# Model Checking II
# Temporal Logic Model Checking

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Specification Language: A propositional temporal logic.

Verification Procedure: Exhaustive search of the state space of the system to determine if the specification is true or not.


- E. M. Clarke and E. A. Emerson. Synthesis of synchronization skeletons for branching time temporal logic. In *Logic of programs: workshop, Yorktown Heights, NY, May 1981*, volume 131 of *Lecture Notes in Computer Science*. Springer-Verlag, 1981.
- J.P. Quielle and J. Sifakis. Specification and verification of concurrent systems in CESAR. In *Proceedings of the Fifth International Symposium in Programming*, volume 137 of *Lecture Notes in Computer Science*. Springer-Verlag, 1981.

# Why Model Checking?

Advantages:

- No proofs!
- Fast
- Counter-examples
- No problem with partial specifications
- Logics can easily express many concurrency properties

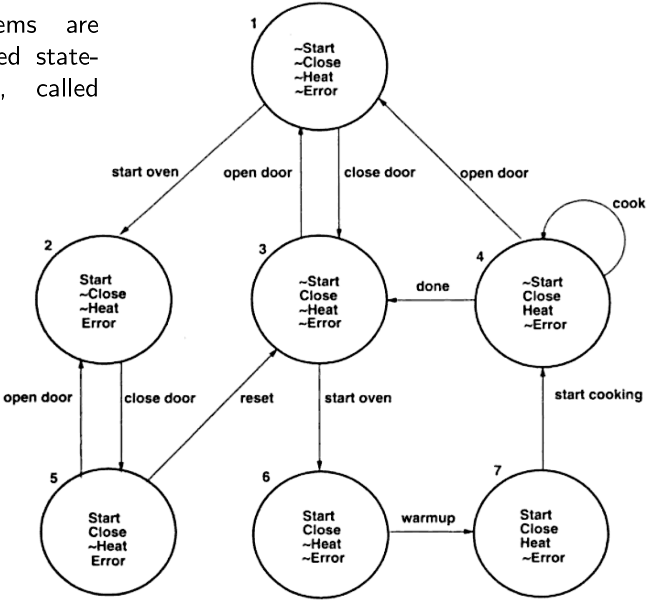
Main Disadvantage: *State Explosion Problem*

- Too many processes
- Data Paths

Much progress has been made on this problem recently!

# Model of Computation; Microwave Example

Finite-state systems are modeled by labeled state-transition graphs, called *Kripke Structures*.

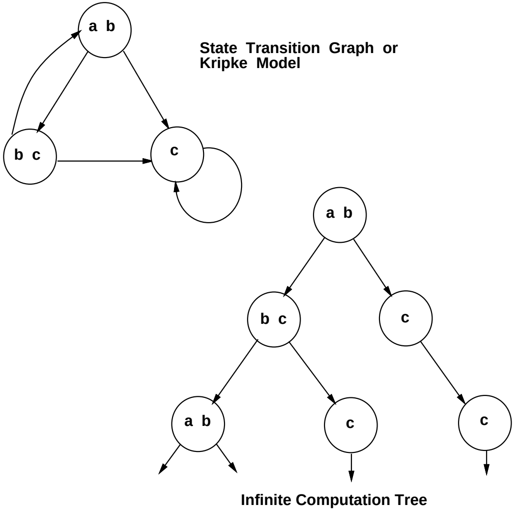


# Model of Computation (Cont.)

If some state is designated as the *initial state*, the structure can be unwound into an infinite tree with that state as the root.

We will refer to this infinite tree as the *computation tree* of the system.

Paths in the tree represent possible computations or behaviors of the program.




(Unwind State Graph to obtain Infinite Tree)

# Model of Computation (Cont.)

Formally, a *Kripke structure* is a triple $M = \langle S, R, L \rangle$, where

- $S$ is the set of states,
- $R \subseteq S \times S$ is the transition relation, and
- $L : S \rightarrow \mathcal{P}(AP)$ gives the set of atomic propositions true in each state.

