Static Single Assignment

15-411 Compiler Design

September 23, 2008

SSA form

In this lecture we introduce **Static Single Assignment (SSA) form**

This is a way of structuring the intermediate representation so that every variable is assigned exactly once

This is formally equivalent to continuation-passing style (CPS) IR

Developed at IBM by Cytron, Ferrante, Rosen, Wegman, and Zadeck

Why use SSA form?

Why do compiler writers use SSA?

- SSA form makes use-def chains explicit in the IR, which in turn helps to simplify some optimizations

Before getting into the details of SSA form, let's look at redundancy elimination as a motivating example

- Redundancy elimination optimizations attempt to remove redundant computations

Warning

SSA form is seductive

- The optimization benefits are real but not significant in simple compilers (like yours)
- It looks easy but it isn’t

My suggestion:

- Think about it but probably not wise to attempt it

Redundancy elimination

Common **redundancy elimination optimizations** are

- value numbering
- conditional constant propagation
- common-subexpression elimination (CSE)
- partial-redundancy elimination

What they do

```
read(i);
j = i + 1;
k = i;
l = k + 1;
```

```
i = 2;
j = i * 2;
k = i + 2;
```

```
     read(i);
     l = 2 * i + i;
     if (i>0) goto L1;
     i = i + 1;
     goto L2;
L1:  k = 2 * i * 1;
L2:
```

value numbering determines that j==l

constant propagation determines that j==k

*CSE determines that 2nd "2*i" is redundant*

Value numbering

Basic idea:

- associate a symbolic value to each computation, in a way that any two computations with the same symbolic value always compute the same value

Congruence of expressions

We define a notion of *congruence* of expressions

- $x \oplus y$ is congruent to $a \otimes b$ if $\oplus$ and $\otimes$ are the same operator, and x is congruent to a and y is congruent to b
- Typically, will also take commutativity into account

Value numbering

Suppose we have

- t1 = t2 + 1

Look up the key "t2+1" in a hash table

- Use a hash function that assigns the same hash value (ie, the same *value number*) to expressions e_1 and e_2 if they are congruent

If key "t2+1" is not in the table, then put it in with value "t1"

- the next time we hit on "t2+1", can replace it in the IR with "t1"

