15-213

"The Class That Gives CMU Its Zip!"

Bits and Bytes
January 15, 2004

Topics

- Why bits?
- Representing information as bits
  - Binary/Hexadecimal
  - Byte representations
    - » numbers
    - » characters and strings
    - » Instructions
- Bit-level manipulations
  - Boolean algebra
  - Expressing in C

Why Don't Computers Use Base 10?

Base 10 Number Representation

- That's why fingers are known as "digits"
- Natural representation for financial transactions
  - Floating point number cannot exactly represent $1.20
- Even carries through in scientific notation
  - 15.213×10^3 (1.5213e4)

Implementing Electronically

- Hard to store
  - ENIAC (First electronic computer) used 10 vacuum tubes / digit
  - IBM 650 used 5+2 bits (1958, successor to IBM's Personal Automatic Computer, PAC from 1956)
- Hard to transmit
  - Need high precision to encode 10 signal levels on single wire
- Messy to implement digital logic functions
  - Addition, multiplication, etc.

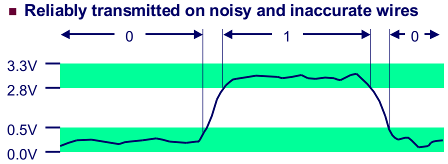
Binary Representations

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



Byte-Oriented Memory Organization

Programs Refer to Virtual Addresses

- Conceptually very large array of bytes
- Actually implemented with hierarchy of different memory types
  - SRAM, DRAM, disk
  - Only allocate for regions actually used by program
- In Unix and Windows NT, address space private to particular "process"
  - Program being executed
  - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation

- Where different program objects should be stored
- Multiple mechanisms: static, stack, and heap
- In any case, all allocation within single virtual address space

Encoding Byte Values

Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
  - First digit must not be 0 in C
- Octal: 000_8 to 0377_8
  - Use leading 0 in C
- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Literary Hex

Common 8-byte hex filler:

- `0xdeadbeef`
- Can you think of other 8-byte fillers?

Machine Words

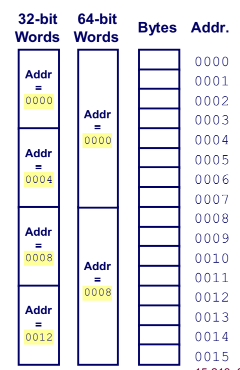
Machine Has "Word Size"

- Nominal size of integer-valued data
  - Including addresses
- Most current machines are 32 bits (4 bytes)
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- High-end systems are 64 bits (8 bytes)
  - Potential address space ≈ 1.8 X 10^{19} bytes
- Machines support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Alpha (RIP)	Typical 32-bit	Intel IA32
unsigned	4	4	4
int	4	4	4
long int	8	4	4
char	1	1	1
short	2	2	2
float	4	4	4
double	8	8	8
long double	8	8	10/12
char *	8	4	4

» Or any other pointer

Byte Ordering

How should bytes within multi-byte word be ordered in memory?

Conventions

- Sun's, Mac's are "Big Endian" machines
  - Least significant byte has highest address
- Alphas, PC's are "Little Endian" machines
  - Least significant byte has lowest address

Byte Ordering Example

Big Endian

- Least significant byte has highest address

Little Endian

- Least significant byte has lowest address

Example

- Variable `x` has 4-byte representation `0x01234567`
- Address given by `&x` is `0x100`

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

Reading Byte-Reversed Listings

Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add $0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl $0x0,0x28(%ebx)

Deciphering Numbers

- Value: `0x12ab`
- Pad to 4 bytes: `0x000012ab`
- Split into bytes: `00 00 12 ab`
- Reverse: `ab 12 00 00`

Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:
`%p`: Print pointer
`%x`: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