We assume that every state has at least one possible successor (i.e., for all states $s \in S$ there exists a state $s' \in S$ such that $(s, s') \in R$).

A *path in M* is an infinite sequence of states, $\pi = s_0, s_1, \ldots$ such that for $i \geq 0$, $(s_i, s_{i+1}) \in R$.

We write $\pi^i$ to denote the *suffix* of $\pi$ starting at $s_i$.

Unless otherwise stated, we assume *finite* Kripke structures.

# Computation Tree Logics

Temporal logics may differ according to how they handle branching in the underlying computation tree.

In a linear temporal logic, operators are provided for describing events along a single computation path.

In a branching-time logic the temporal operators quantify over the paths that are possible from a given state.

# The Logic CTL*

The computation tree logic CTL* (pronounced "CTL star") combines both branching-time and linear-time operators.

In this logic a *path quantifier* can prefix an assertion composed of arbitrary combinations of the usual *linear-time operators*.

1. Path quantifiers:
   - **A** — "for every path"
   - **E** — "there exists a path"
2. Linear-time operators:
   - **X** $p$ — $p$ holds true *next* time.
   - **F** $p$ — $p$ holds true sometime in the *future*
   - **G** $p$ — $p$ holds true *globally* in the future
   - $p$ **U** $q$ — $p$ holds true *until* $q$ holds true

For a path $\pi = (s_0, s_1, \ldots)$, state $s_0$ is considered to be at the present time.

# Path Formulas and State Formulas

The syntax of state formulas is given by the following rules:

- If $p$ is an atomic proposition, then $p$ is a state formula.
- If $f$ and $g$ are state formulas, then $\neg f$ and $f \vee g$ are state formulas.
- If $f$ is a path formula, then $\mathbf{E}(f)$ and $\mathbf{A}(f)$ are state formulas.

Two additional rules are needed to specify the syntax of path formulas:

- If $f$ is a state formula, then $f$ is also a path formula.
  (A state formula $f$ is true for a path $\pi$ if the $f$ is true in the initial state of the path $\pi$.)
- If $f$ and $g$ are path formulas, then $\neg f$, $f \vee g$, $\mathbf{X}\, f$, $\mathbf{F}\, f$, $\mathbf{G}\, f$, and $f\, \mathbf{U}\, g$ are path formulas.

# State Formulas (Cont.)

If $f$ is a state formula, the notation $\boxed{M, s \models f}$ means that $f$ holds at state $s$ in the Kripke structure $M$.

Assume $f_1$ and $f_2$ are state formulas and $g$ is a path formula. The relation $M, s \models f$ is defined inductively as follows:

1. $s \models p$ $\Leftrightarrow$ atomic proposition $p$ is true in $s$ .
2. $s \models \neg f_1$ $\Leftrightarrow$ $s \not\models f_1$.
3. $s \models f_1 \vee f_2$ $\Leftrightarrow$ $s \models f_1$ or $s \models f_2$.
4. $s \models \mathbf{E}(g)$ $\Leftrightarrow$ $g$ holds true for some path $\pi$ starting with $s$
4. $s \models \mathbf{A}(g)$ $\Leftrightarrow$ $g$ holds true for every path $\pi$ starting with $s$

# Path Formulas (Cont.)

If $f$ is a path formula, the notation $\boxed{M, \pi \models f}$ means that $f$ holds true for path $\pi$ in Kripke structure $M$.