Example

```
read(i);
j = i + 1;
k = i;
l = k + 1;
```

i = v1

j = v2

Hash(v1 + 1) → j

k = v1

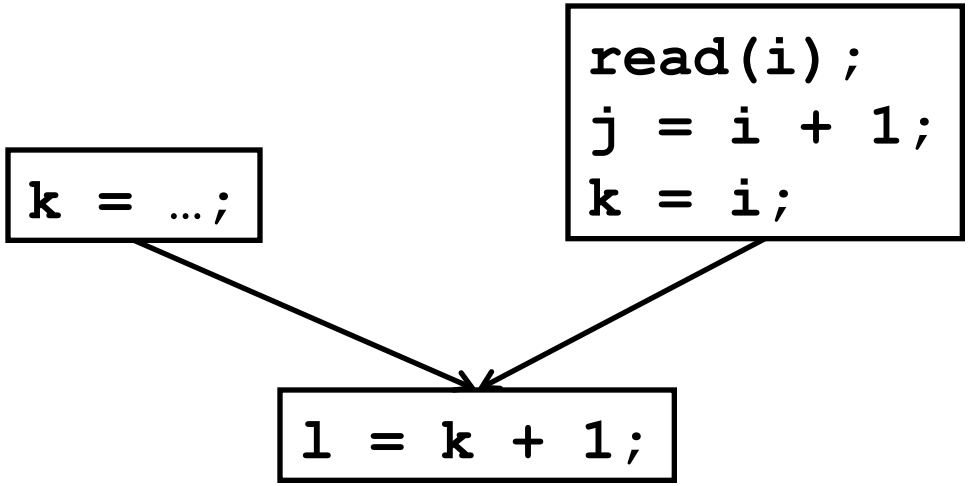
Hash(v1 + 1) → j

Therefore l = j

Global value numbering

Local (ie, within a basic block) value numbering is easy enough

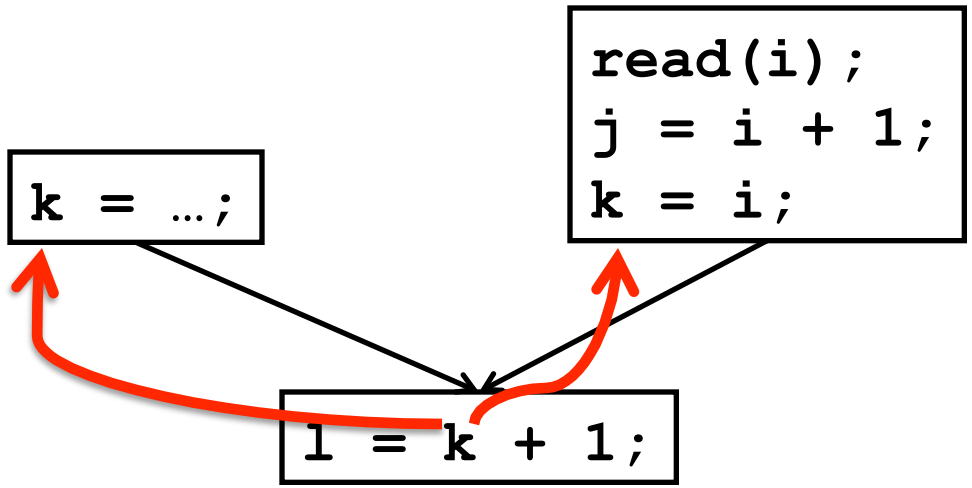
But what about global (ie, within a procedure) value numbering?



Importance of use-def info

Of course, in the global case we must watch out for multiple assignments

We could do a dataflow analysis to extend value numbering to the global case

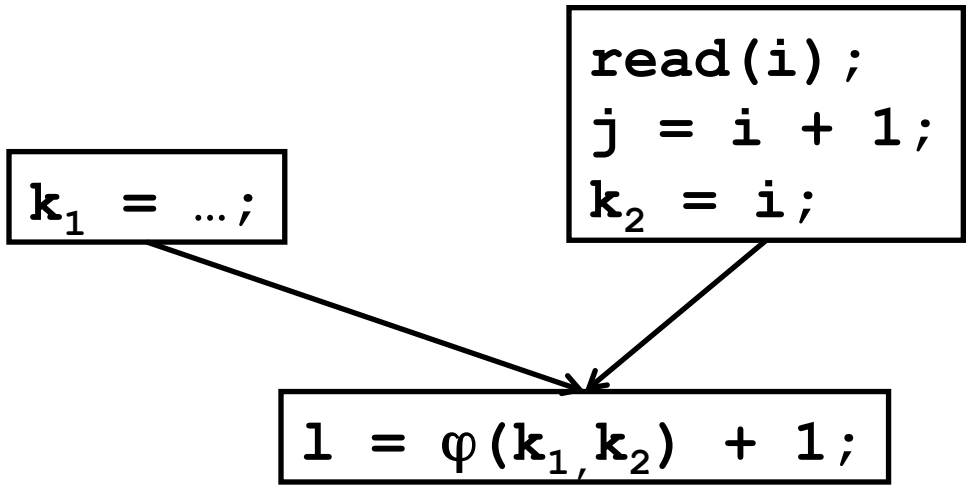


use-def analysis

Embedding use-def into the IR

Use-def information is central to several important optimizations

The point of static single assignment form (SSA form) is to represent use-def information explicitly



