Recitation 7

Combining BSTs

7.1 Announcements

- *FingerLab* has been released, and is due **Friday afternoon**. It’s worth 125 points.
- *RangeLab* will be released on **Friday**.
7.2 Generalized Combination

In lecture, we discussed union, and argued that it has $O(m \log \left( \frac{n}{m} + 1 \right))$ work and $O(\log(n) \log(m))$ span. The latter bound can be improved to $O(\log n + \log m)$ using futures\(^1\), but that is outside the scope of this course.

What about the functions intersection and difference? These can be implemented in a similar fashion as union, and as such have the same cost bounds. In this recitation, we’ll establish this more concretely.

**Task 7.1.** Implement all three functions union, intersection, and difference in terms of a single helper function combine which has $O(m \log \left( \frac{n}{m} + 1 \right))$ work and $O(\log(n) \log(m))$ span for BSTs of size $n$ and $m$, $n \geq m$. Conclude that all three of these functions have the same cost bounds.

Let’s begin by inspecting the code for union.

**Algorithm 7.2.** BST union.

```plaintext
fun union (T1, T2) =
  case (T1, T2) of
  (_, Leaf) ⇒ T1
  | (Leaf, _) ⇒ T2
  | (Node (L1, x, R1), _) ⇒
    let val (L2, _, R2) = split (T2, x)
    in joinMid (L, x, R)
    end
```

What do we have to change to generalize this? Notice that, for example, intersection returns Leaf in both base cases, while difference only returns Leaf in the second case. Next, consider that intersection only keeps the key $x$ if it is also present in $T_2$, and difference specifically removes $x$ if it is present in $T_2$. We can account for all of these differences by introducing new arguments which specify what to do in the base cases, and whether or not we should keep $x$ in the recursive case (based on whether or not it is present in $T_2$).

\(^1\)http://dl.acm.org/citation.cfm?id=258517

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Algorithm 7.3. Generalized BST combine.

```ocaml
fun combine f1 f2 k =
  let
    fun combine' (T1, T2) =
      case (T1, T2) of
        (_, Leaf) ⇒ f1(T1)
      | (Leaf, _) ⇒ f2(T2)
      | (Node (L1, x, R1), _) ⇒
        let val (L2, b, R2) = split (T2, x)
        in
          if k(b) then joinMid (L, x, R)
          else join (L, R)
        end
    end
  in
  combine'
end
val union =
  combine (fn T1 ⇒ T1) (fn T2 ⇒ T2) (fn b ⇒ true)
val intersection =
  combine (fn T1 ⇒ Leaf) (fn T2 ⇒ Leaf) (fn b ⇒ b)
val difference =
  combine (fn T1 ⇒ T1) (fn T2 ⇒ Leaf) (fn b ⇒ not b)
```

Task 7.4. Consider a function `symdiff` where `(symdiff (A, B))` returns a BST containing all keys which are either in A or B, but not both. Implement `symdiff` in terms of `combine`.

```ocaml
val symdiff = combine (fn T1 ⇒ T1) (fn T2 ⇒ T2) (fn b ⇒ not b)
```