Recitation 1

15-213/15-513/18-213 M12

Rick Benua

Outline

- Admin
- Data Lab
- Integers
- Floating Point

Recitations

- Tuesday / Wednesday / Thursday - pick one
- 4:30PM - 6:00PM EST
- Location
- Slides / Videos on course website
- Come with questions!

Office Hours

- TBA
- Any suggestions?
- IRC
  - irc.freenode.net
  - ##213
  - rbenua
- Staff Mailing List
  - 15-213-staff@cs.cmu.edu

Data Lab

- Extended until 23:59 EST Monday
  - 23:59 local time Tuesday for distance students
- Start now if you haven't!

Review of integers

Two types

- **Signed**
- **Unsigned**

Consider a *k*-bit unsigned integer

- **Bit *n* has value ____**
- **Minimum value is ____, and maximum value is ____**

Consider a *k*-bit signed integer

- **Bit *n* has value ____, except for most significant bit, which has value ____**
- **Minimum value is ____, and maximum value is ____**

Shifting

Two directions

- **Left: `x << k`**
- **Right: `x >> k`**

Two types

- **Arithmetic**
- **Logical**

Relationship with signed/unsigned

- **Which type of shift is used with each type of integer?**
- **Why?**

Thinking about shifting

Left shift

- Similar to which arithmetic operation?
- Completely identical?

Right shift

- Similar to which arithmetic operation?
- Completely identical?

The benefit of shifting

- Often faster than performing an actual arithmetic operation
- Good compilers will replace when appropriate on programmer's behalf

Nifty two's complement factoids

Negation

- Does `-x == ~x + 1`?
- Always?

Properties of zero

- What is the value of `x & 0`?
- What is the value of `x | 0`?
- What is the value of `x & (0 – 1)`?
- What is the value of `-0`?

Representation

- Basic format of bit representation (single precision):

Sign	Exponent	Fraction (Mantissa)
1-bit	8-bits	23-bits

$$-1^{s} * M * 2^{E}$$

Where E is based on the Bias

$$Bias = 2^{k-1} - 1 = 2^{8-1} - 1 = 127$$

k = exponent bits

Interpreting the Bits

Exponent →

- 0000 **Denormalized**
  - $E = -Bias + 1$
  - $Fraction = 0.FFF$
- EEEE **Normalized**
  - $E = Exponent - Bias$
  - $Fraction = 1.FFF$
- 1111 **Special**
  - Fraction →
    - 000 **Infinity**
      - Positive/Negative
    - FFF **NaN**

S EEEE FFF

8-bit floating point number

Rounding

- Round to even
  - Like regular rounding except for the exactly half case
  - If the last rounded bit is 1 round up, else round down

1.10 1001	Greater then 0.5, round up	1.11
1.10 0110	Less than 0.5, round down	1.10
1.11 1000	Round to even up	10.00
1.10 1000	Round to even down	1.10

Number to Float

- Convert: -5

S EEEE FFF

8-bit floating point number

Number to Float

- Convert: -5

$$Bias = 2^{4-1} - 1 = 2^3 - 1 = 7$$

- Negative so we know $S = 1$
- Turn 5 to bits:

$$5_{10} = 101_2$$

- Normalized value so lets fit in the leading 1

$$1.01_2$$

- Thus $F = 010$
- Now we figure out the exponent:

$$101_2 \rightarrow 1.01_2 \; x \; 2^2$$

Number to Float

- Calculate the exponent bits:

$$E = Exponent - Bias \rightarrow 2 = Exponent - 7 \rightarrow Exponent = 2 + 7 = 9$$

- So the exponent is 9, in bits:

$$Exponent = 1001_2$$

- Answer: $1\ 1001\ 010_2$

Number to Float

- Convert: 6/512

Number to Float

- Convert: 6/512
- Is it denormalized? Check the largest denormalized:

$$0\ 0000\ 111_2 = 0.111_2\ x\ 2^{-6} = 111_2\ x\ 2^{-9} = 7\ x\ 2^{-9} = 7/512$$