Assume $g_1$ and $g_2$ are path formulas and $f$ is a state formula. The relation $M, \pi \models f$ is defined inductively as follows:

1. $\pi \models f \quad \Leftrightarrow \quad s$ is the first state of $\pi$ and $s \models f$.
2. $\pi \models \neg g_1 \quad \Leftrightarrow \quad \pi \not\models g_1$.
3. $\pi \models g_1 \vee g_2 \quad \Leftrightarrow \quad \pi \models g_1$ or $\pi \models g_2$.
4. $\pi \models \mathbf{X}\, g_1 \quad \Leftrightarrow \quad \pi^1 \models g_1$.
5. $\pi \models \mathbf{F}\, g_1 \quad \Leftrightarrow \quad \pi^k \models g_1$ for some $k \geq 0$
6. $\pi \models \mathbf{G}\, g_1 \quad \Leftrightarrow \quad \pi^k \models g_1$ for every $k \geq 0$
7. $\pi \models g_1 \,\mathbf{U}\, g_2 \quad \Leftrightarrow \quad$ there exists a $k \geq 0$ such that $\pi^k \models g_2$ and $\pi^j \models g_1$ for $0 \leq j < k$.

Recall: For $\pi = (s_0, s_1, \ldots)$, we write $\pi^i$ to denote the suffix starting with $s_i$.

# Time

Notice that $\mathbf{F}p$, $\mathbf{F}\mathbf{F}p$, $\mathbf{F}\mathbf{F}\mathbf{F}p$, etc., hold true for a path $\pi$ even if $p$ holds true at only the initial state in the path $\pi$.

- In CTL$^*$, the 'future' includes the present state. (States have temporal duration, so if we're presently in state $s$ at time $t$, then we'll still be in state $s$ in the future at time $t + dt$ where $dt$ is an infinitesimally small period of time.)

# Relationships between operators

Note the following:

- $\mathbf{A}(f) \equiv \neg\,\mathbf{E}(\neg f)$
- $\mathbf{F}\,f \equiv (true\,\mathbf{U}\,f)$

  (Recall: $\pi \models g_1\,\mathbf{U}\,g_2 \;\Leftrightarrow\;$ there exists a $k \geq 0$ such that $\pi^k \models g_2$ and $\pi^j \models g_1$ for $0 \leq j < k$.)
- $\mathbf{G}\,f \equiv \neg\,\mathbf{F}\,\neg f$

So, given any CTL* formula, we can rewrite it without using the operators **A**, **F**, or **G**.

# The Logic CTL

CTL is a restricted subset of CTL* that permits only branching-time operators—each of the linear-time operators **G**, **F**, **X**, and **U** must be immediately preceded by a path quantifier.

More precisely, CTL is the subset of CTL* that is obtained if the following two rules are used to specify the syntax of path formulas.

- If $f$ and $g$ are state formulas, then $\mathbf{X}\, f$ and $f\ \mathbf{U}\ g$ are path formulas.
- If $f$ is a path formula, then so is $\neg f$.

Example: $\mathbf{AG}(\mathbf{EF}\, p)$

# The Logic LTL

Linear temporal logic (LTL), on the other hand, consists of formulas that have the form **A** $f$ where $f$ is a path formula in which the only state subformulas permitted are atomic propositions.

More precisely, a path formula is either:

- If $p \in AP$, then $p$ is a path formula.
- If $f$ and $g$ are path formulas, then $\neg f$, $f \vee g$, **X** $f$, and $f$ **U** $g$ are path formulas.

Example: $\mathbf{A}(\mathbf{FG}\, p)$

# Expressive Power

It can be shown that the three logics discussed in this section have different expressive powers.

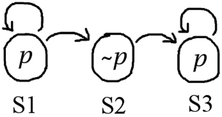
For example, there is no CTL formula that is equivalent to the LTL formula $\mathbf{A}(\mathbf{FG}\, p)$.

Likewise, there is no LTL formula that is equivalent to the CTL formula $\mathbf{AG}(\mathbf{EF}\, p)$.

The disjunction $\mathbf{A}(\mathbf{FG}\, p) \vee \mathbf{AG}(\mathbf{EF}\, p)$ is a CTL$^*$ formula that is not expressible in either CTL or LTL.

