable matrix product */
int var_prod_ele
  (int *a, int *b,
   int i, int k, int n)
{
  int j;
  int result = 0;
  for (j = 0; j < n; j++)
    result +=
      a[i*n+j] * b[j*n+k];
  return result;
}
```

Optimizing Dynamic Array Multiplication

- **Optimizations**
  - Performed when set optimization level to `-O2`
- **Code Motion**
  - Expression `i*n` can be computed outside loop
- **Strength Reduction**
  - Incrementing `j` has effect of incrementing `j*n+k` by `n`
- **Operations count**
  - 4 adds, 1 mult
- **Compiler can optimize regular access patterns**

```
{
  int j;
  int result = 0;
  for (j = 0; j < n; j++)
    result +=
      a[i*n+j] * b[j*n+k];
  return result;
}
```

```
{
  int j;
  int result = 0;
  int iTn = i*n;
  int jTnPk = k;
  for (j = 0; j < n; j++) {
    result +=
      a[iTn+j] * b[jTnPk];
    jTnPk += n;
  }
  return result;
}
```

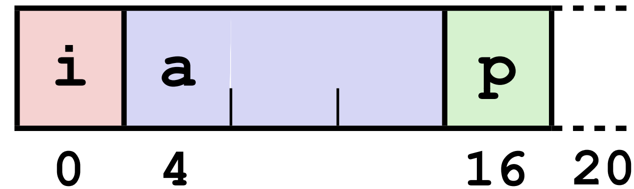
Today

- Procedures (x86-64)
- Arrays
  - One-dimensional
  - Multi-dimensional (nested)
  - Multi-level
- **Structures**

Structures

```
struct rec {
  int i;
  int a[3];
  int *p;
};
```

Memory Layout



- **Concept**
  - Contiguously-allocated region of memory
  - Refer to members within structure by names
  - Members may be of different types
- **Accessing Structure Member**

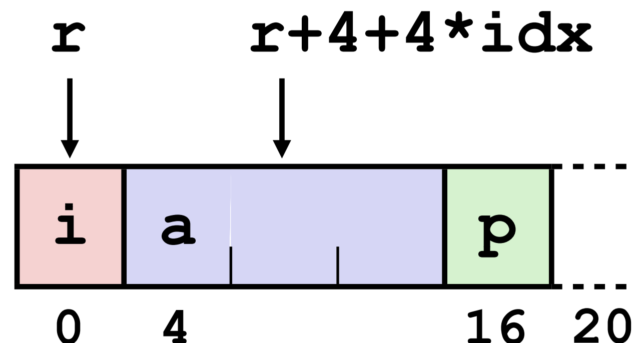
```
void
set_i(struct rec *r,
      int val)
{
  r->i = val;
}
```

IA32 Assembly

```
# %eax = val
# %edx = r
movl %eax,(%edx)    # Mem[r] = val
```

Generating Pointer to Structure Member

```
struct rec {
  int i;
  int a[3];
  int *p;
};
```



- **Generating Pointer to Array Element**
  - Offset of each structure member determined at compile time

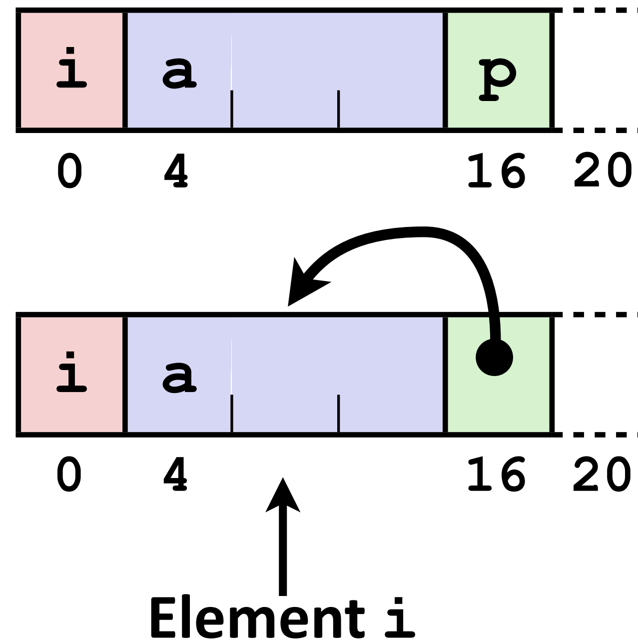
```
int *find_a
 (struct rec *r, int idx)
{
  return &r->a[idx];
}
```

```
# %ecx = idx
# %edx = r
leal 0(,%ecx,4),%eax    # 4*idx
leal 4(%eax,%edx),%eax # r+4*idx+4
```

Structure Referencing (Cont.)

- C Code

```
struct rec {
  int i;
  int a[3];
  int *p;
};
```




```
void
set_p(struct rec *r)
{
  r->p =
   &r->a[r->i];
}
```

```
 # %edx = r
movl  (%edx),%ecx          # r->i
leal  0(,%ecx,4),%eax      # 4*(r->i)
leal  4(%edx,%eax),%eax    # r+4+4*(r->i)
movl  %eax,16(%edx)        # Update r->p
```

Today

- **Procedures (x86-64)**
- **Arrays**
  - One-dimensional
  - Multi-dimensional (nested)
  - Multi-level
- **Structures**

- **Next Time**
  - Structures
  - Unions
  - Alignment
  - Floating point (x87 and SSE3)