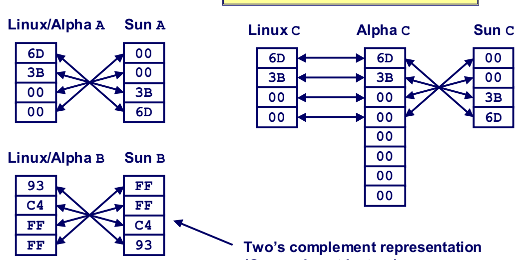
Result (Linux):

```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```


Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

Decimal:	15213
Binary:	0011 1011 0110 1101
Hex:	3 B 6 D



Two's complement representation
(Covered next lecture)

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Alpha P: A0 FC FF FF 01 00 00 00

Alpha Address

Hex:	1	F	F	F	F	F	C	A	0
Binary:	0001	1111	1111	1111	1111	1111	1100	1010	0000

Sun P: EF FF FB 2C

Sun Address

Hex:	E	F	F	F	F	B	2	C
Binary:	1110	1111	1111	1111	1111	1011	0010	1100

Linux P: D4 F8 FF BF

Linux Address

Hex:	B	F	F	F	F	8	D	4
Binary:	1011	1111	1111	1111	1111	1000	1101	0100

Different compilers & machines assign different locations to objects

Representing Floats

`Float F = 15213.0;`

Linux/Alpha F	Sun F
00	46
B4	6D
6D	B4
46	00

IEEE Single Precision Floating Point Representation

Hex:	4	6	6	D	B	4	0	0
Binary:	0100	0110	0110	1101	1011	0100	0000	0000
15213:			110	1101	1011	01		

Not same as integer representation, but consistent across machines

Can see some relation to integer representation, but not obvious

Representing Strings

`char S[6] = "15213";`

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code 0x30
    - » Digit *i* has code 0x30+*i*
- String should be null-terminated
  - Final character = 0

Linux/Alpha S	Sun S
31	31
35	35
32	32
31	31
33	33
00	00

Compatibility

- Byte ordering not an issue
- Text files generally platform independent
  - Except for different conventions of line termination character(s)!
    - » Unix (`'\n' = 0x0a = ^J`)
    - » Mac (`'\r' = 0x0d = ^M`)
    - » DOS and HTTP (`'\r\n' = 0x0d0a = ^M^J`)

Machine-Level Code Representation

Encode Program as Sequence of Instructions

- Each simple operation
  - Arithmetic operation
  - Read or write memory
  - Conditional branch
- Instructions encoded as bytes
  - Alpha's, Sun's, Mac's use 4 byte instructions
    - » Reduced Instruction Set Computer (RISC)
  - PC's use variable length instructions
    - » Complex Instruction Set Computer (CISC)
- Different instruction types and encodings for different machines
  - Most code not binary compatible

Programs are Byte Sequences Too!

Representing Instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
  - Use differing numbers of instructions in other cases
- PC uses 7 instructions with lengths 1, 2, and 3 bytes
  - Same for NT and for Linux
  - NT / Linux not fully binary compatible

Alpha sum	Sun sum	PC sum
00	81	55
00	C3	89
30	E0	E5
42	08	8B
01	90	45
80	02	0C
FA	00	03
6B	09	45
		08
		89
		EC
		5D
		C3

Different machines use totally different instructions and encodings

Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Exclusive-Or (Xor)

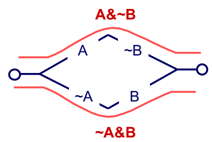
- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

Applied to Digital Systems by Claude Shannon

- 1937 MIT Master's Thesis
- Reason about networks of relay switches
  - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Integer Algebra

Integer Arithmetic

- ⟨Z, +, *, –, 0, 1⟩ forms a "ring"
- Addition is "sum" operation
- Multiplication is "product" operation
- – is additive inverse
- 0 is identity for sum
- 1 is identity for product

Boolean Algebra

Boolean Algebra

- ⟨{0,1}, |, &, ~, 0, 1⟩ forms a "Boolean algebra"
- Or is "sum" operation
- And is "product" operation
- ~ is "complement" operation (not additive inverse)
- 0 is identity for sum
- 1 is identity for product

Boolean Algebra ≈ Integer Ring

- *Commutativity*
  - A | B = B | A — A + B = B + A
  - A & B = B & A — A * B = B * A
- *Associativity*
  - (A | B) | C = A | (B | C) — (A + B) + C = A + (B + C)
  - (A & B) & C = A & (B & C) — (A * B) * C = A * (B * C)
- *Product distributes over sum*
  - A & (B | C) = (A & B) | (A & C) — A * (B + C) = A * B + B * C
- *Sum and product identities*
  - A | 0 = A — A + 0 = A
  - A & 1 = A — A * 1 = A
- *Zero is product annihilator*
  - A & 0 = 0 — A * 0 = 0
- *Cancellation of negation*
  - ~ (~ A) = A — – (– A) = A

Boolean Algebra ≠ Integer Ring

- Boolean: *Sum distributes over product*
  - A | (B & C) = (A | B) & (A | C) — A + (B * C) ≠ (A + B) * (B + C)
- Boolean: *Idempotency*
  - A | A = A — A + A ≠ A
  - "A is true" or "A is true" = "A is true"
  - A & A = A — A * A ≠ A
- Boolean: *Absorption*
  - A | (A & B) = A — A + (A * B) ≠ A
  - "A is true" or "A is true and B is true" = "A is true"
  - A & (A | B) = A — A * (A + B) ≠ A
- Boolean: *Laws of Complements*
  - A | ~A = 1 — A + –A ≠ 1
  - "A is true" or "A is false"
- Ring: *Every element has additive inverse*
  - A | ~A ≠ 0 — A + –A = 0

Properties of & and ^

Boolean Ring

- ⟨{0,1}, ^, &, I, 0, 1⟩
- Identical to integers mod 2
- I is identity operation: I (A) = A
  - A ^ A = 0

Property	Boolean Ring
Commutative sum	A ^ B = B ^ A
Commutative product	A & B = B & A
Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
Associative product	(A & B) & C = A & (B & C)
Prod. over sum	A & (B ^ C) = (A & B) ^ (B & C)
0 is sum identity	A ^ 0 = A
1 is prod. identity	A & 1 = A
0 is product annihilator	A & 0 = 0
Additive inverse	A ^ A = 0

Relations Between Operations

DeMorgan's Laws

- Express & in terms of |, and vice-versa
  - A & B = ~(~A | ~B)
    - » A and B are true if and only if neither A nor B is false
  - A | B = ~(~A & ~B)
    - » A or B are true if and only if A and B are not both false

Exclusive-Or using Inclusive Or

- A ^ B = (~A & B) | (A & ~B)
  - » Exactly one of A and B is true
- A ^ B = (A | B) & ~(A & B)
  - » Either A is true, or B is true, but not both

General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

All of the Properties of Boolean Algebra Apply

Representing & Manipulating Sets

Representation

- Width w bit vector represents subsets of {0, …, w–1}
- $a_j = 1$ if $j \in A$

