15-213
"The Class That Gives CMU Its Zip!"

Bits and Bytes
Aug. 29, 2002

Topics
- Why bits?
- Representing information as bits
  - Binary/Hexadecimal
  - Byte representations
    - » numbers
    - » characters and strings
    - » Instructions
- Bit-level manipulations
  - Boolean algebra
  - Expressing in C

Why Don't Computers Use Base 10?

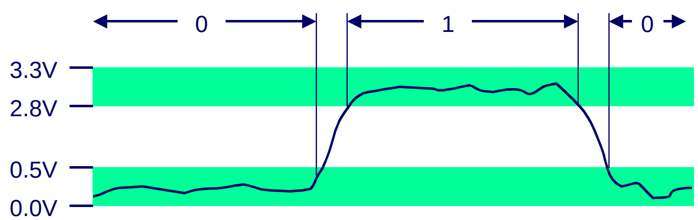
Base 10 Number Representation
- That's why fingers are known as "digits"
- Natural representation for financial transactions
  - Floating point number cannot exactly represent \$1.20
- Even carries through in scientific notation
  - 1.5213×10^4

Implementing Electronically
- Hard to store
  - ENIAC (First electronic computer) used 10 vacuum tubes / digit
- Hard to transmit
  - Need high precision to encode 10 signal levels on single wire
- Messy to implement digital logic functions
  - Addition, multiplication, etc.

Binary Representations

Base 2 Number Representation
- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Electronic Implementation
- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



Byte-Oriented Memory Organization

Programs Refer to Virtual Addresses
- Conceptually very large array of bytes
- Actually implemented with hierarchy of different memory types
  - SRAM, DRAM, disk
  - Only allocate for regions actually used by program
- In Unix and Windows NT, address space private to particular "process"
  - Program being executed
  - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation
- Where different program objects should be stored
- Multiple mechanisms: static, stack, and heap
- In any case, all allocation within single virtual address space

Encoding Byte Values

Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Machine Words

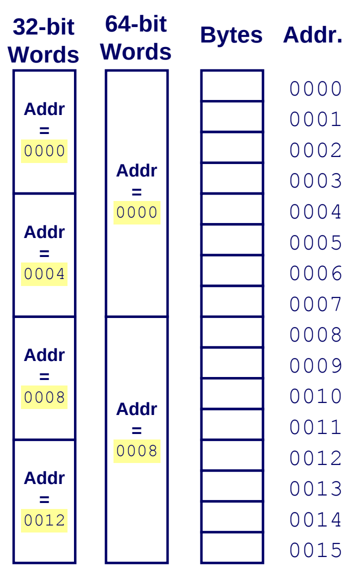
Machine Has “Word Size”

- Nominal size of integer-valued data
  - Including addresses
- Most current machines are 32 bits (4 bytes)
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- High-end systems are 64 bits (8 bytes)
  - Potentially address ≈ 1.8×10^{19} bytes
- Machines support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Compaq Alpha	Typical 32-bit	Intel IA32
int	4	4	4
long int	8	4	4
char	1	1	1
short	2	2	2
float	4	4	4
double	8	8	8
long double	8	8	10/12
char *	8	4	4

» Or any other pointer

Byte Ordering

How should bytes within multi-byte word be ordered in memory?

Conventions

- Sun's, Mac's are "Big Endian" machines
  - Least significant byte has highest address
- Alphas, PC's are "Little Endian" machines
  - Least significant byte has lowest address

Byte Ordering Example

Big Endian

- Least significant byte has highest address

Little Endian

- Least significant byte has lowest address

Example

- Variable `x` has 4-byte representation `0x01234567`
- Address given by `&x` is `0x100`

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

Reading Byte-Reversed Listings

Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add $0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl $0x0,0x28(%ebx)

Deciphering Numbers

- Value: `0x12ab`
- Pad to 4 bytes: `0x000012ab`
- Split into bytes: `00 00 12 ab`
- Reverse: `ab 12 00 00`

Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:
- `%p`: Print pointer
- `%x`: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

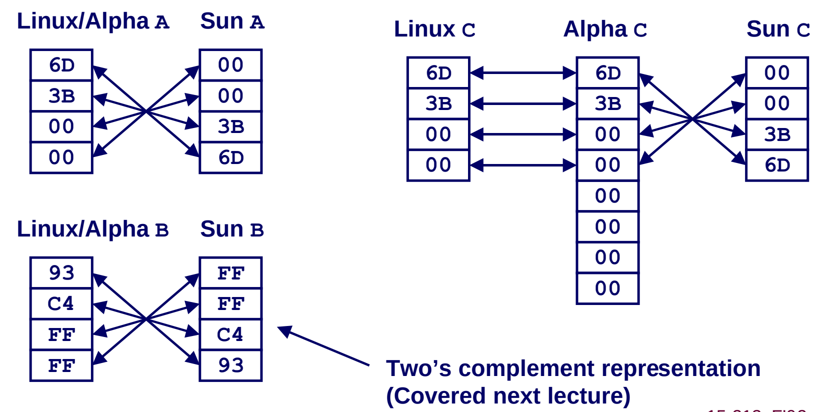
Result (Linux):

```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```


Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

Decimal:	15213
Binary:	0011 1011 0110 1101
Hex:	3 B 6 D



Representing Pointers

