Review for first midterm

February 3, 1998
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15-212 Section B

1. *Turing machines*: Give an informal description of a Turing machine which solves the *elemental distinctness problem*: given a list of strings over $\{0,1\}$ separated by #s, accept if and only if all the strings are different. That is,

$$E = \{\#x_1\#x_2\#x_3\ldots\#x_l \mid \text{ each } x_i \in \{0,1\}^\star \text{ and } x_i \neq x_j \text{ for each } i \neq j\}$$

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2. *Ambiguity in grammars*: Consider the following grammar:

$$E \rightarrow E + E \mid E \times E \mid (E) \mid a$$

This grammar is ambiguous. Provide a string which may be parsed in multiple ways and show two different parses. Can we "fix" this grammar to make it non-ambiguous?

3. *Infinities*: Let $\mathcal{Q}$ be the set of all rational numbers (i.e. fractions). Is $\mid \mathcal{Q} \mid = \aleph_0$? Why or why not? Now let L be the set of all languages over $\{0,1\}$. Is $\mid L \mid = \aleph_0$?

4. *Decidability*: Let $\infty_{\mathrm{DFA}} = \{M \mid L(M) \text{ is an infinite language }\}$. Show that ∞_{DFA} is decidable.

5. *Pumping lemma*: State precisely the pumping lemma for regular languages. Provide an informal proof.

6. Show that none of the following are any more powerful (in terms of the class of languages accepted) than a normal Turing machine:

   - A Turing machine with a two-way infinite tape
   - A k-tape Turing machine
   - A random-access Turing machine
   - a Turing machine with one tape, but multiple heads
   - a non-deterministic Turing machine

7. *A toughie*: Say that a **write-twice Turing machine** is a single-tape TM that can alter each tape square at most twice. Show that this kind of TM is as powerful as the standard TM.