# AF(AG(p)) vs A(FG(p))

Consider the following Kripke structure:




Are there any paths starting with $S_1$ for which **G** $p$ is true?
Starting with $S_2$?
Starting with $S_3$?

At which states does **AG** $p$ hold true?

At which states does **AF AG** $p$ hold true?

Does **F G** $p$ hold true for all paths starting with $S_1$?

# Basic CTL Operators

There are eight basic CTL operators:

- **AX** and **EX**,
- **AG** and **EG**,
- **AF** and **EF**,
- **AU** and **EU**

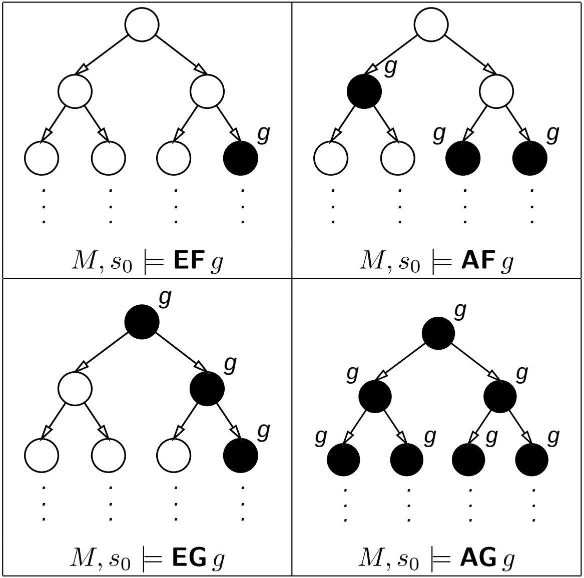
Each of these can be expressed in terms of **EX**, **EG**, and **EU**:

- $\mathbf{AX}\, f = \neg\, \mathbf{EX}(\neg f)$
- $\mathbf{AG}\, f = \neg\, \mathbf{EF}(\neg f)$
- $\mathbf{AF}\, f = \neg\, \mathbf{EG}(\neg f)$
- $\mathbf{EF}\, f = \mathbf{E}[true\ \mathbf{U}\ f]$
- $\mathbf{A}[f\ \mathbf{U}\ g] \equiv \neg\, \mathbf{E}[\neg g\ \mathbf{U}\ \neg f \wedge \neg g] \wedge \neg\, \mathbf{EG}\, \neg g$

# Basic CTL Operators (Cont.)

The four most widely used CTL operators are illustrated here.

Each computation tree has the state $s_0$ as its root.



# Typical CTL* formulas

- $\textbf{EF}(Started \wedge \neg Ready)$: It is possible to get to a state where *Started* holds but *Ready* does not hold.
- $\textbf{AG}(Req \rightarrow \textbf{AF}\, Ack)$: If a request occurs, then it will be eventually acknowledged.
- $\textbf{AG}(\textbf{AF}\, DeviceEnabled)$: The proposition *DeviceEnabled* holds infinitely often on every computation path.
- $\textbf{AG}(\textbf{EF}\, Restart)$: From any state it is possible to get to the *Restart* state.
- $\textbf{A}(\textbf{GF}\, enabled \Rightarrow \textbf{GF}\, executed)$: if a process is infinitely-often *enabled*, then it is infinitely-often *executed*.

Note that the first four formulas are CTL formulas.

# Model Checking Problem

Let $M$ be the state–transition graph obtained from the concurrent system.

Let $f$ be the specification expressed in temporal logic.

