Recitation 12

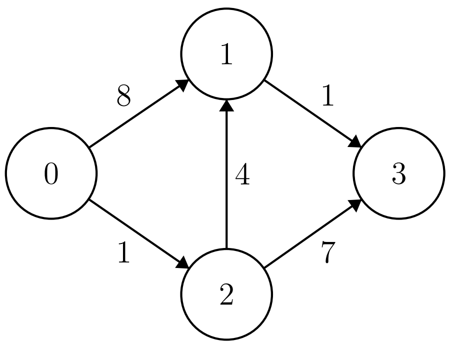
Dynamic Programming

12.1 Announcements

- *Midterm 2* is on **Friday**.

12.2 Can We Solve SSSP With Dynamic Programming?

Task 12.1. *Let $\delta(v, k)$ be the shortest path distance between the source and v using at most k edges. For the example graph shown, fill in the table below with values $\delta(v, k)$ using 0 as the source. If a vertex v is unreachable from the source using at most k edges, then write $\delta(v, k) = \infty$.*



		v			
		0	1	2	3
	0				
	1				
k	2				
	3				

Task 12.2. *What are the values $\delta(v, 0)$ for every v?*

Task 12.3. *Write $\delta(v, k)$ in terms of $\delta(\cdot, k-1)$. (Intuition: if we know shortest path distances using at most $k-1$ edges, can we easily calculate all shortest path distances which use at most k edges? We only need to extend each shortest path by one edge.)*

Task 12.4. *Assuming the graph contains no negative cycles, prove the following statement:*

> *For each vertex, there exists a shortest path to it from the source using at most $|V| - 1$ edges.*

What does this statement tell us about using $\delta(v, k)$ to solve the SSSP problem?

Task 12.5. *Using what's been established above, write a top-down dynamic programming solution to the SSSP problem. Note that your result will essentially be a top-down version of the well-known* Bellman-Ford *algorithm.*

12.2.1 Cost Analysis

Task 12.6. *Describe the DAG of dependencies between subproblems $\delta(v, k)$; i.e., each vertex is a pair (v, k), and there is an arc from (v', k') to (v, k) if we need to know the value $\delta(v', k')$ in order to calculate $\delta(v, k)$.*

Draw the dependency DAG for the example graph given above.

Task 12.7. *Identify a bottom-up ordering of the Bellman-Ford DAG which maximizes parallelism while respecting subproblem dependencies. Use this bottom-up ordering to determine the work and span of Bellman-Ford in terms of n, the number of vertices, and m, the number of edges. You may assume constant-time access to subproblems you have already computed.*

12.2.2 Is Bellman-Ford A Useful Algorithm?

Task 12.8. *Recall that the work bound of Dijkstra's algorithm as presented in class was $O(m \log n)$. For certain priority queue implementations, this can be reduced even to $O(m + n \log n)$. Bellman-Ford is clearly much slower. In what situations might you want to use Bellman-Ford instead of Dijkstra's algorithm?*

Task 12.9. *Describe a simple modification to Bellman-Ford's algorithm which can be used to detect the presence of a negative cycle.*

Task 12.10. *Describe an optimization for Bellman-Ford which (assuming there are no negative cycles) causes it terminate after ℓ rounds, where ℓ is the maximum number of edges used in any of the shortest paths.*

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