```
01101001    { 0, 3, 5, 6 }
76543210

01010101    { 0, 2, 4, 6 }
76543210
```

Operations

- & Intersection 01000001 { 0, 6 }
- | Union 01111101 { 0, 2, 3, 4, 5, 6 }
- ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
- ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - `long, int, short, char, unsigned`
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- `~0x41 --> 0xBE`
  $\sim 01000001_2$ --> 10111110_2
- `~0x00 --> 0xFF`
  $\sim 00000000_2$ --> 11111111_2
- `0x69 & 0x55 --> 0x41`
  01101001_2 & 01010101_2 --> 01000001_2
- `0x69 | 0x55 --> 0x7D`
  01101001_2 | 01010101_2 --> 01111101_2

Contrast: Logic Operations in C

Contrast to Logical Operators

- `&&, ||, !`
  - View 0 as "False"
  - Anything nonzero as "True"
  - Always return 0 or 1
  - Early termination

Examples (char data type)

- `!0x41 --> 0x00`
- `!0x00 --> 0x01`
- `!!0x41 --> 0x01`
- `0x69 && 0x55 --> 0x01`
- `0x69 || 0x55 --> 0x01`
- `p && *p` (avoids null pointer access)

Shift Operations

Left Shift: `x << y`

- Shift bit-vector x left y positions
  - Throw away extra bits on left
  - Fill with 0's on right

Right Shift: `x >> y`

- Shift bit-vector x right y positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on right
  - Useful with two's complement integer representation

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Cool Stuff with Xor

- Bitwise Xor is form of addition
- With extra property that every value is its own additive inverse

  A ^ A = 0

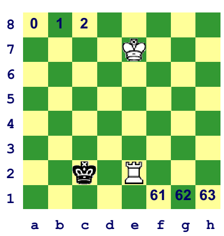
```
void funny(int *x, int *y)
{
    *x = *x ^ *y;    /* #1 */
    *y = *x ^ *y;    /* #2 */
    *x = *x ^ *y;    /* #3 */
}
```

	*x	*y
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A
3	(A^B)^A = B	A
End	B	A

More Fun with Bitvectors

Bit-board representation of chess position:

`unsigned long long blk_king, wht_king, wht_rook_mv2,…;`




```
wht_king     = 0x0000000000001000ull;
blk_king     = 0x0004000000000000ull;
wht_rook_mv2 = 0x10ef101010101010ull;
...
/*
 * Is black king under attach from
 * white rook ?
 */
if (blk_king & wht_rook_mv2)
      printf("Yes\n");
```


More Bitvector Magic

Count the number of 1's in a word

MIT Hackmem 169:

```
int bitcount(unsigned int n)
{
    unsigned int tmp;

    tmp = n - ((n >> 1) & 033333333333)
            - ((n >> 2) & 011111111111);
    return ((tmp + (tmp >> 3)) & 030707070707)%63;
}
```

Some Other Uses for Bitvectors

Representation of small sets

Representation of polynomials:
- Important for error correcting codes
- Arithmetic over finite fields, say GF(2^n)
- Example 0x15213 : $x^{16} + x^{14} + x^{12} + x^9 + x^4 + x + 1$

Representation of graphs
- A ‘1’ represents the presence of an edge

Representation of bitmap images, icons, cursors, …
- Exclusive-or cursor patent

Representation of Boolean expressions and logic circuits

Summary of the Main Points

It’s All About Bits & Bytes
- Numbers
- Programs
- Text

Different Machines Follow Different Conventions for
- Word size
- Byte ordering
- Representations

Boolean Algebra is the Mathematical Basis
- Basic form encodes “false” as 0, “true” as 1
- General form like bit-level operations in C
  - Good for representing & manipulating sets