Model Checking Assignment 3a (Due Oct 15)

Recall:

- We identify a predicate with the set of states in which the predicate is true.
- X is a fixed point of τ iff $\tau[X] = X$.
- τ is *monotonic* iff $P \subseteq Q$ implies $\tau[P] \subseteq \tau[Q]$.
- τ is $\cap$-*continuous* iff $P_1 \supseteq P_2 \supseteq \ldots$ implies $\tau[\cap_i P_i] = \cap_i \tau[P_i]$.

Note: Treat this assignment as an open-book take-home quiz. You may refer to your notes, the textbook, and the lecture slides, but don't discuss solutions to these problems with other people. Your grade on this assignment will count toward your mid-semester grade.

Exercise 1. Show that if τ is an $\cap$-continuous predicate transformer then τ is monotonic.

Exercise 2. Consider a Kripke structure M, in which the set of states is $\{S_1, S_2, S_3\}$, and a monotonic predicate transformer τ. (The domain of τ is $\mathcal{P}(\{S_1, S_2, S_3\})$.) Suppose that both $\{S_1, S_2\}$ and $\{S_1, S_3\}$ are fixed points of τ, but $\{S_1\}$ is not a fixed point of τ. What is the least fixed point of τ? What is the greatest fixed point of τ? Justify your answers.

Exercise 3. Write a CTL* path formula for each of the following descriptions of a path π. The atomic proposition `req` means "we receive a request" and `ack` means "we send an acknowledgement". (You may use all the std logical operators (e.g., "$\wedge$", "$\vee$", "$\Rightarrow$", etc.).)

Example: There exists a $k \geq 0$ such that $\pi^k \models p$. **Solution:** $\mathbf{F}\, p$

a. There exists a $k \geq 2$ such that $\pi^k \models p$.

b. There exists a $k \geq 2$ such that $\pi^k \models \neg p$ and $\pi^j \models p$ for all $j > k$.

c. Whenever we receive a request, we send an acknowledgement on the next state. (More precisely, for every $i \geq 0$, if $[\pi^i \models \texttt{req}]$ is true then $[\pi^{i+1} \models \texttt{ack}]$ is true.)

d. We never send an acknowledgement unless a request was received on the previous state. (More precisely, for every $i \geq 1$, $[\pi^i \models \texttt{ack}]$ is false unless $[\pi^{i-1} \models \texttt{req}]$ is true.)

e. We will never permanently stop receiving requests. (More precisely, for every $k \geq 0$, there exists a $j \geq k$ such that $\pi^j \models \texttt{req}$.)

Exercise 4. Draw a 4-state Kripke structure in which all paths from the start state s_0 satisfy **c**, **d**, and **e** from Exercise 3 and $[s_0 \models \mathbf{AG}\,[\,[\mathbf{EX}\,\texttt{req}] \wedge [\mathbf{EF}(\neg\texttt{req} \wedge \neg\texttt{ack})]\,]]$ is true.

Exercise 5. **(A)** Draw a (reduced) OBDD for $(a \wedge b) \oplus (a \vee c)$ with the ordering $[a, b, c]$. **(B)** Again, but with the ordering $[b, a, c]$.