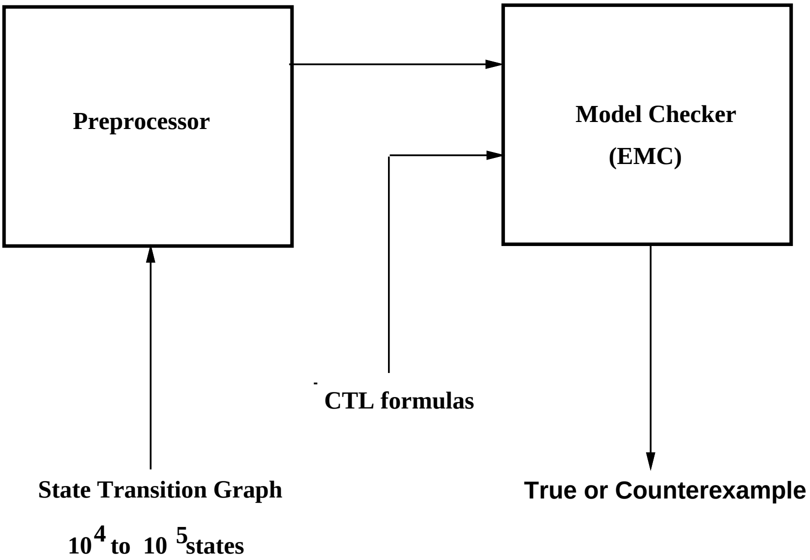
Find all states $s$ of $M$ such that

$$M, s \models f.$$

There exist very efficient model checking algorithms for the logic CTL.

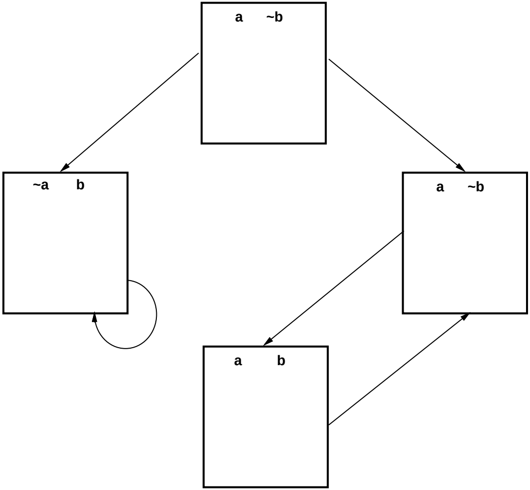

- E. M. Clarke, E. A. Emerson, and A. P. Sistla. Automatic verification of finite-state concurrent systems using temporal logic specifications. *ACM Trans. Programming Languages and Systems*, 8(2):pages 244–263, 1986.

# The EMC Verification System



# Basic Model Checking Algorithm

- $M, s_0 \models \textbf{EG}\, a \wedge \textbf{AF}\, b$?
- $M, s_0 \models \neg\, \textbf{AF}\, \neg a \wedge \textbf{AF}\, b$?



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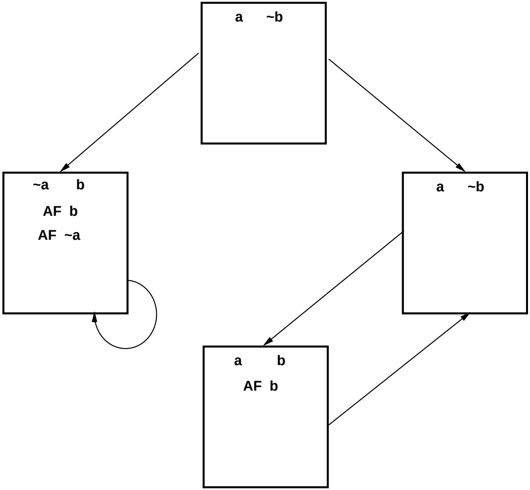

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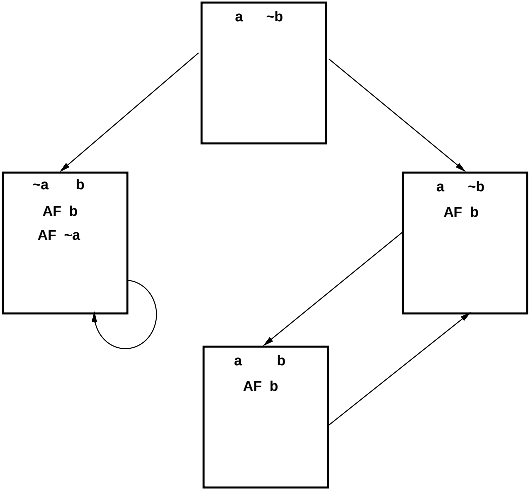

