1. Verification of the equivalence of circuits

In the following we present two standard ways to implement boolean functions by digital circuits. In both cases the implemented function is

\[ f(x_1, x_2, x_3) \leftrightarrow \text{exactly one of } \{x_1, x_2, x_3\} \text{ is 1} \]

The function \( f \) is the exclusive or for three arguments. The first implementation will be via a conjunctive normal form and the second will be via a disjunctive normal form of the boolean formula denoted by \( f \). We will show that the resulting circuits are equivalent by showing that their OBDDs are the same. In all OBDDs keep the variable ordering \([x_1, x_2, x_3]\).

A) Conjunctive normal form

A literal is a disjunction of variables or their negations. A formula is in conjunctive normal form (CNF) if it is a conjunction of literal. In our case the literals are:

\[
\begin{align*}
A &= x_1 + x_2 + x_3 \\
B &= \overline{x_1} + \overline{x_2} \\
C &= \overline{x_1} + \overline{x_3} \\
D &= \overline{x_2} + \overline{x_3}
\end{align*}
\]

The CNF representation of \( f \) is

\[ f(x_1, x_2, x_3) = A \cdot B \cdot C \cdot D \]

1. Construct and the OBDDs for \( A, B, C \) and \( D \)

2. Construct and simplify the OBDDs for
   - \( B \cdot C \)
   - \((B \cdot C) \cdot D\)
   - \( A \cdot (B \cdot C) \cdot D\)
**B) Disjunctive normal form**

A *monom* is a conjunction of variables or their negations. A formula is in *disjunctive normal form* (DNF) if it is a disjunction of monoms. In our case the monoms are

\[
A = x_1 \cdot \overline{x}_2 \cdot \overline{x}_3 \\
B = \overline{x}_1 \cdot x_2 \cdot \overline{x}_3 \\
C = \overline{x}_1 \cdot \overline{x}_2 \cdot x_3
\]

The DNF representation of \( f \) is

\[
f(x_1, x_2, x_3) = A + B + C
\]

1. Construct the OBDDs for \( A, B \) and \( C \)
2. Construct and simplify the OBDDs for
   - \( A + B \)
   - \( (A + B) + C \)

Now compare the resulting OBDD with the one from A)!

**2. Optimal OBDDs**

Find an ordering on the variables \( x_1, \ldots, x_4 \) s.th. the OBDD for the following DNF-formula is optimal:

\[
A = x_1 x_2 + x_3 x_4 + x_2 \overline{x}_4 + \overline{x}_2 x_4
\]

Hint: The solution has 5 variable nodes.

**Have a nice thanksgiving!**