15-817 Assignment 1

Exercise 1. Warm Up. Let $f(x, y, z) \overset{\text{def}}{=} y + \overline{z} \cdot x + z \cdot \overline{y} + y \cdot x$ be a boolean formula. Compute f's Shannon expansion with respect to

(a) x

(b) y

(c) z

Exercise 2. Reduced OBDDs. Given the truth table

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

compute the reduced OBDD with respect to the following ordering of variables:

(a) $[x, y, z]$

(b) $[z, y, x]$

(c) $[y, z, x]$

(d) $[x, z, y]$

Exercise 3. The package CUDD. In this problem you'll get an idea of how to use the BDD package CUDD. It is important that you become familiar with this package, since future assignments in the course may use the package in more non-trivial ways.

(a) Google and get CUDD (for building BDDs) and Graphviz (for printing graphs).

(b) Read enough of the manual of CUDD to understand the provided template program "`cudd-assignment.cc`". Compile and run the template program. You can find it at `http://www.cs.cmu.edu/~emc/15817-f09/cudd-assignment.cc`.

(c) Modify the program to generate BDDs for the following three formulas:

$$\begin{aligned} f &= a \rightarrow (b \rightarrow a) \\ g &= (a \rightarrow (b \rightarrow c)) \rightarrow ((a \rightarrow b) \rightarrow (a \rightarrow c)) \\ h &= f \leftrightarrow g \end{aligned}$$

(d) Look up the "`Cudd_DumpDot()`" function in the CUDD manual and source code. Figure out how to print out BDDs into a ".dot" file that can be read by Graphviz.

(e) Use Graphviz to print out three graphs for the formulas given in **(c)**.

Submit the graphs.

Exercise 4.

(a) Show whether the QBF formula $\forall x\, \exists y\, (x \oplus y)$, where "$\oplus$" denotes the XOR operator, means the same thing when you change the order of quantification: $\exists y\, \forall x\, (x \oplus y)$.

(b) Consider the QBF formula

$$\forall u_1\, \exists e_2\, \forall u_3\, \exists e_4 \, ... \forall u_{n-1}\, \exists e_n.\, f$$

where f is a propositional formula in which the only logical operators are "$\wedge$" and "$\vee$". Prove or disprove the following: Changing the order of the quantifiers won't affect the value of the QBF formula. (Hint: if f is false under $e_i = 1$, can f possibly be true under $e_i = 0$?)

Bonus Exercise. Let f_n be the n-bit comparator function.

$$f_n(a_1, \ldots, a_n, b_1, \ldots, b_n) = (a_1 \leftrightarrow b_1) \wedge \ldots \wedge (a_n \leftrightarrow b_n)$$

Prove that in the reduced OBDD for f_n:

(a) if we use the ordering $a_1 < b_1 < \ldots < a_n < b_n$, the number of vertices will be $3n + 2$.

(b) if we use the ordering $a_1 < \ldots < a_n < b_1 < \ldots < b_n$, the number of vertices is $3 \cdot 2^n - 1$.