```
int B = -15213;
int *P = &B;
```

Alpha P: A0 FC FF FF 01 00 00 00

Alpha Address									
Hex:	1	F	F	F	F	F	C	A	0
Binary:	0001	1111	1111	1111	1111	1111	1100	1010	0000

Sun P: EF FF FB 2C

Sun Address								
Hex:	E	F	F	F	F	B	2	C
Binary:	1110	1111	1111	1111	1111	1011	0010	1100

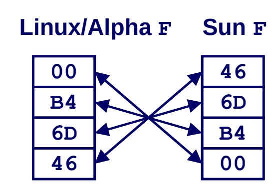
Linux P: D4 F8 FF BF

Linux Address								
Hex:	B	F	F	F	F	8	D	4
Binary:	1011	1111	1111	1111	1111	1000	1101	0100

Different compilers & machines assign different locations to objects

Representing Floats

```
Float F = 15213.0;
```



IEEE Single Precision Floating Point Representation								
Hex:	4	6	6	D	B	4	0	0
Binary:	0100	0110	0110	1101	1011	0100	0000	0000
15213:			1110	1101	1011	01		

Not same as integer representation, but consistent across machines

Can see some relation to integer representation, but not obvious

Representing Strings

```
char S[6] = "15213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Other encodings exist, but uncommon
  - Character "0" has code `0x30`
    - » Digit *i* has code `0x30`+*i*
- String should be null-terminated
  - Final character = 0

Linux/Alpha `S`		Sun `S`
31	↔	31
35	↔	35
32	↔	32
31	↔	31
33	↔	33
00	↔	00

Compatibility

- Byte ordering not an issue
  - Data are single byte quantities
- Text files generally platform independent
  - Except for different conventions of line termination character(s)!

Machine-Level Code Representation

Encode Program as Sequence of Instructions

- Each simple operation
  - Arithmetic operation
  - Read or write memory
  - Conditional branch
- Instructions encoded as bytes
  - Alpha's, Sun's, Mac's use 4 byte instructions
    - » Reduced Instruction Set Computer (RISC)
  - PC's use variable length instructions
    - » Complex Instruction Set Computer (CISC)
- Different instruction types and encodings for different machines
  - Most code not binary compatible

Programs are Byte Sequences Too!

Representing Instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
  - Use differing numbers of instructions in other cases
- PC uses 7 instructions with lengths 1, 2, and 3 bytes
  - Same for NT and for Linux
  - NT / Linux not fully binary compatible

Alpha `sum`	Sun `sum`	PC `sum`
00	81	55
00	C3	89
30	E0	E5
42	08	8B
01	90	45
80	02	0C
FA	00	03
6B	09	45
		08
		89
		EC
		5D
		C3

Different machines use totally different instructions and encodings

Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode "True" as 1 and "False" as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

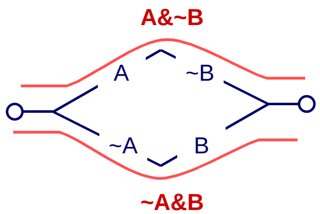
- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

Applied to Digital Systems by Claude Shannon

- 1937 MIT Master's Thesis
- Reason about networks of relay switches
  - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Integer Algebra

Integer Arithmetic

- ⟨Z, +, *, –, 0, 1⟩ forms a "ring"
- Addition is "sum" operation
- Multiplication is "product" operation
- – is additive inverse
- 0 is identity for sum
- 1 is identity for product

Boolean Algebra

Boolean Algebra

- ⟨{0,1}, |, &, ~, 0, 1⟩ forms a "Boolean algebra"
- Or is "sum" operation
- And is "product" operation
- ~ is "complement" operation (not additive inverse)
- 0 is identity for sum
- 1 is identity for product

Boolean Algebra ≈ Integer Ring

- *Commutativity*

A | B = B | A	A + B = B + A
A & B = B & A	A * B = B * A

- *Associativity*

(A | B) | C = A | (B | C)	(A + B) + C = A + (B + C)
(A & B) & C = A & (B & C)	(A * B) * C = A * (B * C)

