Model Checking Assignment 3b

Due Oct 22

Recall:

- We identify a predicate with the set of states in which the predicate is true.
- X is a fixed point of τ iff $\tau[X] = X$.
- A predicate transformer τ is *monotonic* iff $P \subseteq Q$ implies $\tau[P] \subseteq \tau[Q]$.
- A predicate transformer τ is $\cup$-*continuous* iff $P_1 \subseteq P_2 \subseteq \ldots$ implies $\tau[\cup_i P_i] = \cup_i \tau[P_i]$.
- A predicate transformer τ is $\cap$-*continuous* iff $P_1 \supseteq P_2 \supseteq \ldots$ implies $\tau[\cap_i P_i] = \cap_i \tau[P_i]$.

Exercise 0. Read the relevant sections in the textbook. See the course website for details.

Note: Your answers to Exercises 1 and 2 will be very similar to the proofs that we will post for the corresponding least-fixpoint lemmas. We expect you to first read the LFP proofs so that you understand them and then to try to complete Exercises 1 and 2 without further reference to the LFP proofs. If you simply copy the LFP proofs without understanding them, you defeat the purpose of these exercises.

Exercise 1. Show that $\mathbf{gfp}\, Z \left[\tau(Z)\right] = \cup\{Z \mid \tau(Z) = Z\}$ whenever τ is monotonic.

Exercise 2. Show that $\mathbf{gfp}\, Z \left[\tau(Z)\right] = \cap_i \tau^i(\mathit{True})$ whenever τ is $\cap$-continuous.