- Denormalized, we know the exponent is going to be:

$$E = -Bias + 1 = -7 + 1 = -6$$

- So we know the form of the answer is going to be:

$$0.FFF\ x\ 2^{-6}$$

- Lets remove the decimal point to make it a bit easier:

$$FFF\ x\ 2^{-9}$$

- The fraction bits are the top of the fraction:

$$6_{10} = 110_2$$

Number to Float

- Why does that work? Lets remove the decimal point to make it a bit easier to see:

$$0.110\ x\ 2^{-6} = 110\ x\ 2^{-9}$$

- Remembering that these are fractions:

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$

- We can see the table in terms on 512ths:

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}
$\frac{256}{512}$	$\frac{128}{512}$	$\frac{64}{512}$	$\frac{32}{512}$	$\frac{16}{512}$	$\frac{8}{512}$	$\frac{4}{512}$	$\frac{2}{512}$	$\frac{1}{512}$

- We want 6 512ths which are the bits we used.

Number to Float

- Putting it all together: 0 0000 110

Number to Float

- Convert: 27

Number to Float

- Convert: 27
- Positive so we know $S = 0$
- Turn 27 to bits:

$$27_{10} = 11011_2$$

- Normalized value so lets fit in the leading 1

$$1.1011_2$$

- But we only have 3 fraction bits so we must round. Digits after rounding equal half, last rounding digit is 1 so we round up.

$$1.1011_2 = 1.110_2$$

- Thus $F = 110$

Number to Float

- Calculate the exponent :

$$11100_2 \rightarrow 1.1100_2 \; x \; 2^4$$

- Calculate the exponent bits:

$$E = Exponent - Bias \rightarrow 4 = Exponent - 7 \rightarrow Exponent = 4 + 7 = 11$$

- So the exponent is 11, in bits:

$$Exponent = 1011_2$$

- Answer: $0\ 1011\ 110_2$

Float to Number

- Convert: $1\ 1001\ 010_2$

Float to Number

- Convert: $1\ 1001\ 010_2$
- Sign bit tells us it is negative
- We know it is normalized (non-zero exponent) so lets figure out the exponent:

$$1001_2 = 9_{10}$$

$$E = Exponent - Bias \rightarrow 9 - 7 = 2$$

- Now the fraction (remember the leading 1):

$$1.010_2$$

- Put it all together:

$$1.010_2\ x\ 2^2 = 101_2 = 5_{10}$$

- Answer: -5

Float to Number

- Convert: 0 0000 110

Float to Number

- Convert: 0 0000 110
- Sign bit tells us its positive
- It is denormalized because of the 0 exponent so lets figure out the exponent:

$$E = -Bias + 1 \rightarrow -7 + 1 = -6$$

- Now the fraction (remember the leading 0):

$$0.110 \; x \; 2^{-6}$$

- Put it all together:

$$0.110_2 \; x \; 2^{-6} = 0.000000110_2$$

Float to Number

- Put it all together:

$$0.110_2 \; x \; 2^{-6} = 0.000000110_2$$

- Now lets examine our fraction chart:

$\mathbf{2^{-1}}$	$\mathbf{2^{-2}}$	$\mathbf{2^{-3}}$	$\mathbf{2^{-4}}$	$\mathbf{2^{-5}}$	$\mathbf{2^{-6}}$	$\mathbf{2^{-7}}$	$\mathbf{2^{-8}}$	$\mathbf{2^{-9}}$
$\frac{256}{512}$	$\frac{128}{512}$	$\frac{64}{512}$	$\frac{32}{512}$	$\frac{16}{512}$	$\frac{8}{512}$	$\frac{4}{512}$	$\frac{2}{512}$	$\frac{1}{512}$

- Answer: 6/512

Float to Number

- Convert: $0\ 1011\ 110_2$

Float to Number

- Convert: $0\ 1011\ 110_2$
- Sign bit tells us it is positive
- We know it is normalized (non-zero exponent) so lets figure out the exponent:

$$1011_2 = 11_{10}$$

$$E = Exponent - Bias \rightarrow 11 - 7 = 4$$

- Now the fraction (remember the leading 1):

$$1.110_2$$

- Put it all together:

$$1.110_2\ x\ 2^4 = 11100_2 = 28_{10}$$

- Answer: 28

Questions?