Tractability and Intractability in Model Checking

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A Puzzle
Move #1
Move #2
Perform Moves #1 and #2 any number of times. Keep all the coins at the end.

How much money can you make?
$100?
$1000?
$1 MILLION?
MORE MONEY THAN BILL GATES?
INFINITE?
Not Infinite...

Each move decreases the value in lexicographic ordering
So no matter what you do, the process will terminate
Not Infinite...

Each move decreases the value in lexicographic ordering.
So no matter what you do, the process will terminate.
Three Cups
Three Cups

\[(a, b, 0) \rightarrow (a, 0, 2b) \rightarrow (a-1, 2b, 0) \rightarrow^* (0, 2^{a\cdot b}, 0)\]

\[(a, 0, 0) \rightarrow^* (0, 2^a, 0)\]
Four Cups

\[(a, b, 0, 0) \rightarrow^* (a, 0, 2^b, 0) \rightarrow (a-1, 2^b, 0, 0)\]

\[(a, 0, 0, 0) \rightarrow^* (0, 2^{^a}, 0, 0)\]
Knuth’s Up Arrow

\[ a \uparrow\uparrow 0 = 1 \]
\[ a \uparrow\uparrow (b + 1) = a \uparrow (a \uparrow\uparrow b) \]

\[ 2 \uparrow\uparrow 5 = 2 \uparrow 65536 \approx 10 \uparrow 20033 \]

\[ 2 \uparrow\uparrow 6 = 2 \uparrow \ldots \]

Bill Gates has < $10^{10}$
Knuth’s Up Arrow

\[ a \uparrow^0 b = a \cdot b \]

\[ a \uparrow^n 0 = 1 \quad (\text{if } n \geq 1) \]

\[ a \uparrow^n b = a \uparrow^{n-1} (a \uparrow^n (b - 1)) \]

(Ackermann, non-primitive recursive)
N Cups

\[(a, 0, \ldots, 0) \Rightarrow (0, 2^{N-2}a, 0, \ldots, 0)\]
Where is Model Checking?

Finite state spaces defined by simple transitions can be very large!

Decidable != Practical

Undecidable != Impractical
Oh and One More Thing...

Theorem: There is a family of Petri nets with finite but non-primitive recursive reachable state space

[Mayr & Meyer 1981]
Petri Net?
Move #2?
Thank You