- *Product distributes over sum*

A & (B | C) = (A & B) | (A & C)	A * (B + C) = A * B + B * C

- *Sum and product identities*

A | 0 = A	A + 0 = A
A & 1 = A	A * 1 = A

- *Zero is product annihilator*

A & 0 = 0	A * 0 = 0

- *Cancellation of negation*

~ (~ A) = A	– (– A) = A

Boolean Algebra ≠ Integer Ring

- Boolean: *Sum distributes over product*
  - A | (B & C) = (A | B) & (A | C) A + (B * C) ≠ (A + B) * (B + C)
- Boolean: *Idempotency*
  - A | A = A A + A ≠ A
  - "A is true" or "A is true" = "A is true"
  - A & A = A A * A ≠ A
- Boolean: *Absorption*
  - A | (A & B) = A A + (A * B) ≠ A
  - "A is true" or "A is true and B is true" = "A is true"
  - A & (A | B) = A A * (A + B) ≠ A
- Boolean: *Laws of Complements*
  - A | ~A = 1 A + –A ≠ 1
  - "A is true" or "A is false"
- Ring: *Every element has additive inverse*
  - A | ~A ≠ 0 A + –A = 0

Properties of & and ^

Boolean Ring

- ⟨{0,1}, ^, &, *I*, 0, 1⟩
- Identical to integers mod 2
- *I* is identity operation: *I* (A) = A
  - A ^ A = 0

Property	Boolean Ring
Commutative sum	A ^ B = B ^ A
Commutative product	A & B = B & A
Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
Associative product	(A & B) & C = A & (B & C)
Prod. over sum	A & (B ^ C) = (A & B) ^ (B & C)
0 is sum identity	A ^ 0 = A
1 is prod. identity	A & 1 = A
0 is product annihilator	A & 0 = 0
Additive inverse	A ^ A = 0

Relations Between Operations

DeMorgan's Laws

- Express & in terms of |, and vice-versa
  - A & B = ~(~A | ~B)
    - » A and B are true if and only if neither A nor B is false
  - A | B = ~(~A & ~B)
    - » A or B are true if and only if A and B are not both false

Exclusive-Or using Inclusive Or

- A ^ B = (~A & B) | (A & ~B)
  - » Exactly one of A and B is true
- A ^ B = (A | B) & ~(A & B)
  - » Either A is true, or B is true, but not both

General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

All of the Properties of Boolean Algebra Apply

Representing & Manipulating Sets

Representation

- Width *w* bit vector represents subsets of {0, …, *w*–1}
- $a_j = 1$ if $j \in A$

  `01101001` { 0, 3, 5, 6 }
  `76543210`

  `01010101` { 0, 2, 4, 6 }
  `76543210`

Operations

- & Intersection `01000001` { 0, 6 }
- | Union `01111101` { 0, 2, 3, 4, 5, 6 }
- ^ Symmetric difference `00111100` { 2, 3, 4, 5 }
- ~ Complement `10101010` { 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - `long, int, short, char`
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- `~0x41 --> 0xBE`

  ~01000001_2 --> 10111110_2
- `~0x00 --> 0xFF`

  ~00000000_2 --> 11111111_2
- `0x69 & 0x55 --> 0x41`

  01101001_2 & 01010101_2 --> 01000001_2
- `0x69 | 0x55 --> 0x7D`

  01101001_2 | 01010101_2 --> 01111101_2

Contrast: Logic Operations in C

Contrast to Logical Operators

- `&&, ||, !`
  - View 0 as "False"
  - Anything nonzero as "True"
  - Always return 0 or 1
  - Early termination

Examples (char data type)

- `!0x41 --> 0x00`
- `!0x00 --> 0x01`
- `!!0x41 --> 0x01`

- `0x69 && 0x55 --> 0x01`
- `0x69 || 0x55 --> 0x01`
- `p && *p` (avoids null pointer access)

Shift Operations

Left Shift: `x << y`

- Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
  - Fill with 0's on right

Right Shift: `x >> y`

- Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on right
  - Useful with two's complement integer representation

Argument `x`	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument `x`	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

Cool Stuff with Xor

- Bitwise Xor is form of addition
- With extra property that every value is its own additive inverse

  A ^ A = 0

```
void funny(int *x, int *y)
{
   *x = *x ^ *y;    /* #1 */
   *y = *x ^ *y;    /* #2 */
   *x = *x ^ *y;    /* #3 */
}
```

	*x	*y
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A
3	(A^B)^A = B	A
End	B	A

Main Points

It's All About Bits & Bytes

- Numbers
- Programs
- Text

Different Machines Follow Different Conventions

- Word size
- Byte ordering
- Representations

Boolean Algebra is Mathematical Basis

- Basic form encodes "false" as 0, "true" as 1
- General form like bit-level operations in C
  - Good for representing & manipulating sets