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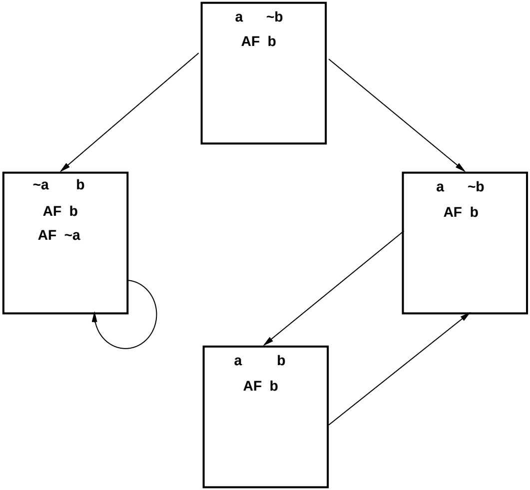

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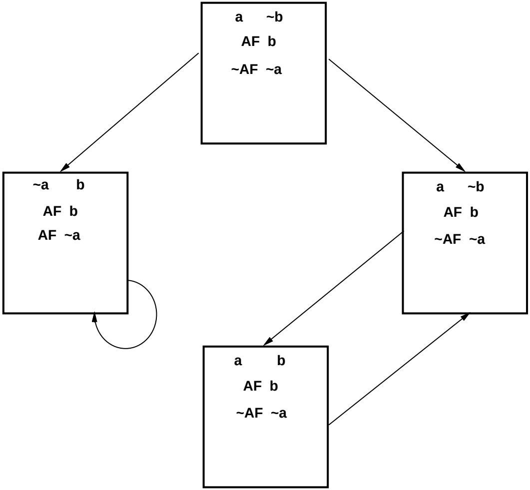
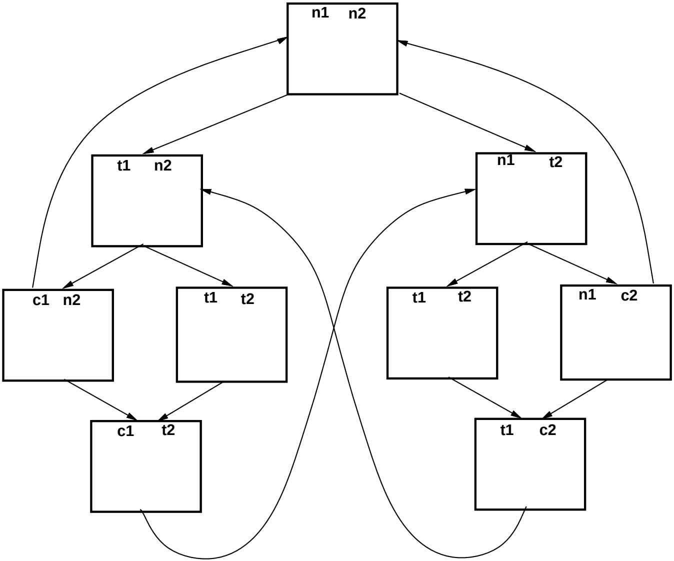