SSA Form

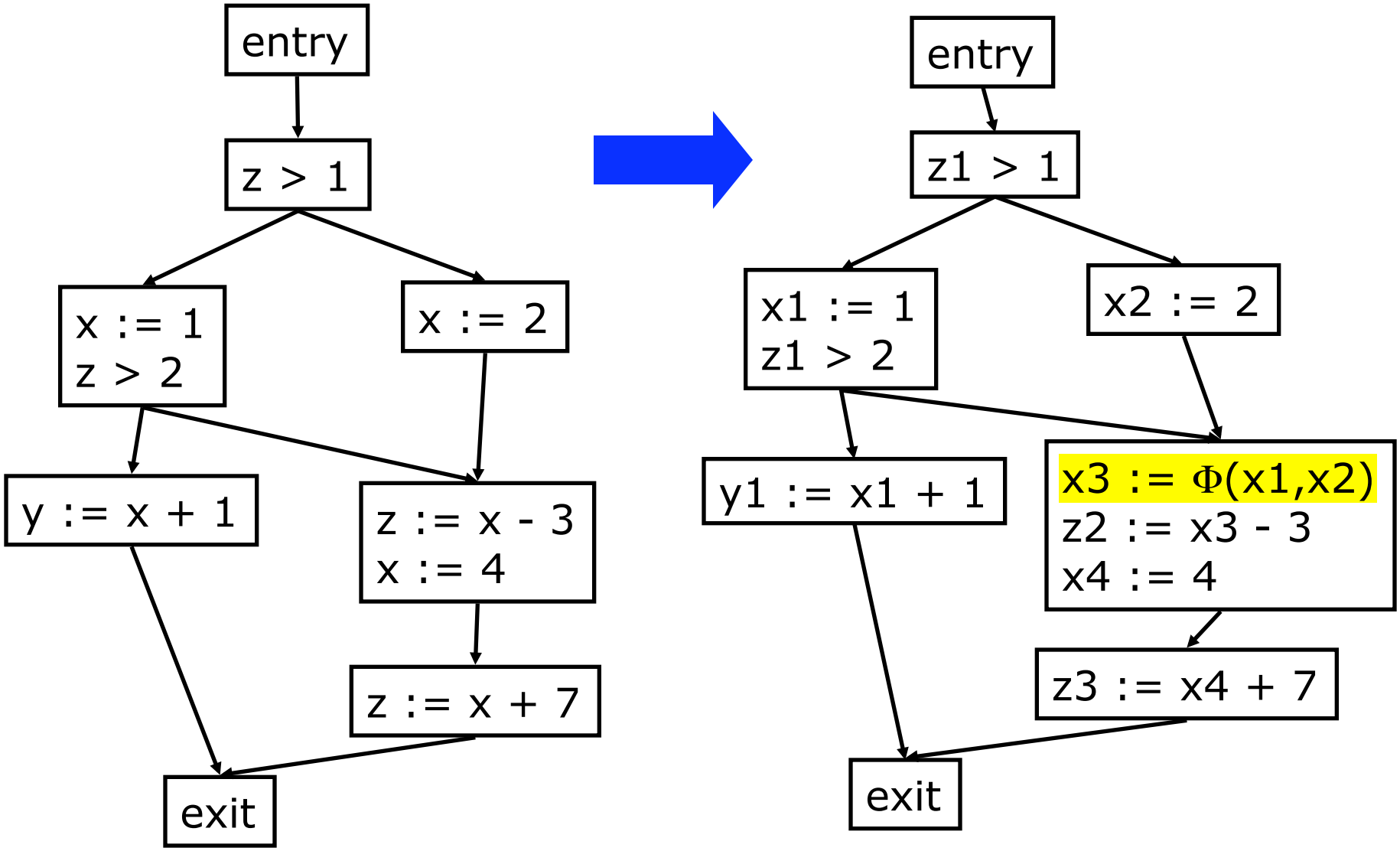
SSA form

Static single-assignment form arranges for every value computed by a program to have a unique assignment (aka, “definition”)

A procedure is in SSA form if every variable has (statically) exactly one definition

SSA form simplifies several important optimizations, including various forms of redundancy elimination

Example


entry
z > 1
x := 1
z > 2
x := 2
y := x + 1
z := x - 3
x := 4
z := x + 7
exit
entry
z1 > 1
x1 := 1
z1 > 2
x2 := 2
y1 := x1 + 1
x3 := Φ(x1,x2)
z2 := x3 - 3
x4 := 4
z3 := x4 + 7
exit

Value numbering in SSA

In SSA form, if x and a are variables, they are congruent only if they are both live and they are the same variable

Or if they are provably the same value (by constant or copy propagation)

SSA and chordal graphs

Note also that the interference graph for an SSA form IR is always chordal (can you see why?)

Assuming no pre-colored registers, the register allocation algorithm you have implemented is provably optimal

