Recitation 9

Graph Search: BFS and DFS

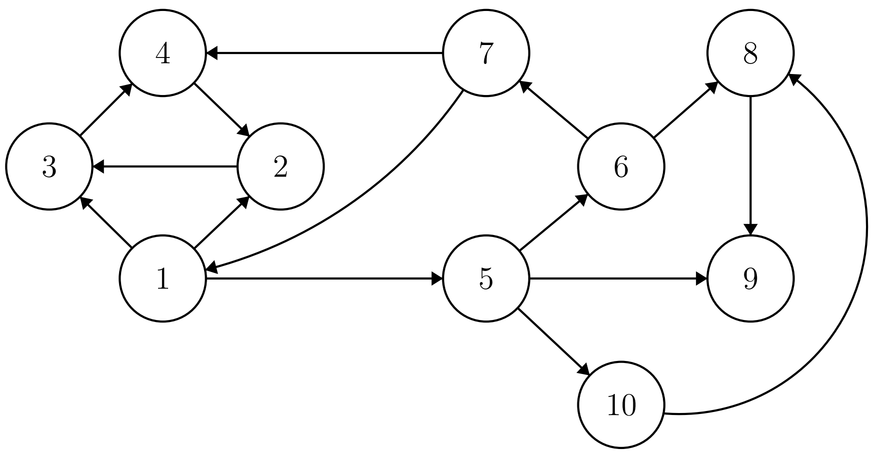
9.1 Announcements

- *BridgeLab* has been released, and is worth 140 points. The due dates are a bit wonky because of Spring Break: the written section is due at Friday at 5pm, while the programming portion is due Sunday night.
- *ShortLab* will be released on **Friday**.

9.2 DFS Trees and Numberings

Task 9.1. *Starting at vertex 1, execute DFS on the following graph, visiting vertices in increasing order. Trace the process by doing each of the following.*

1. *Draw the resulting DFS tree. Draw tree edges as solid lines, and include non tree edges in your drawing as dashed lines.*
2. *Classify each non tree edge as one of* ***forward***, ***back***, *or* ***cross***.
3. *Label each vertex with its discovery and finish times.*



Task 9.2. *Suppose DFS is run on a directed graph, and consider some edge (x, y). Using the discovery and finish times of x and y, attempt to classify this edge as one of* tree, forward, back, *or* cross.

9.2.1 Higher-Order DFS

Recall the following code from the textbook:

Algorithm 9.3. *Directed, generalized DFS.*

```
directedDFS (revisit,discover,finish) (G,Σ_0,s) =
  let
    DFS p ((X,Σ),v) =
      if (v ∈ X) then (X,revisit (Σ,v,p)) else
      let
        Σ' = discover (Σ,v,p)
        X' = X ∪ {v}
        (X'',Σ'') = iterate (DFS v) (X',Σ') (N_G^+(v))
        Σ''' = finish (Σ',Σ'',v,p)
      in
        (X'',Σ''')
      end
  in
    DFS s (({},Σ_0),s)
  end
```

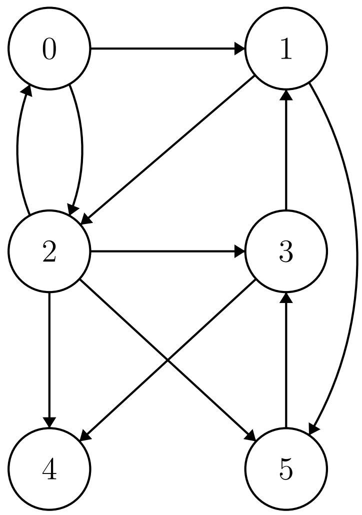
Task 9.4. *Define* Σ_0, `revisit`, `discover`, *and* `finish` *to calculate DFS numberings.*

Task 9.5. *Modify the given generalized DFS code to work with undirected graphs.*

(Hint: We only want to traverse each edge once! Try implementing undirected cycle detection with the above algorithm and see where it fails.)

9.3 BFS

9.3.1 An Example



Task 9.6. *Run BFS on the example graph above, starting at vertex 1. Draw the resulting BFS tree. Draw tree edges as solid lines and non-tree edges as dashed lines.*

9.3.2 Implementation

Consider the following code, which computes the BFS tree of an enumerated graph represented by an adjacency sequence. For brevity, we'll write `NONE` as $\boxed{}$ and (`SOME` x) as $\boxed{x}$.

Algorithm 9.7. *Computing BFS trees on adjacency sequences.*

```
fun BFS (G, s) =
  let
    fun BFS' (X_i, F_i) =
      if |F_i| = 0 then STSeq.toSeq X_i else
      let
        val N_i =
          Seq.flatten ⟨⟨(u, [v]) : u ∈ G[v] | X_i[u] = [ ]⟩ : v ∈ F_i⟩
        val X_{i+1} = STSeq.inject (X_i, N_i)
        val F_{i+1} = ⟨u : (u, v) ∈ N_i | X_{i+1}[u] = [v]⟩
      in
        BFS' (X_{i+1}, F_{i+1})
      end

    val init = STSeq.fromSeq ⟨[ ] : 0 ≤ i < |G|⟩
    val X_0 = STSeq.update (init, (s, [s]))
    val F_0 = ⟨s⟩
  in
    BFS' (X_0, F_0)
  end
```

Task 9.8. *Execute this code on the example graph given in the first section, starting with vertex 1 as the source. Trace the process by writing down the values X_i, F_i, and N_i for $i = 0, 1, 2, 3$.*

Task 9.9. *Analyze the work and span of this implementation in terms of n (the number of vertices), m (the number of edges), and d (the diameter of the graph).*

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