# Mutual Exclusion Example



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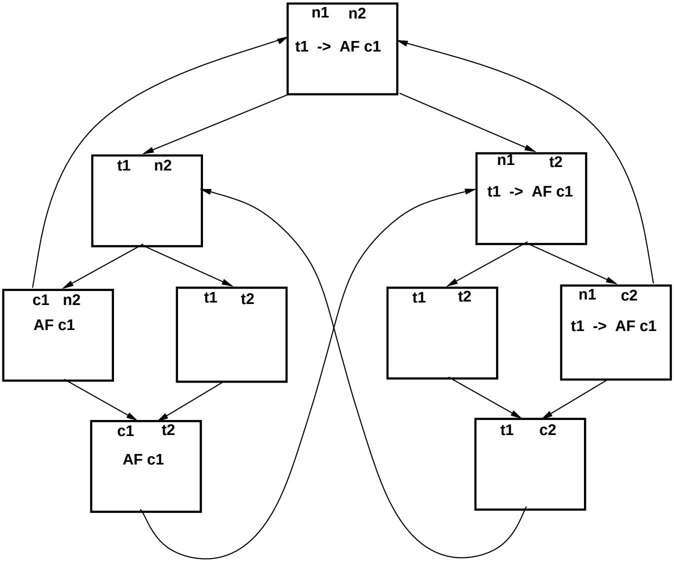

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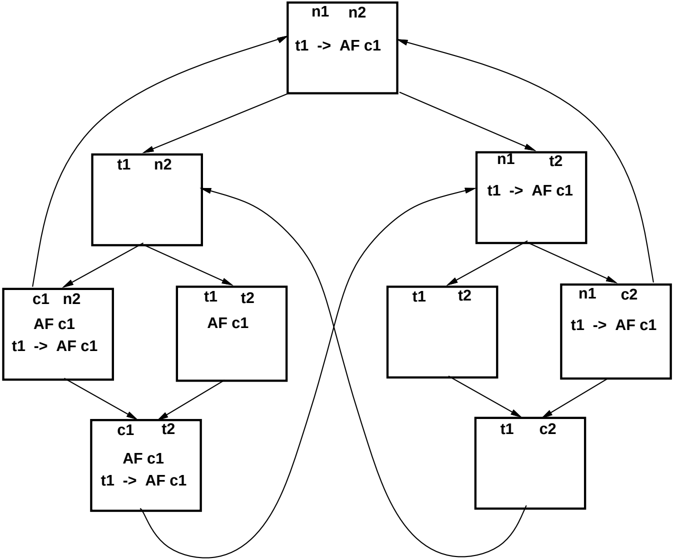

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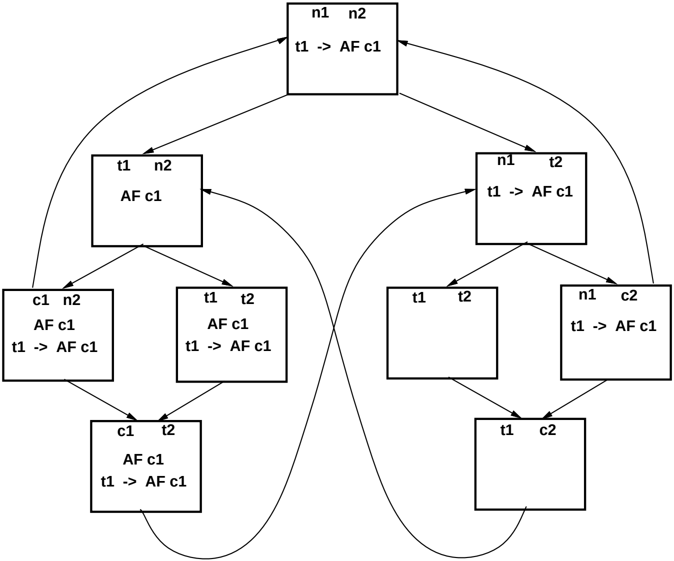

# The Kyoto University Verifier

Vectorized version of *EMC* algorithm on Fujitsu FACOM VP400E using an explicit representation of the state–transition graph.

State Machine size:

- 131,072 states
- 67,108,864 transitions
- 512 transitions from each state on the average.

CTL formula:

- 113 different subformulas.

Time for model checking:

- 225 seconds!!