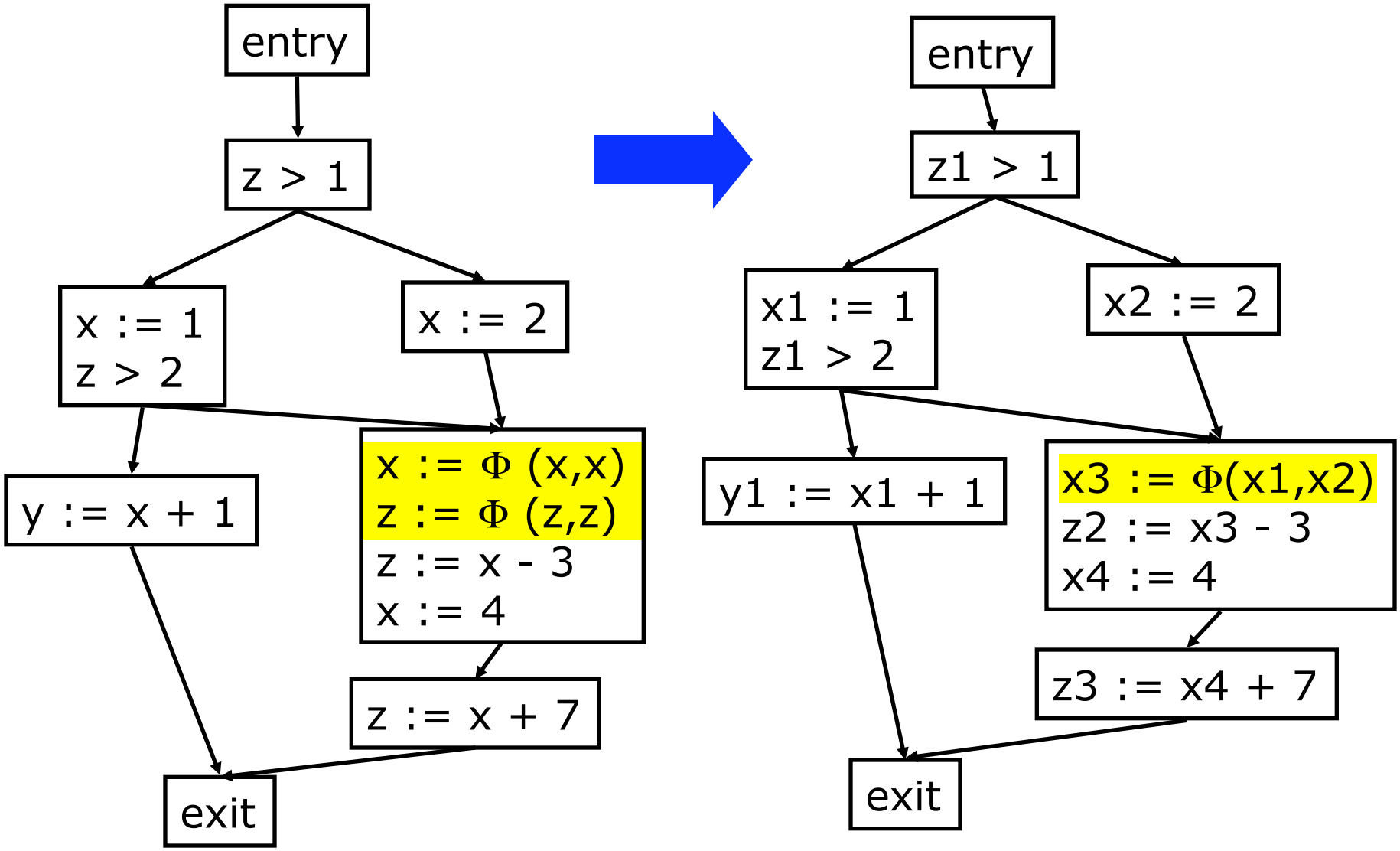
Creating SSA form

To translate into SSA form:

- Insert trivial Φ functions at join points for each live variable
  - $\Phi(t,t,\ldots,t)$, where the number of t's is the number of incoming flow edges
- Globally analyze and rename definitions and uses of variables to establish SSA property

After we are done with our optimizations, we can throw away all of the statements involving Φ functions (ie, "unSSA")

Example


entry
z > 1
x := 1
z > 2
x := 2
y := x + 1
x := Φ (x,x)
z := Φ (z,z)
z := x - 3
x := 4
z := x + 7
exit
entry
z1 > 1
x1 := 1
z1 > 2
x2 := 2
y1 := x1 + 1
x3 := Φ(x1,x2)
z2 := x3 - 3
x4 := 4
z3 := x4 + 7
exit

SSA form for general graphs

An SSA form with the minimum number of Φ functions can be created by using *dominance frontiers*

Definitions:

- In a flowgraph, node *a* dominates node *b* ("*a dom b*") if every possible execution path from *entry* to *b* includes *a*
- If *a* and *b* are different nodes, we say that *a strictly dominates b* ("*a sdom b*")
- If *a sdom b*, and there is no *c* such that *a sdom c* and *c sdom b*, we say that *a* is the immediate dominator of *b* ("*a idom b*")

