Announcements

- Graded:
  - Programming Assignment 1 – Ian or Michael
    - » Grades in file in your turnin directory
  - Written Assignment – Michael
  - Derivation for Assignment 2 – Ian
- Programming Assignment 2 due on Thursday – questions?
- Written Assignment 2 out on Thursday

Polygon Meshes and Implicit Surfaces

Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry

Watt: Chapter 2

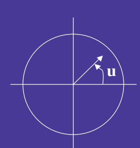
What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

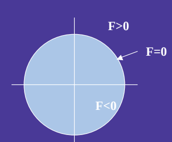
No one technique solves all problems

Two Ways to Define a Circle

Parametric



Implicit



x = f(u) = r cos (u)
y = g(u) = r sin (u)

$F(x,y) = x^2 + y^2 - r^2$

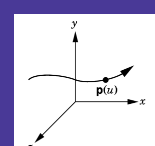
Curve Representations

- Explicit: y = f(x)

  $y = mx + b \qquad y = x^2$
  - must be a function (single-valued):
  - big limitation—vertical lines?
- Parametric: (x,y) = (f(u),g(u))

  $(x, y) = (\cos u, \sin u)$
  - \+ easy to specify, modify, control
  - – extra "hidden" variable u, the *parameter*
- Implicit: f(x,y) = 0

  $x^2 + y^2 - r^2 = 0$
  - \+ y can be multiple valued function of x
  - – hard to specify, modify, control



Surface Representations

- Parametric surface — x(u,v), y(u,v), z(u,v)
  - e.g. plane, sphere, cylinder, torus, bicubic surface, swept surface
  - parametric functions let you *iterate* over the surface by incrementing u and v in nested loops
  - great for making polygon meshes, etc
  - terrible for intersections: ray/surface, point-inside-boundary, etc.
- Implicit surface: F(x,y,z) = 0
  - e.g. plane, sphere, cylinder, quadric, torus, blobby models
  - terrible for iterating over the surface
  - great for intersections, morphing
- Subdivision surfaces
  - defined by a control mesh and a recursive subdivision procedure
  - good for interactive design

Modeling Complex Shapes

- We want to build models of very complicated objects
- An equation for a sphere is possible, but how about an equation for a telephone, or a face, or a cloud?
- Complexity is achieved using simple pieces
  - polygons, parametric surfaces, or implicit surfaces
- Goals
  - Model *anything* with arbitrary precision (in principle)
  - Easy to build and modify
  - Efficient computations (for rendering, collisions, etc.)
  - Easy to implement (a minor consideration...)

Polygon Meshes

- Any shape can be modeled out of polygons
  - if you use enough of them…
- Polygons with how many sides?
  - Can use triangles, quadrilaterals, pentagons, … n-gons
  - Triangles are most common.
  - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.

- Polygon meshes are built out of
  - *vertices* (points)
  - *edges* (line segments between vertices)
  - *faces* (polygons bounded by edges)

Frontfacing / Backfacing

•A polygon has two sides, of course.

•Customary in CG to use the right hand rule to pick one side to call the *front face*.

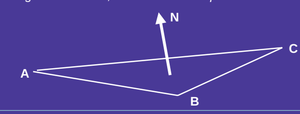
•Counterclockwise = front, clockwise = back

•Important for:

–lighting

–backface culling

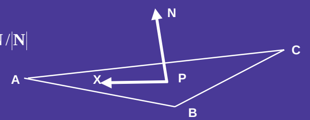
–for the triangle ABC below, the front face is *up*.



Normals and Plane Equations

- Need normals for shading, plane eqns for intersection tests
- A *normal* to a plane is a vector that is perpendicular to that plane (two possible choices)
- A plane is specified by a point P and a normal vector N
- N•(X-P) = 0 if and only if X lies in the plane; this is an *implicit equation* for the plane
  - Expand this out: 0 = N•X - N•P = ax + by + cz + d
- 3 vertices define a plane, its normal is: N=(B-A) x (C-A)
- Unit normal

$\hat{\mathbf{N}} = \mathbf{N} / |\mathbf{N}|$



Polygon Models in OpenGL

- for faceted shading

```
< calculate face normal n
  using cross product rule >
glNormal3fv(n);
glBegin(GL_POLYGONS);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();
```

- for smooth shading

```
glBegin(GL_POLYGONS);
glNormal3fv(normal1);
glVertex3fv(vert1);
glNormal3fv(normal2);
glVertex3fv(vert2);
glNormal3fv(normal3);
glVertex3fv(vert3);
glEnd();
```


Data Structures for Polygon Meshes

- Simplest (but dumb)
  - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  - redundant: each vertex stored multiple times
- Vertex List, Face List
  - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

  Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.
- Fancier schemes:

  Store more topological info so adjacency queries can be answered in O(1) time.

  Winged-edge data structure – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

A File Format for Polygon Models: OBJ

```
# OBJ file for a 2x2x2 cube
v -1.0  1.0  1.0   - vertex 1
v -1.0 -1.0  1.0   - vertex 2
v  1.0 -1.0  1.0   - vertex 3
v  1.0  1.0  1.0   - …
v -1.0  1.0 -1.0
v -1.0 -1.0 -1.0
v  1.0 -1.0 -1.0
v  1.0  1.0 -1.0
f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3
```

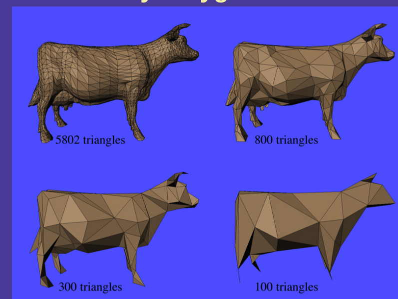
Syntax:

v *x y z* - *a vertex at (x,y,z)*

f $v_1\ v_2 \ldots v_n$
a face with vertices $v_1, v_2, \ldots v_n$

 # anything - *comment*

How Many Polygons to Use?



Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without
•losing key features
•getting artifacts in the silhouette
•popping

Surface Representations

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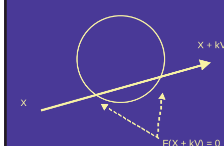
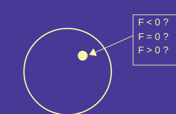
Sets of Points, Surfaces and Solids

- Implicit surface: set of all points that satisfy F(x,y,z)=0
- The points that satisfy F(x,y,z)<0 define a solid (or solids) bounded by the surface
- The solid is directly defined (unlike definitions using parametric surfaces)
- Example
  - An infinitely long (solid) cylinder with radius r:
    $F = x^2 + y^2 - r^2$
  - To limit cylinder to length L, abs(z) < L/2 and keep the function implicit use max:
    $F = \max\left(\text{abs}(z)-L/2,\ x^2 + y^2 - r^2\right)$
- Implicit functions for a cube? Any convex polyhedron?

What Implicit Functions are Good For



Ray - Surface Intersection Test

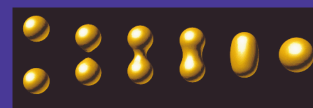


Inside/Outside Test

Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
  - constant value surfaces
  - level sets
  - isosurfaces
- Nice Feature: you can add them! (and other tricks)
  - this merges the shapes
  - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs*, or *Soft Objects*. Great for modeling animals.

Blobby Models

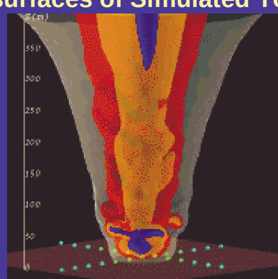


- Implicit function is the sum of Gaussians centered at several points in space, minus a threshold
- varying the standard deviations of the Gaussians makes each blob bigger
- varying the threshold makes blobs merge or separate

How to draw implicit surfaces?

- It's easy to ray trace implicit surfaces
  - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
  - Divide space into cubes
  - Evaluate implicit function at each cube vertex
  - Do root finding or linear interpolation along each edge
  - Polygonize on a cube-by-cube basis

Isosurfaces of Simulated Tornado



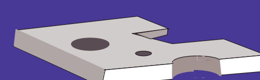
Constructive Solid Geometry (CSG)

Generate complex shapes with basic building blocks

machine an object - saw parts off, drill holes

glue pieces together

This is sensible for objects that are actually made that way (human-made, particularly machined objects)

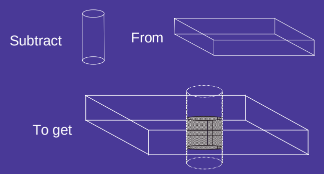


A CSG Train

Brian Wyvill & students, Univ. of Calgary

Negative Objects

- Use point-by-point boolean functions
  - remove a volume by using a negative object
  - e.g. drill a hole by subtracting a cylinder



Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))

Set Operations

- UNION: Inside(A) || Inside(B)
  — Join A and B
- INTERSECTION: Inside(A) && Inside(B)
  — Chop off any part of A that sticks out of B.
- SUBTRACTION: Inside(A) && (! Inside(B))
  — Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

Implicit Functions for Booleans

- Recall the implicit function for a solid: F(x,y,z)<0
- Boolean operations are replaced by arithmetic:
  - MINUS replaces NOT(unary subtraction)
  - MAX replaces AND (intersection)
  - MIN replaces OR (union)
- Thus
  - F(Subtract(A,B)) = MAX(F(A), -F(B))
  - F(Intersect(A,B)) = MAX(F(A),F(B))
  - F(Union(A,B)) = MIN(F(A),F(B))

You *can* try this at home

- Drawing boolean objects - combine parametric and implicit functions
- The boolean object has surfaces from all its constituent objects
- Draw using polygonal meshes, test before drawing using implicit function
  - for a hole drilled in a block - the surface of the hole is given by the cylinder used to drill it, the rest of the object's surface is defined by the block
  - draw points on the block if they are outside the cylinder
  - draw points on the cylinder if they are inside the block
- Implementing union:
  - draw both objects, use hidden-surface algorithms to take care of visibility
- Implementing intersection:
  - draw points only if they are inside both objects
- Implementing subtraction
  - points on the *positive* object's surface are visible outside the negative object
  - points on the *negative* object's surface are visible inside the positive object
- Draw using parametric functions, trim using implicit functions
  - And that's where the tricky part comes in

3-D Object Representation

- Individual elements are *voxels* (volume elements)
- Compression is almost mandatory
- Use octrees (3-D version of quadtree)
  - adaptively subdivide a cube into 8 sub-cubes forming a tree
  - stop dividing when the whole cube is entirely full or empty, or the minimum resolution is reached
  - at minimum resolution fill the block if majority is full
  - combine sibling cubes if they all have the same state
- Partially full cubes are nodes, full or empty cubes are leaves
- Data space requirement is proportional to the surface area of the object (except a few worst cases)

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