Dominance frontier

For a node *a*, the dominance frontier of *a*, DF[*a*], is the set of all nodes *b* such that *a* strictly dominates an immediate precedessor of *b* but not *b* itself

More formally:

- DF[*a*] = {*b* | (∃*c*∈Pred(*b*) such that *a* *dom c* but not *a sdom b*}

Computing DF[a]

A naïve approach to computing DF[a] for all nodes a would require quadratic time

However, an approach that usually is linear time involves cutting into parts:

- $DF_l[a] = \{b \in Succ(a) \mid idom(b) \neq a\}$
- $DF_u[a,c] = \{b \in DF[c] \mid idom(c)=a \wedge idom(b) \neq a\}$

Then:

- $DF[a] = DF_l[a] \cup \bigcup_{c \in G\ (idom(c)=a)} DF_u[a,c]$

DF computation, cont'd

What we want, in the end, is the set of nodes that need Φ functions, for each variable

So we define DF[S], for a set of flowgraph nodes S:

- $DF[S] = \bigcup_{a \in S} DF[a]$

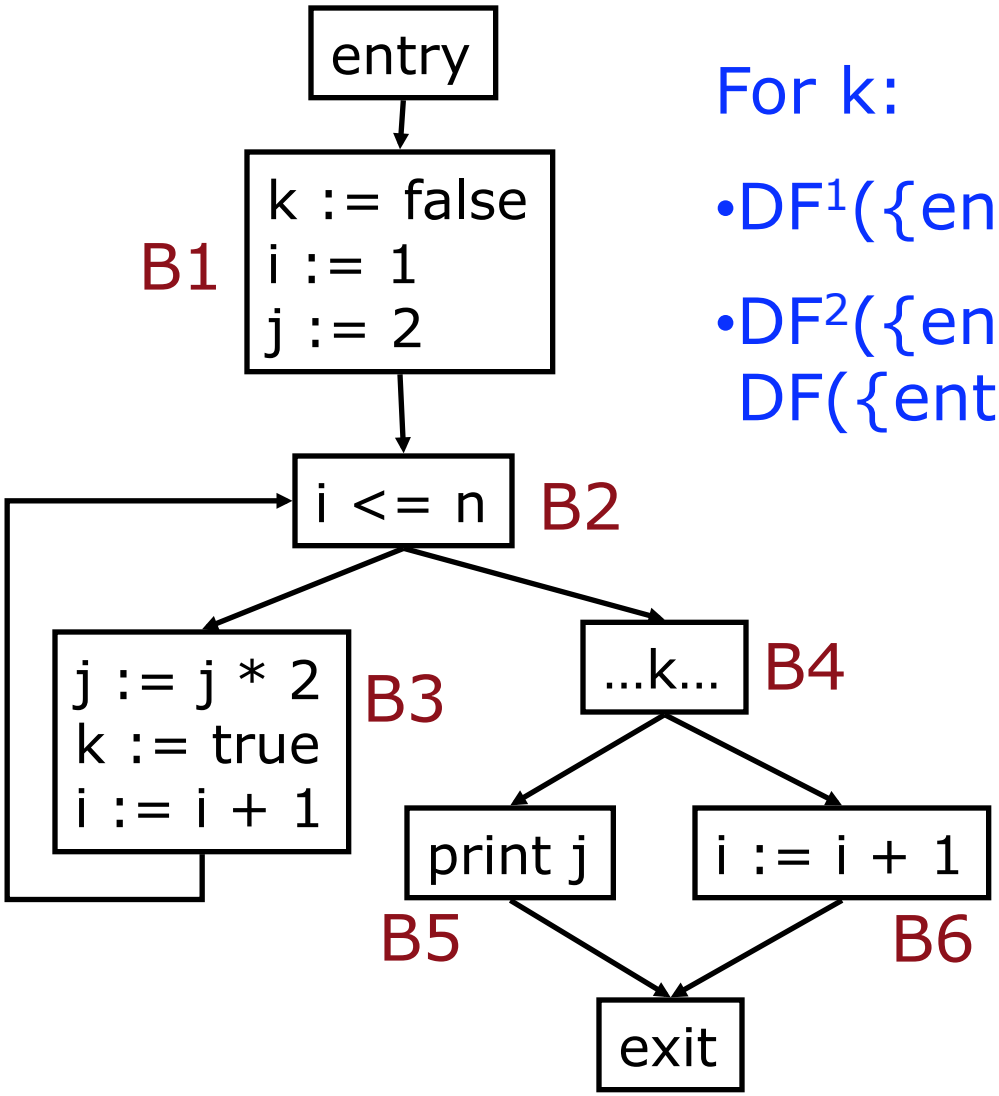
Iterated DF

Then, the iterated dominance frontier is defined as follows:

- $DF^{+}[S] = \lim_{i \to \infty} DF^{i}[S]$
- where
  - $DF^{1}[S] = DF[S]$
  - $DF^{i+1}[S] = DF[S \cup DF^{i}[S]]$

If S is the set of nodes that assign to variable t, then $DF^{+}[S \cup \{entry\}]$ is the set of nodes that need Φ functions for t

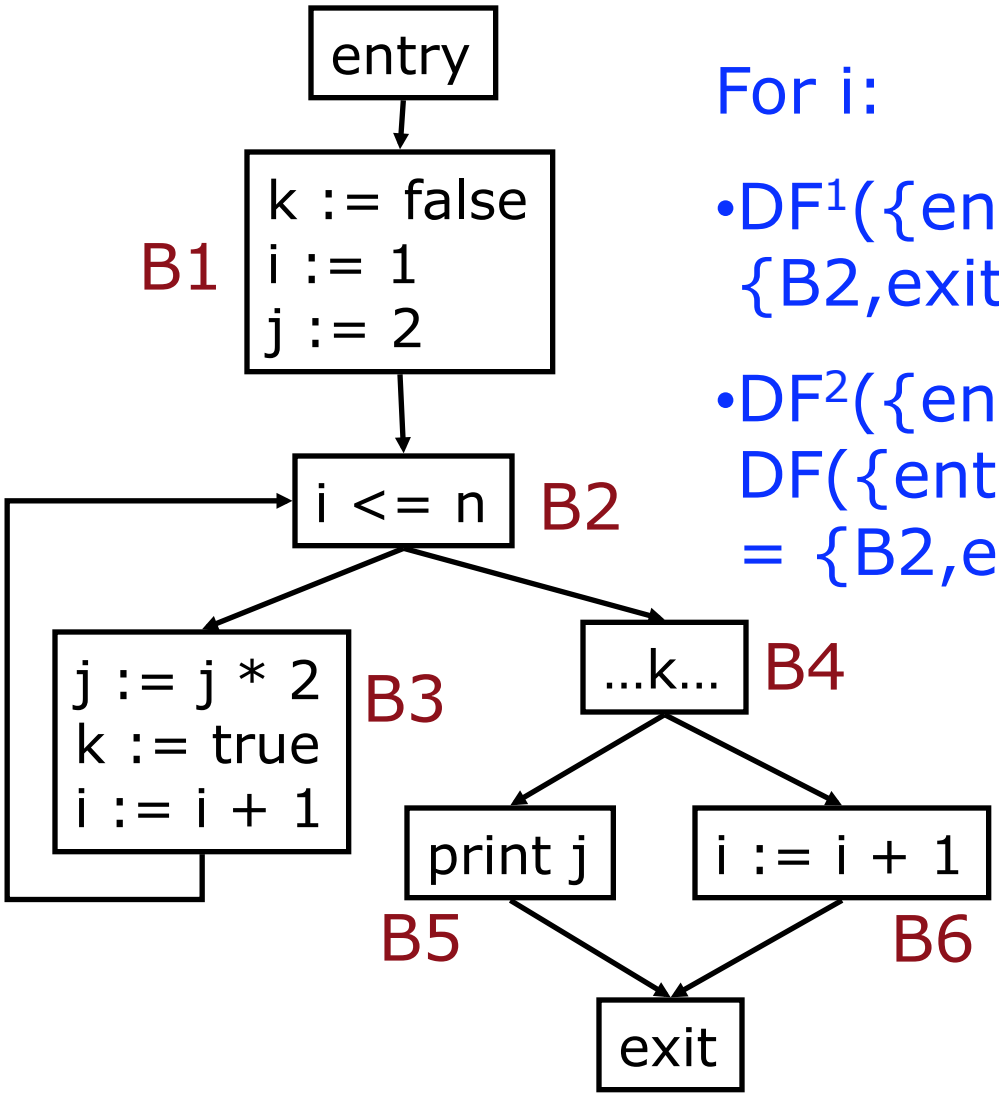
Example



For k:

- $DF^1(\{entry,B1,B3\}) = \{B2\}$
- $DF^2(\{entry,B1,B3\}) = DF(\{entry,B1,B2,B3\}) = \{B2\}$

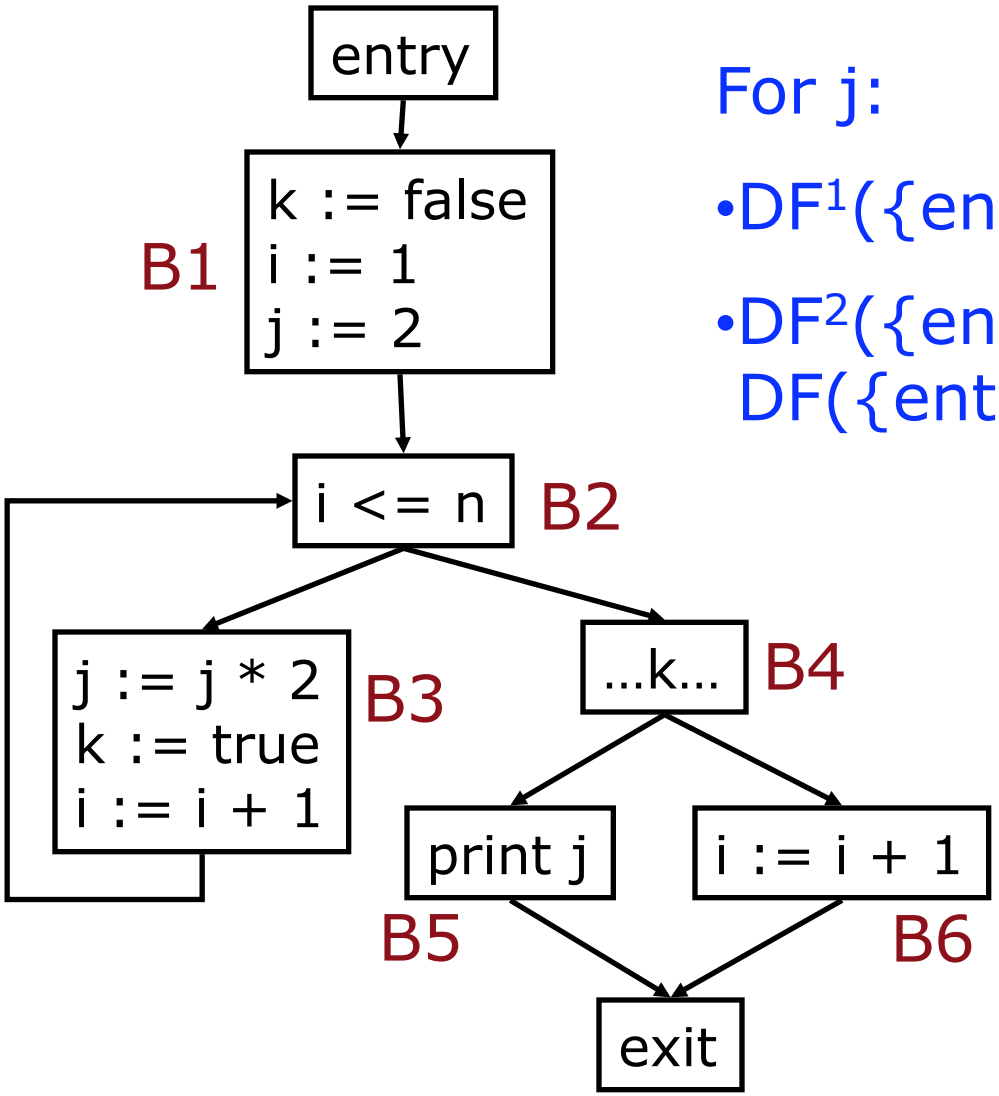
Example



For i:

- $DF^1(\{entry,B1,B3,B6\}) = \{B2,exit\}$
- $DF^2(\{entry,B1,B3,B6\}) = DF(\{entry,B1,B2,B3,B6,exit\}) = \{B2,exit\}$

Example



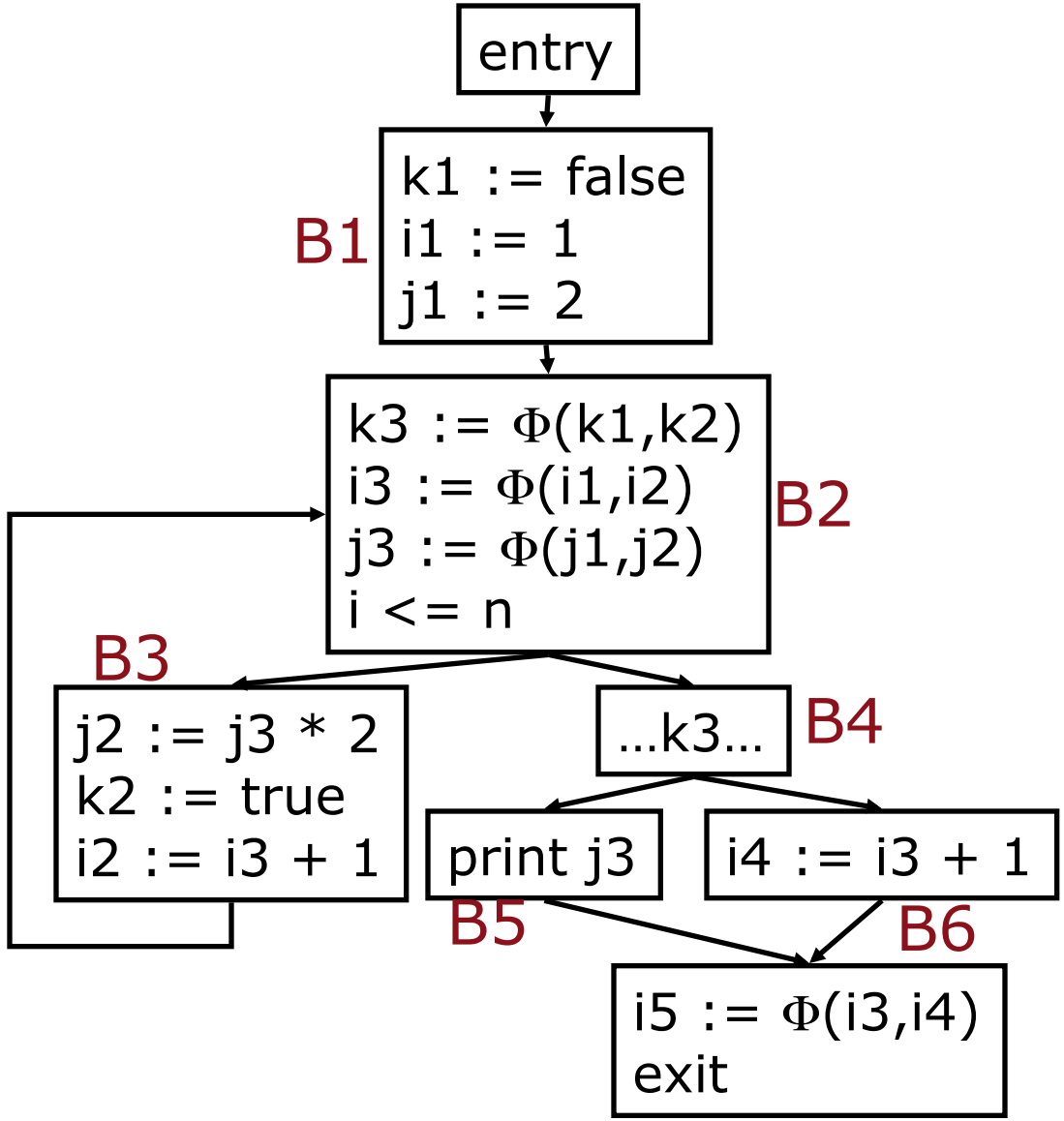
For j:

- $DF^1(\{entry,B1,B3\}) = \{B2\}$
- $DF^2(\{entry,B1,B3\}) =$ $DF(\{entry,B1,B2,B3\}) = \{B2\}$

Example, cont'd

So, Φ nodes for i, j, and k are needed in B2, and i also needs one in exit

- exit Φ nodes are usually pruned



Other ways to get SSA

Although computing iterated dominance frontiers will result in the minimal SSA form, there are easier ways that work well for simple languages

Without knowing the details of your project, I would guess that your translator always knows when it is creating a join point and can keep track of the immediate dominator

If so, it can also create the necessary Φ nodes during translation

Project advice

The bottom line for your project:

- You don’t need to generate SSA form for your project
- However, if you decide to do this, then it is advisable to simplify matters by generating SSA directly during translation

Summary

SSA form has had a huge impact on compiler design

Most modern production compilers use SSA form (including, for example, gcc, suif, llvm, hotspot, …)

Compiler frameworks (ie, toolkits for creating compilers) all use SSA form